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MULTIPLE EXCHANGES IN LEPTON PAIR PRODUCTION IN HIGH-ENERGY HEAVY ION COLLISIONS

E. Bartoš[*], *S. R. Gevorkyan*[**], *E. A. Kuraev*[***]

Joint Institute for Nuclear Research
141980, Dubna, Russia

N. N. Nikolaev[****]

Landau Institute for Theoretical Physics
142432, Chernogolovka, Moscow Region, Russia

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As an archetype reaction for pQCD multigluon hard processes in collisions of ultrarelativistic nuclei, we analyse generic features of lepton pair production via multiphoton processes in peripheral heavy ion scattering. We report explicit results for collisions of two photons from one nucleus with two photons from the other nucleus, $2\gamma + 2\gamma \to l^+l^-$. The results found suggest that the familiar eikonalization of Coulomb distortions breaks down for the oppositely moving Coulomb centers. The breaking of eikonalization in QED suggests that multigluon pQCD processes cannot be described in terms of the collective nuclear gluon distributions. We discuss a logarithmic enhancement of the contribution from the $2\gamma + 2\gamma \to l^+l^-$ process to production of lepton pairs with large transverse momentum; similar enhancement is absent for the $n\gamma + m\gamma \to l^+l^-$ processes with $m, n > 2$. We comment on the general structure of multiphoton collisions and properties of higher-order terms that cannot be eikonalized.

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1. INTRODUCTION

The exact theory of Coulomb distortions of the spectrum of ultrarelativistic lepton pairs photoproduced in the Coulomb field of a nucleus has been developed by Bethe and Maximon [1]. It is based on the description of leptons by exact solutions of the Dirac equation in the Coulomb field (see, e.g., textbook [2]). In the Feynman diagram language, one has to sum multiphoton exchanges between the produced electrons and positrons and the target nucleus. For ultrarelativistic leptons, the result of this summation is the eikonal factors in the impact parameter representation. In the momentum space, the same eikonal form leads to simple recursive relations between the $(n+1)$- and n-photon exchange amplitudes [3], where the incoming photon can be either real or virtual. There are two fundamental points behind these simple results.

i) The lightcone momenta of ultrarelativistic leptons are conserved in a multiple scattering process (i. e., if the nucleus moves along the n_--lightcone and the produced leptons move along the n_+-lightcone, then the p_+-components of the lepton momenta are conserved).

ii) The s-channel helicity of leptons is conserved in high-energy QED (see textbook [2]). It is the last property by which distortions reduce to a simple eikonal factor.

The same properties allow one to expcess the pair production cross section in the dipole representation [4]. They also underlie the color dipole perturbative Quantum Chromo Dynamics (pQCD) analysis of nuclear distortions and the derivation of nonlinear $k_\perp$-factorization for multijet hard processes in DIS off nuclei [5].

As shown in [6], the so-called Abelianization takes

[*]E-mail: bartos@thsun1.jinr.ru, Department of Theoretical Physics, Comenius University, 84248, Bratislava, Slovakia.
[**]E-mail: gevs@nusun.jinr.ru, Yerevan Physics Institute, 375036, Yerevan, Armenia.
[***]E-mail: kuraev@thsun1.jinr.ru
[****]E-mail: n.nikolaev@fr-juelich.de, Institut für Kernphysik, Forschungszentrum Jülich, D-52425, Jülich, Germany.

place in certain cases of practical interest. Specifically, the hard dijet production in a hadron–nucleus collision is dominated by a hard collision of an isolated parton from the beam hadron simultaneously with many gluons from the nucleus, which belong to different nucleons of a target nucleus. Nevertheless, at least for single-particle spectra, the interaction with a large number of nuclear gluons can be reduced to that with a single gluon from the collective gluon field of a nucleus, i.e., the nonlinear $k_\perp$-factorization reduces to the linear one, and in terms of the collective glue, one only needs to evaluate the familiar Born cross sections. Extending the nonlinear $k_\perp$-factorization for hard processes from hadron–nucleus collisions to collisions of ultrarelativistic nuclei is a formidable task that has not been properly addressed so far. The lightcone QED and QCD share many properties, and we here address a much simpler, Abelian problem of Coulomb distortions of lepton pairs produced in peripheral collisions of relativistic nuclei.

