ЖЭТФ, 2005, том 127, вып. 1, стр. 37–44

COMPTON AND DOUBLE COMPTON SCATTERING PROCESSES AT COLLIDING ELECTRON–PHOTON BEAMS

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Submitted 12 July 2004

Radiative corrections to the Compton scattering cross section are calculated in the leading and next-to-leading logarithmic approximations in the case of colliding high-energy photon-electron beams. Radiative corrections to the double Compton scattering cross section in the same experimental set-up are calculated in the leading logarithmic approximation. We consider the case where no pairs are created in the final state. We show that the differential cross section can be written in the form of the Drell–Yan process cross-section. Numerical values of the K-factor and the leading-order distribution on the scattered electron energy fraction and scattering angle are presented.

PACS: 11.10.Gh, 12.20.Ds, 13.60.Fz

1. INTRODUCTION

The Compton scattering process

$$\begin{aligned} &\gamma(k_1) + e^-(p_1) \to \gamma(k_2) + e^-(p_2), \\ &k_1^2 = k_2^2 = 0, \quad p_1^2 = p_2^2 = m^2, \\ &\kappa_1 = 2p_1k_1 = 4\epsilon_1\omega_1, \quad \kappa_1' = 2p_2k_1 = 2\epsilon_2\omega_1(1-c), \\ &s_1 = 2p_1p_2 = 2\epsilon_1\epsilon_2(1+c), \\ &\kappa_1 \sim \kappa_1' \sim s_1 \gg m^2, \quad \epsilon_2 = \frac{2\epsilon_1\omega_1}{\omega_1(1-c)+\epsilon_1(1+c)} \end{aligned} \tag{1}$$

(where $\epsilon_{1,2}, \omega_1$ are the energies of the initial and scattered electrons and the initial photon, $c = \cos\theta$, and θ is the angle between $\mathbf{p}_2$ and $\mathbf{k}_1$) plays an important role as a possible calibration process at high-energy photon–electron colliders [1]. Obtaining a radiation-corrected cross section of this process is the motivation of this paper. Modern methods based on the renormalization group approach in combination with the lowest-order radiative corrections (RC) allows obtaining a differential cross section in the leading approximation (where $((\alpha/\pi)L)^n \sim 1$, with the «large logarithm» $L = \ln(s_1/m^2)$) and in the next-to-leading approximation (where terms of the order of $(\alpha/\pi)^n L^{n-1}$ are kept). The accuracy of the formulas given below is therefore determined by terms of the order of

$$\frac{m^2}{\kappa_1}, \quad \frac{\alpha^2}{\pi^2}, \quad \alpha\frac{s}{M_Z^2} \tag{2}$$

compared with the terms of the order of unity and is at the level of per-mille for typical experimental conditions [1] $\theta \sim 1$, $\kappa_1 \sim 10$ GeV^2. Terms of order (2) are systematically omitted in what follows. We consider the energies of initial particles to be much less than the Z-boson mass M_Z, and therefore the weak corrections to the Compton effect are beyond our accuracy.

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The first papers devoted to cancellation of radiative corrections to Compton scattering were published in 1952 by Brown and Feynman [2] (virtual and soft real photon emission contribution) and Mandl and Skyrme [3] (emission of an additional hard photon). In the work of Veltman [4], the lowest-order radiative corrections to the polarized Compton scattering were calculated in nonrelativistic kinematics. This case of kinematics was also considered in the paper of Swartz [5]. In the papers of Denner and Dittmaier [6], the lowest-order radiative corrections in the framework of the Standard model were calculated in the case of polarized electron and photon.

In this paper, we consider the case of high-energy electron and photon Compton scattering (with the cms energy supposed to be much higher than the electron mass but much less than the Z-boson mass). We find that the cross section with radiative corrections of all orders of the perturbation theory taken into account can be written in the form of the Drell–Yan process. Both leading and next-to-leading contributions are derived explicitly.

We consider the kinematics where the initial photon and electron move along the z axis in the opposite directions. The energy of the scattered electron is a function of its scattering angle:

$$z_0 = \frac{\varepsilon_2}{\omega_1} = \frac{2\rho}{a}, \quad a = a(c,\rho) = 1 - c + \rho(1+c), \quad \rho = \frac{\varepsilon_1}{\omega_1}. \tag{3}$$

We now consider the kinematic case where $\rho < 1$. The case where $\rho > 1$ is considered in Appendix B.