The process of lepton pair production in the Coulomb fields of two colliding ultrarelativistic heavy ions was intensely investigated recently [7–14]. Such an activity is mainly connected with new practical interest in pair production opened with operation of the facilities such as RHIC and LHC. Despite the high activity in this area, the issue of correct allowance for the final-state interaction of produced leptons with the colliding ion Coulomb field remains open. The main results obtained so far in this direction are as follows.

i) The produced high-energy lepton pair interacts strongly with the Coulomb field of heavy ions, and the corresponding corrections have a noticeable impact on the cross section of the process [10].

ii) The perturbation series corresponding to a multiple interaction of a produced pair with Coulomb fields can be summed and the result can be expressed in an eikonal-like form [14] if one restricts oneself to terms growing with the energy in the cross section [12]. In QED, such an approximation can be considered satisfactory, but it is not warranted in QCD, and the problem of higher-order corrections in pair production requires further investigation.

In our paper [12], we cited the amplitude $M_{(2)}^{(2)}$, which is irrelevant in the leading and next-to-leading logarithmic approximations in QED. Nevertheless, the knowledge of contributions of this type becomes important for similar processes in QCD with multigluon exchanges between the color constituents of each of the colliding hadrons and the created quark–antiquark pair. This is the main motivation for our interest in multiple exchanges and their impact on the lepton pair yield in the ultrarelativistic heavy ion collisions, an issue that is not only useful in understanding the electromagnetic processes but also broadly applicable in QCD.

We skip the previously studied case where one of the ions radiates a single photon and the other radiates an arbitrary number of photons absorbed by the created pair [14]. The photon exchanges between the ions were not taken into account either [13].

This paper is organized as follows. In Sec. 2, we consider the case where each of the colliding ions radiates two photons, which create a lepton pair. We derive the relevant amplitude $M_{(2)}^{(2)}$ using the powerful Sudakov technique, well suited for calculations of processes at high energies. In Sec. 3, we study the wide-angle limit in pair production kinematics corresponding to the case of large transverse momenta of pair components. In this limit, the results are much more transparent than in the general case, as can be seen from the form of the differential cross section given below. In Sec. 4, we discuss the generalization of the process under consideration to the case where the number of photons exchanged by each ion exceeds two.

2. THE LEPTON PAIR PRODUCTION

We are interested in the process of lepton pair production in the collision of two relativistic nuclei A and B with charge numbers Z_1 and Z_2,

$$\mathrm{A}(p_1)+\mathrm{B}(p_2) \to l^-(q_-)+l^+(q_+)+\mathrm{A}(p_1')+\mathrm{B}(p_2'), \quad (1)$$

with the kinematical invariants

$$\begin{gathered} s=(p_1+p_2)^2, \quad q_1^2=(p_1-p_1')^2, \\ q_2^2=(p_2-p_2')^2, \quad s_1=(q_++q_-)^2, \\ p_1^2={p_1'}^2=M_1^2, \quad p_2^2={p_2'}^2=M_2^2, \quad q_\pm^2=m^2. \end{gathered} \quad (2)$$

We are interested in peripheral kinematics, i.e.,

$$s \gg M_1^2,\, M_2^2,\, |q_1^2|,\, |q_2^2| \gg m^2, \quad (3)$$

which corresponds to small scattering angles of ions A and B.