The differential cross section in the Born approximation is given by

$$\frac{d\sigma_B}{dc}(p_1,\theta) = \frac{\pi\alpha^2 U_0}{\omega_1^2 a^2}, \quad U_0 = \frac{a}{1-c} + \frac{1-c}{a}. \tag{4}$$

In taking RC of higher orders (arising from emission of both virtual and real photons) into account, the simple relation between the scattered electron energy and the scattering angle changes, and the differential cross section in general depends on the energy fraction z of the scattered electron. Accepting the Drell–Yan form of the cross section, we can write it in the form

$$\frac{d\sigma}{dc\,dz}(p_1,\theta,z) = \int_0^1 dx D(x,L) \times \\ \times \int_z^{z_0} \frac{dt}{t} D\left(\frac{z}{t},L\right) \frac{d\sigma_h}{dcdt}(xp_1,\theta,t)\left(1+\frac{\alpha}{\pi}K\right), \tag{5}$$

where the structure function $D(x,L)$ (specified below) describes the probability to find the electron (considered as a parton) inside the electron, K is the so-called K-factor, which can be calculated from the lowest RC orders, K is specified below (see Eqs. (8), (19), and (26)), and the «hard» cross section is

$$\frac{d\sigma_h}{dcdt}(xp_1,\theta,t) = \frac{d\sigma_B}{dc}(xp_1,\theta)\delta(t-t(x)), \tag{6}$$

$$\frac{d\sigma_B(xp_1,\theta)}{dc} = \frac{\pi\alpha^2}{\omega_1^2}\frac{1}{(1-c+\rho x(1+c))^2} \times \\ \times \left(\frac{1-c}{1-c+\rho x(1+c)} + \frac{1-c+\rho x(1+c)}{1-c}\right),$$

$$t(x) = \frac{2x\rho}{1-c+\rho x(1+c)}.$$

The cross section written in the Drell–Yan form explicitly satisfies the Kinoshita–Lee–Nauenberg theorem [7]. Indeed, being integrated over the scattered electron energy fraction z, the structure function corresponding to the scattered electron turns to unity because

$$\int_0^1 dz \int_z^1 \frac{dt}{t} D\left(\frac{z}{t},L\right) f(t) = \int_0^1 dt\, f(t). \tag{7}$$

Mass singularities associated with the initial lepton structure function remain.

Therefore, our master formula for the cross section with RC taken into account is

$$\frac{d\sigma}{dz\,dc}(p_1,p_2) = \\ = \int_{x_0}^1 \frac{dx}{t(x)} D(x,L)\frac{d\sigma_B}{dc}(xp_1,\theta)D\left(\frac{z}{t(x)},L\right) + \\ + \frac{\alpha}{\pi}\frac{d\sigma_B(p_1,\theta)}{dc}\Big[K_{SV}\delta(z-z_0)+K_h\Big], \tag{8}$$

$$z = \frac{\epsilon_2'}{\omega_1} < z_0, \quad x_0 = \frac{z(1-c)}{\rho(2-z(1+c))},$$

$$L = \ln\frac{2\omega_1^2 z_0\rho(1+c)}{m^2},$$

with the nonsinglet structure function D defined as [8]

$$D(z,L) = \delta(1-z) + \sum_{k=1}^{\infty} \frac{1}{k!}\left(\frac{\alpha L}{2\pi}\right)^k P_1(z)^{\otimes k},$$

$$P_1(z)^{\otimes k} = \underbrace{P_1 \otimes \cdots \otimes P_1(z)}_{k},$$

$$P_1 \otimes P_1(z) = \int_z^1 P_1(t) P_1\left(\frac{z}{t}\right)\frac{dt}{t}, \qquad (9)$$

$$P_1(z) = \frac{1+z^2}{1-z}\theta(1-z-\Delta) + $$
$$+\delta(1-z)\left(2\ln\Delta + \frac{3}{2}\right), \quad \Delta \ll 1.$$

In Conclusion (see Eq. (30)), we give the so-called «smoothed» form of the structure function.

The second term in the right-hand side of (8) collects all the nonleading contributions from the emission of virtual, soft, and hard photons, with K_{SV} given in Sec. 2, where the virtual and soft real contributions are considered. In Secs. 3 and 4, we consider the contribution from an additional hard photon emission and introduce an auxiliary parameter θ_0 to distinguish the collinear and noncollinear kinematics of photon emission. We also give the expression for the hard photon contribution K_h. The results of numerical estimation of the K-factor and leading contributions are given in Sec. 5. In Appendix A, we demonstrate the explicit cancellation of the θ_0 dependence. In Appendix B, we consider the kinematic case $\varepsilon_1 > \omega_1$.

2. CONTRIBUTION OF VIRTUAL AND SOFT REAL PHOTONS

To obtain the explicit form of the K-factor, we reproduce the lowest-order RC. It consists of the virtual photon emission contribution and the contribution from the real (soft and hard) photon emission. The virtual and soft photon emission contribution was first calculated in the famous paper by Brown and Feynman [2]. The result is

$$\frac{d\sigma_{virt}}{d\sigma_B} = -\frac{\alpha}{\pi}\frac{U_1}{U_0}, \qquad (10)$$

with (see [2], kinematic case II)

$$\frac{U_1}{U_0} = (1-L)\left(\frac{3}{2}+2\ln\frac{\lambda}{m}\right) + \frac{1}{2}L^2 - \frac{\pi^2}{6} - K_V,$$
$$U_0 = \frac{\kappa_2}{\kappa_1} + \frac{\kappa_1}{\kappa_2}, \qquad (11)$$

where K_V (the virtual photon contribution to the K-factor) is

$$K_V = -\frac{1}{U_0} \times$$
$$\times \left[\left(1-\frac{\kappa_2}{2\kappa_1}-\frac{\kappa_1}{\kappa_2}\right)\left(\ln^2\frac{s_1}{\kappa_1} - \ln\frac{s_1}{\kappa_1} + 2\ln\frac{\kappa_2}{\kappa_1}\right) + \right.$$
$$\left. + \left(1-\frac{\kappa_1}{2\kappa_2}-\frac{\kappa_2}{\kappa_1}\right)\left(\ln^2\frac{s_1}{\kappa_2} - \ln\frac{s_1}{\kappa_1} - \ln\frac{\kappa_1}{\kappa_2} + \pi^2\right)\right], \qquad (12)$$

and

$$\frac{\kappa_2}{\kappa_1} = \frac{z_0(1-c)}{2\rho}, \qquad (13)$$

$$\frac{s_1}{\kappa_1} = \frac{z_0(1+c)}{2}, \quad \frac{s_1}{\kappa_2} = \frac{\rho(1+c)}{1-c}. \qquad (14)$$

The soft photon emission for our kinematics has the form

$$\frac{d\sigma_{soft}}{d\sigma_B} = -\frac{4\pi\alpha}{16\pi^3} \times$$
$$\times \int \frac{d^3k}{\omega}\left(\frac{p_1}{p_1 k} - \frac{p_2}{p_2 k}\right)^2_{\omega=\sqrt{\mathbf{k}^2+\lambda^2}<\Delta\epsilon\ll\epsilon_1\sim\epsilon_2}. \qquad (15)$$

Standard calculations lead to the result

$$\frac{d\sigma_{soft}}{d\sigma_B} = \frac{\alpha}{\pi}\left((L-1)\ln\frac{m^2\Delta\varepsilon^2}{\lambda^2\varepsilon_1\varepsilon_2} + \frac{1}{2}L^2 - \right.$$
$$\left. - \frac{1}{2}\ln^2\frac{\varepsilon_1}{\varepsilon_2} - \frac{\pi^2}{3} + \mathrm{Li}_2\frac{1-c}{2}\right). \qquad (16)$$

The resulting contribution to the cross section from virtual and soft real photons is independent of the fictitious «photon mass» λ and the L^2-type terms. It can be written as

$$\left(\frac{d\sigma}{dz\,dc}\right)_{sv} = \frac{d\sigma_{virt}+d\sigma_{soft}}{dc}\delta(z-z_0) =$$
$$= \frac{\alpha}{2\pi}\frac{d\sigma_B(p_1,\theta)}{dc} \times$$
$$\times \left[(L-1)(P_{1\Delta}+P_{2\Delta}) + 2K_{SV}\right]\delta(z-z_0), \qquad (17)$$

where we introduce the notation

$$P_{1\Delta} = \frac{3}{2} + 2\ln\frac{\Delta\varepsilon}{\varepsilon_1}, \quad P_{2\Delta} = \frac{3}{2} + 2\ln\frac{\Delta\varepsilon}{\varepsilon_2}. \qquad (18)$$

We can see that the terms proportional to the «large» logarithm L have the form conforming with the renormalization group prescription of the structure function. The contribution of nonleading terms is

$$K_{SV} = -\frac{\pi^2}{6} + \mathrm{Li}_2\frac{1-c}{2} - \frac{1}{2}\ln^2\frac{z_0}{\rho} + K_V. \qquad (19)$$

3. CONTRIBUTION OF THE HARD COLLINEAR REAL PHOTON EMISSION

The dependence on the auxiliary parameter $\Delta\epsilon$ is eliminated when the emission of a real additional hard photon with 4-momentum k and the energy ω exceeding $\Delta\epsilon$ is taken into account.