It is convenient to use the Sudakov parameterization for all 4-momenta entering process (1),

$$\begin{aligned} q_1 &= a_1\tilde p_2 + b_1\tilde p_1 + q_{1\perp}, \\ q_2 &= a_2\tilde p_2 + b_2\tilde p_1 + q_{2\perp}, \\ k_1 &= \alpha_1\tilde p_2 + \beta_1\tilde p_1 + k_{1\perp}, \\ k_2 &= \alpha_2\tilde p_2 + \beta_2\tilde p_1 + k_{2\perp}, \\ q_\pm &= \alpha_\pm\tilde p_2 + \beta_\pm\tilde p_1 + q_{\pm\perp}, \end{aligned} \quad (4)$$

with lightcone 4-vectors $\tilde p_{1,2}$ obeying the conditions

$$\tilde p_1^2=\tilde p_2^2=0, \quad \tilde p_{1,2}\cdot q_\perp=0, \quad 2\tilde p_1\cdot\tilde p_2=s.$$

2.1. The pair production by 4-photons

We consider the creation of a lepton pair by four virtual photons (Fig. 1). The photons with momenta k_1 and q_1-k_1 (referred to as photons 1 and 2 hereafter) are emitted by ion A and the photons with momenta k_2 and q_2-k_2 (referred as the photons 3 and 4) by ion B. The leading contribution to the cross section comes from the following regions of the Sudakov variables:

$$\begin{aligned} &\alpha_1 \ll \beta_1 \sim b_1, \quad \beta_+ + \beta_- = b_1, \\ &\beta_2 \ll \alpha_2 \sim a_2, \quad \alpha_+ + \alpha_- = a_2, \\ &|a_1| \ll a_2, \quad |b_2| \ll b_1, \quad q_{i\perp} = \mathbf{q}_i, \\ &\mathbf{q}_1 + \mathbf{q}_2 = \mathbf{q}_+ + \mathbf{q}_-, \\ &\alpha_\pm = \frac{\mathbf{q}_\pm^2}{s\beta_\pm}, \quad \mathbf{q}_\pm^2 \gg m^2. \end{aligned} \tag{5}$$

Hereinafter, the boldface $\mathbf{q}_i$ denotes the two-dimensional transverse part of any considered 4-momentum. For definiteness, we assume $\beta_+, \beta_- > 0$, which corresponds to the situation where the pair moves along the momentum of ion A (the momentum p_1). With a possible extension to pQCD in mind, we neglect the lepton masses whenever appropriate.

The contribution to the matrix element of such a set of the Feynman diagrams (FD) is given by

$$\begin{aligned} M_{(2)}^{(2)} = is\frac{(Z_1Z_2)^2(4\pi\alpha)^4}{(2\pi)^8} \times \\ \times \int \frac{d^4k_1 d^4k_2}{k_1^2k_2^2(q_1-k_1)^2(q_2-k_2)^2} \times \\ \times \frac{1}{s}\bar{u}^\eta(p_1')O_1^{\mu_1\nu_1}u^\eta(p_1)\bar{u}^\lambda(p_2')O_2^{\rho_1\sigma_1}u^\lambda(p_2) \times \\ \times \bar{u}(q_-)T^{\mu\nu\rho\sigma}v(q_+)g_{\mu\mu_1}g_{\nu\nu_1}g_{\rho\rho_1}g_{\sigma\sigma_1}, \end{aligned} \tag{6}$$

where u and v are the leptonic Dirac bispinors and

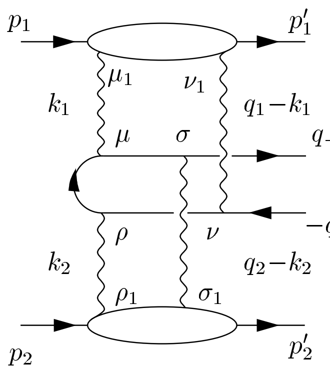

Fig. 1. A typical Feynman diagram for the amplitude $M_{(2)}^{(2)}$

O_1, O_2, and T are the corresponding tensors of the upper, down, and pair production blocks. To see the proportionality of matrix element (6) to the invariant energy s, we use the Gribov representation for the virtual photon Green's functions

$$g_{\mu\mu_1}g_{\nu\nu_1}g_{\rho\rho_1}g_{\sigma\sigma_1} \approx \\ \approx \left(\frac{2}{s}\right)^4 p_{1\mu}p_{1\nu}p_{1\rho_1}p_{1\sigma_1}p_{2\mu_1}p_{2\nu_1}p_{2\rho}p_{2\sigma}. \tag{7}$$