It is convenient to consider the kinematics in which this additional photon moves within a narrow cone of the angular size $m/\epsilon_1 \ll \theta_0 \ll 1$ along the directions of the initial or scattered electrons. The contribution of these kinematic regions can be obtained using the «quasireal electron method» [9] instead of the general (rather cumbersome) expression for the cross section of the double Compton (DC) scattering process [3].

In the case where the collinear photon is emitted along the initial electron, the result is

$$\left(\frac{d\sigma}{dz\,dc}\right)_{\mathbf{k}||\mathbf{p}_1} = \frac{\alpha}{2\pi}\int\limits_0^{1-\Delta\epsilon/\epsilon_1} dx\frac{d\sigma_B}{dc}(xp_1,\theta) \times \times\left[\frac{1+x^2}{1-x}(L_1-1)+1-x\right]\delta(z-t(x)), \quad (20)$$

$$L_1 = \ln\frac{\theta_0^2\epsilon_1^2}{m^2} = L + \ln\frac{\theta_0^2\rho}{2z_0(1+c)}.$$

When the photon is emitted along the scattered electron, we have

$$\left(\frac{d\sigma}{dz\,dc}\right)_{\mathbf{k}||\mathbf{p}_2} = \frac{\alpha}{2\pi}\frac{d\sigma_B}{dc}(p_1,\theta)\int\limits_{z(1+\Delta\epsilon/\epsilon_2)}^{z_0}\frac{dt}{t}\delta(t-z_0)\times \times\left[\frac{1+z^2/t^2}{1-z/t}(L_2-1)+1-\frac{z}{t}\right], \quad (21)$$

$$L_2 = \ln\frac{\epsilon_2'^2\theta_0^2}{m^2} = L + \ln\frac{\theta_0^2 z^2}{2\rho(1+c)z_0},$$

where $z = \varepsilon_2'/\omega_1 < z_0$ is the energy fraction of the scattered electron (after emission of the collinear photon).

It is convenient to write the contribution of the collinear kinematics in the form

$$\left(\frac{d\sigma_h}{dz\,dc}\right)_{coll} = \frac{\alpha}{2\pi}(L-1)\left[\int\limits_0^1 dx\frac{1+x^2}{1-x}\times \times\theta(1-x-\Delta_1)\frac{d\sigma_B(xp_1,\theta)}{dc}\delta(z-t(x)) + +\int\limits_z^{z_0}\frac{dt}{t}\frac{1+z^2/t^2}{1-z/t}\theta\left(1-\frac{z}{t}-\Delta_2\right)\frac{d\sigma_B(p_1,\theta)}{dc}\delta(t-z_0)\right] + +\frac{df^{(1)}}{dz\,dc}+\frac{df^{(2)}}{dz\,dc}, \quad (22)$$

where

$$\frac{df^{(1)}}{dz\,dc} = \frac{\alpha^3}{4\rho(1-c)\omega_1^2}\times \times\left(\frac{2-z(1+c)}{2}+\frac{2}{2-z(1+c)}\right)\times \times\left[\frac{1+x^2}{1-x}\ln\frac{\rho\theta_0^2}{2z_0(1+c)}+1-x\right]_{x=x_0}\times \times\theta(1-x-\Delta_1), \quad (23)$$

$$\frac{df^{(2)}}{dz\,dc} = \frac{\alpha^3}{4\rho a\omega_1^2}\left(\frac{1-c}{a}+\frac{a}{1-c}\right)\times \times\left[\frac{1+z^2/t^2}{1-z/t}\ln\frac{z^2\theta_0^2}{2\rho(1+c)z_0}+1-\frac{z}{t}\right]_{t=z_0}\times \times\theta\left(1-\frac{z}{t}-\Delta_2\right), \quad \Delta_{1,2} = \frac{\Delta\varepsilon}{\varepsilon_{1,2}}.$$

We here use the relation

$$\delta(z-t(x)) = \frac{2x_0^2\rho}{z^2(1-c)}\delta(x-x_0).$$

Again, we can see that the terms containing the large logarithm L have the form conforming with the structure function. Our ansatz (5) is therefore confirmed.

The dependence on the auxiliary parameter θ_0 vanishes when the contribution of noncollinear kinematics of the additional hard photon emission is taken into account (see Sec. 6).

4. NONCOLLINEAR KINEMATICS CONTRIBUTION. DOUBLE COMPTON SCATTERING PROCESS

The general expression for the cross section of the DC scattering process

$$\gamma(k_1)+e^-(p_1)\to\gamma(k_2)+\gamma(k)+e^-(p_2), \quad \kappa = 2kp_1, \quad \kappa' = 2kp_2, \quad \kappa_2 = 2k_2p_1, \quad \kappa_2' = 2k_2p_2, \quad (24)$$

Table 1. The value of K_h as a function of z and $\cos\theta$ (calculated for $\rho = 0.4$)

$z \setminus \cos\theta$	−0.8	−0.6	−0.4	−0.2	0	0.2	0.4	0.6	0.8
0.1	−2.82	−2.61	−2.39	−2.19	−2.09	−1.89	−1.87	−2.06	−2.75
0.2	−2.77	−2.47	−2.17	−1.90	−1.65	−1.46	−1.39	−1.56	−2.30
0.3	−3.43	−2.98	−2.55	−2.14	−1.77	−1.47	−1.30	−1.38	−2.13
0.4	−4.96	−3.87	−3.23	−2.65	−2.13	−1.67	−1.34	−1.30	−2.02

Table 2. The value of $\tilde{K}_h$ as a function of y and $\cos\theta$ (calculated for $\eta = 0.064$)

$y \setminus \cos\theta$	−0.8	−0.6	−0.4	−0.2	0	0.2	0.4	0.6	0.8
0.05	0.70	−1.97	−7.41	−15.54	−26.90	−42.70	−65.40	−100.64	−166.21
0.10	0.36	−3.20	−9.85	−18.38	−18.35				
0.15	0.03	−3.38	−1.34						
0.20	−0.20	0.29							
0.25	−0.25								

was obtained years ago by Mandl and Skyrme [3]. The expression for the cross section presented in this paper is exact but, unfortunately, too complicated. Instead, we use the expression for the differential cross section calculated (by the methods of chiral amplitudes [10]) under the assumption that all kinematic invariants are large compared with the electron mass squared, $\kappa \sim \kappa' \sim \kappa_i \sim \kappa_i' \gg m^2$:

$$\begin{aligned} &\frac{\varepsilon_2 d\sigma_0^{DC}}{d^3p_2} = \frac{1}{2!}\,\frac{\alpha^3}{2\pi^2\kappa_1} R\, d\Phi, \\ &d\Phi = \frac{d^3k_2}{\omega_2}\,\frac{d^3k}{\omega}\delta^4(p_1+k_1-p_2-k_2-k), \\ &R = s_1 \times \\ &\times \frac{\kappa\kappa'(\kappa^2+\kappa'^2)+\kappa_1\kappa_1'(\kappa_1^2+\kappa_1'^2)+\kappa_2\kappa_2'(\kappa_2^2+\kappa_2'^2)}{\kappa\kappa'\kappa_1\kappa_1'\kappa_2\kappa_2'}. \end{aligned} \tag{25}$$

The explicit expression for the contribution to the K-factor from hard photon emission K_h is

$$\frac{\alpha}{\pi}\frac{d\sigma_B}{dc}K_h = \frac{d\sigma_{\theta_0}^{DC}}{dz\,dc} + \frac{df^{(1)}}{dz\,dc} + \frac{df^{(2)}}{dz\,dc}, \tag{26}$$

where

$$\frac{d\sigma_{\theta_0}^{DC}}{dz\,dc} = \frac{\alpha^3 z}{2!4\pi\rho}\int R\,d\Phi, \tag{27}$$

and the phase volume $d\Phi$ is restricted by the conditions $\omega, \omega_2 > \Delta\epsilon$ and the requirement that the angles between the 3-vectors $\mathbf{k}_2, \mathbf{k}$ and the 3-vectors $\mathbf{p}_1$, $\mathbf{p}_2$ exceed θ_0.

The values of K_h calculated numerically are given in Tables 1 and 2. We find the independence of K_h from the auxiliary parameters θ_0 and $\Delta\varepsilon$ numerically and analytically (see Appendix A).

The cross section of the DC scattering process in an inclusive experimental set-up with the leading logarithmic approximation in terms of the structure functions has the form

$$\begin{aligned} d\sigma^{DC}(p_1,k_1;p_2,k,k_2) &= \int_0^1 dx\, D(x,L)\times \\ &\times \int_z^1 D\left(\frac{z}{t}\right)\frac{dt}{t}d\sigma_0^{DC}\left(xp_1,k_1;\frac{tp_2}{z},k,k_2\right), \end{aligned} \tag{28}$$

with the structure functions given above and

$$\begin{aligned} d\sigma_0^{DC}(p_1,k_1;p_2,k,k_2) &= \frac{\alpha^3}{4\pi^2\kappa_1}\times \\ &\times R\frac{d^3k_2 d^3k d^3p_2}{\omega_2\omega\epsilon_2}\delta^4(p_1+k_1-p_2-k_2-k). \end{aligned} \tag{29}$$

5. CONCLUSION

The characteristic form «reverse radiative tail» (see Tables 3 and 4) of the differential cross section vs. the energy fraction z can be reproduced if one uses the «smoothed» expression for nonsinglet structure functions, which includes the virtual electron pair production [11]

Table 3. The value of $\omega_1^2/\alpha^2 d\sigma/(dc\,dz)$ (the leading contribution, the first term in the right-hand side of master formula (8)) as a function of z and $\cos\theta$ (calculated for $\rho = 0.4$, $\omega_1 = 5$ GeV)

$z \setminus \cos\theta$	−0.8	−0.6	−0.4	−0.2	0	0.2	0.4	0.6	0.8
0.1	0.211	0.237	0.265	0.299	0.345	0.413	0.526	0.754	1.450
0.2	0.337	0.357	0.378	0.405	0.445	0.508	0.618	0.850	1.576
0.3	0.703	0.669	0.643	0.634	0.644	0.685	0.782	1.013	1.784
0.4	3.883	2.153	1.554	1.264	1.113	1.054	1.090	1.296	2.122

Table 4. The value of $(\varepsilon_1^2/\alpha^2)\, d\tilde{\sigma}/dc\,dy$ (the leading contribution, the first term in the right-hand side of master formula (39)) as a function of y and $\cos\theta$ (calculated for $\omega_1 = 400$ MeV, $\varepsilon_1 = 6$ GeV)

$y \setminus \cos\theta$	−0.8	−0.6	−0.4	−0.2	0	0.2	0.4	0.6
0.05	9.658	11.110	13.626	17.513	23.678	34.116	53.669	98.208
0.10	11.350	15.024	22.633	39.297	86.017			
0.15	13.839	23.190	56.097					
0.20	17.735	45.672						
0.25	24.303							

$$D(x,L) = \frac{\beta}{2}(1-x)^{\beta/2-1}\left(1+\frac{3}{8}\beta\right) - \\ - \frac{\beta}{4}(1+x) + O(\beta^2), \quad \beta = \frac{2\alpha}{\pi}(L-1), \quad (30)$$

$$O(\beta^2) = \frac{\beta}{2}(1-x)^{\beta/2-1}\left(-\frac{1}{48}\beta^2\left(\frac{1}{3}L+\pi^2-\frac{47}{8}\right)\right) + \\ + \frac{1}{32}\beta^2\left(-4(1+x)\ln(1-x) - \frac{1+3x^2}{1-x}\ln x - 5 - x\right).$$

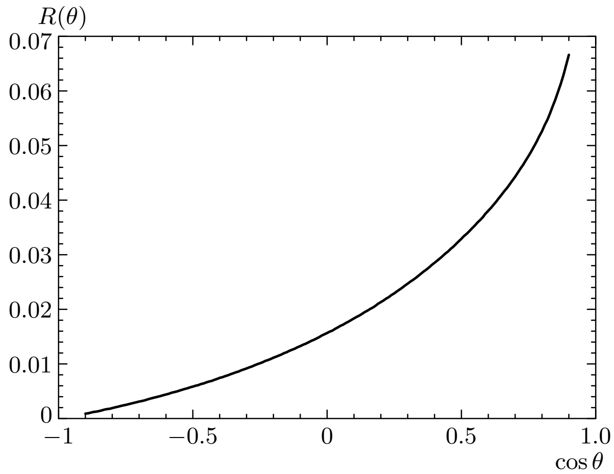
In the Figure, we give the magnitude of RC in the leading approximation

$$R(\theta) = \left(\frac{d\sigma_B}{dc}\right)^{-1}\left(\int dz \frac{d\sigma}{dz\,dc} - \frac{d\sigma_B}{dc}\right). \quad (31)$$

The results given above refer to the experimental set-up without additional $e^+e^-, \mu^+\mu^-, \pi^+\pi^-$ real pairs in the final state.

The accuracy of the formulas given above is determined by the order of magnitude of the terms omitted (see (2)) compared to the terms of the order of unity, i.e., is of the order of 0.1% for typical experimental conditions. In particular, this is the reason why we omit the evolution effect of the K-factor terms.

The numerical value of K_h, leading contributions, and the Born cross section for different kinematic regions are presented as functions of z and c in Tables 1–6.



The leading-order radiative corrections as a $\cos\theta$-distribution (see formulas (31))

Two of us (E. A. K) and (V. V. B) are grateful to the RFBR (grant № 03-02-17077) for supporting this work. We are grateful to S. Gevorkyan for collaboration at the beginning of this work and S. Dittmaier for reminding us of the valuable set of previously published papers.

Table 5. Born cross section (4) (without the factor α^2/ω_1^2) for $\rho = 0.4$

$\cos\theta$	−0.8	−0.6	−0.4	−0.2	0	0.2	0.4	0.6	0.8
$\frac{\omega_1^2}{\alpha^2}\frac{d\sigma_B}{dc}$	1.779	2.038	2.365	2.796	3.389	4.266	5.721	8.669	17.881

Table 6. Born cross section (40) (without the factor α^2/ω_1^2) for $\omega_1 = 400$ MeV and $\varepsilon_1 = 6$ GeV

$\cos\theta$	−0.8	−0.6	−0.4	−0.2	0	0.2	0.4	0.6	0.8
$\frac{\varepsilon_1^2}{\alpha^2}\frac{d\tilde{\sigma}_B}{dc}$	93.317	60.706	49.428	44.994	44.351	47.084	54.584	72.444	129.944

Table 7. The value of y_0 and z_0 as a function of c for $\eta = 0.064$ and $\rho = 0.4$

$\cos\theta$	−0.8	−0.6	−0.4	−0.2	0	0.2	0.4	0.6	0.8
y_0	0.417	0.263	0.192	0.152	0.125	0.106	0.093	0.082	0.074
z_0	0.423	0.455	0.489	0.526	0.571	0.625	0.690	0.769	0.870

APPENDIX A

Integrating the phase volume over k_2,

$$d\Phi = \frac{d^3k}{\omega}\frac{d^3k_2}{\omega_2}\delta^4(Q-k-k_2), \quad Q = p_1+k_1-p_2, \quad (32)$$

we can put it in the form

$$d\Phi = \frac{\omega d\omega}{\omega_1^2}\frac{2dc_1dc_2}{\sqrt{D}}\delta\left[2\rho - \rho z(1+c) - z(1-c) - \right.$$
$$\left. - \frac{\omega}{\omega_1}(\rho(1-c_1) - z(1-c_2) + 1 + c_1)\right], \quad (33)$$

where $D = 1 - c_1^2 - c_2^2 - c^2 - 2cc_1c_2$ and c_1, c_2 are the cosines of the respective angles between $\mathbf{k}$ and $\mathbf{p}_1, \mathbf{p}_2$.

For collinear kinematics, the following relations can be useful:

1. $k \approx (1-x)p_1$,

$$R_1 = R|_{\mathbf{k}||\mathbf{p}_1} = \left(\frac{2x\rho}{z(1-c)} + \frac{z(1-c)}{2x\rho}\right) \times$$
$$\times \frac{1+x^2}{(1-x)^2}\frac{1}{2\rho^2(1-c_1)x\omega_1^2},$$
$$d\Phi_1 = d\Phi|_{\mathbf{k}||\mathbf{p}_1} = 2\frac{d^3k}{\omega}\delta((xp_1+k_1-p_2)^2) =$$
$$= 2\pi\frac{\rho(1-x)dx\,dc_1}{2-z(1+c)}\delta(x-x_0), \quad (34)$$
$$\frac{d\sigma_h^1}{dz\,dc} = \frac{\alpha^3 z}{2!4\pi\rho}\int R_1 d\Phi_1 = \frac{\alpha^3}{4\rho\omega_1^2(1-c)} \times$$
$$\times \frac{1+x_0^2}{1-x_0}\left(\frac{2x_0\rho}{z(1-c)} + \frac{z(1-c)}{2x_0\rho}\right)\ln\frac{4}{\theta_0^2}.$$

In the last equation, we take the same contribution from the region $k_2 \approx (1-x)p_1$ into account.

2. In the case where $k \approx (t/z - 1)p_2$, we obtain

$$R_2 = R|_{\mathbf{k}||\mathbf{p}_2} = \left(\frac{2x\rho}{z(1-c)} + \frac{z(1-c)}{2x\rho}\right) \times$$
$$\times \frac{1+x^2}{(1-x)^2}\frac{1}{2\rho^2(1-c_1)x\omega_1^2},$$
$$d\Phi_1 = d\Phi|_{\mathbf{k}||\mathbf{p}_1} = 2\frac{d^3k}{\omega}\delta((xp_1+k_1-p_2)^2) =$$
$$= 2\pi\frac{\rho(1-x)dx\,dc_1}{2-z(1+c)}\delta(x-x_0). \quad (35)$$

Therefore, the contribution in the case where $\mathbf{k}||\mathbf{p}_2$ ($\mathbf{k}_2||\mathbf{p}_2$) has the form

$$\frac{d\sigma_h^2}{dz\,dc} = \frac{\alpha^3 z}{2!4\rho\omega_1^2}\int R_1\,d\Phi_1 =$$
$$= \frac{\alpha^3}{4\rho a\omega_1^2}\left(\frac{1-c}{a} + \frac{a}{1-c}\right)\frac{1+\frac{z^2}{t^2}}{1-\frac{z}{t}}\ln\frac{4}{\theta_0^2}. \quad (36)$$

Comparing formulas (34) and (36) with (23), we can see explicit cancellation of the θ_0 dependence.

APPENDIX B

Here, we describe the different cases of kinematic regions for ρ and z.

All the above formulas were considered for $\rho < 1$, and the possible region for the variable z was determined by the inequality $x_0 < 1$,

$$z \le \frac{2\rho}{1 - c + \rho(1+c)}, \tag{37}$$

which means that the lower integration limit in formula (8) is less than 1. In the case where $\rho > 1$, it is convenient to introduce the new variables

$$\eta = \frac{\omega_1}{\varepsilon_1}, \quad y = \frac{\varepsilon_2'}{\varepsilon_1}, \quad y_0 = \frac{\varepsilon_2}{\varepsilon_1} = \frac{2\eta}{1+c+\eta(1-c)}, \quad \eta < 1. \tag{38}$$

For $\rho > 1$ (or $\eta < 1$), master equation (8) becomes

$$\frac{d\tilde{\sigma}}{dy\,dc}(p_1,p_2) = \int\limits_{\tilde{x}_0}^{1} \frac{dx}{\tilde{t}(x)} D(x,\tilde{L}) \frac{d\tilde{\sigma}_B(xp_1,\theta)}{dc} \times \times D\left(\frac{y}{\tilde{t}(x)}, \tilde{L}\right) + + \frac{\alpha}{\pi}\frac{d\tilde{\sigma}_B(p_1,\theta)}{dc}\left[\tilde{K}_{SV}\delta(y-y_0) + \tilde{K}_h\right], \tag{39}$$

$$\tilde{x}_0 = \frac{y\eta(1-c)}{2\eta - y(1+c)}, \quad \tilde{L} = \ln\frac{2\varepsilon_1^2 y_0(1+c)}{m^2}, \quad \tilde{t}(x) = \frac{2\eta x}{x(1+c)+\eta(1-c)},$$

with the possible values for the energy fraction y of the scattered electron given by $y \le y_0$. Born cross section (4) and (6) and formulas for hard photon emission, $\tilde{K}_{SV}$, $\tilde{K}_h$ for $\rho > 1$ follow just by the appropriate substitution $\rho \to \eta^{-1}$:

$$\frac{d\tilde{\sigma}_B(xp_1,\theta)}{dc} = \frac{\pi\alpha^2}{\varepsilon_1^2}\frac{1}{(\eta(1-c)+x(1+c))^2} \times \times \left(\frac{\eta(1-c)}{\eta(1-c)+x(1+c)} + \frac{\eta(1-c)+x(1+c)}{\eta(1-c)}\right). \tag{40}$$

Large values of the leading contribution (see Table 4) near the kinematic bound (see Table 7) can be understood as a manifestation of the $\delta(y - y_0)$-character of the differential cross section. The y_0, z_0 dependence is given in Table 7.

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