The numerators of the Green's functions of nucleus A can be written as s^2N_1 with

$$N_1 = \frac{1}{s}\bar{u}^\eta(p_1')\hat{p}_2u^\eta(p_1), \quad \sum_\eta |N_1|^2 = 2,$$

and a similar expression exists for nucleus B. The denominators of the virtual photon Green's functions in the considered kinematics depend only on the transverse components of the corresponding 4-vectors, and therefore

$$k_1^2k_2^2(q_1-k_1)^2(q_2-k_2)^2 = \mathbf{k}_1^2\mathbf{k}_2^2(\mathbf{q}_1-\mathbf{k}_1)^2(\mathbf{q}_2-\mathbf{k}_2)^2.$$

There are 24 FD contributing to $M_{(2)}^{(2)}$. Instead of them, it is convenient to consider $24\cdot2\cdot2 = 96$ FD with all possible permutations of emission and absorption points of the exchanged photons by the nuclei (Fig. 2). Then the result must be divided by $(2!)^2$. This trick [15] provides the convergence of integrals over β_2,

$$\frac{1}{2\pi i}\int_{-\infty}^{\infty} d\beta_2 \left[\frac{s}{s\beta_2-c+i0} + \frac{s}{-s\beta_2-d+i0}\right] = -1, \tag{8}$$

and of a similar integral over α_1. After all operations, we can write the matrix element as

$$M_{(2)}^{(2)} = is\frac{(16\pi\alpha^2Z_1Z_2)^2N_1N_2}{(2!)^2} \times \\ \times \int \frac{d^2k_1d^2k_2}{\pi^2} \frac{\bar{u}(q_-)Rv(q_+)}{\mathbf{k}_1^2\mathbf{k}_2^2(\mathbf{q}_1-\mathbf{k}_1)^2(\mathbf{q}_2-\mathbf{k}_2)^2}, \tag{9}$$

where

$$R = \frac{1}{s}\int \frac{d\beta_1 d\alpha_2}{(2\pi i)^2} p_{1\mu}p_{1\nu}p_{2\rho}p_{2\sigma}T^{\mu\nu\rho\sigma}.$$

2.2. The classification of Feynman diagrams

It is convenient to classify FD by the ordering of the exchanged photons absorbed by the lepton world line (Fig. 3). We label them as R_{ijkl}, $R = \sum R_{ijkl}$, with pairwise distinct integers i, j, k, l from one to four, counting from a negative lepton emission point.

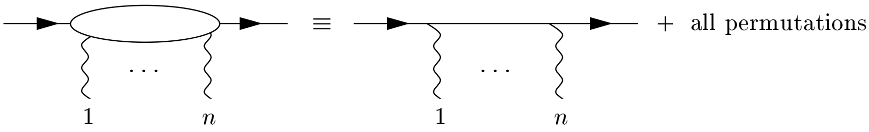

Fig. 2. The notation for the permutations of n virtual photons emitted by a heavy ion

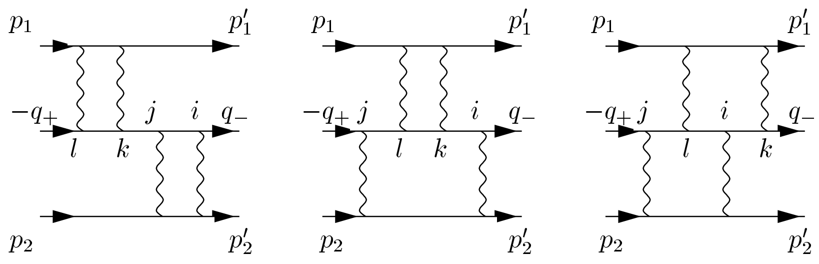

Fig. 3. The set of basic Feynman diagrams for the amplitude $M_{(2)}^{(2)}$

a) We first consider the set of four FD (Fig. 4*a*), labeled R_{1234}, R_{2134}, R_{1243}, and R_{2143}, in which the interactions with two nuclei are ordered consecutively against the lepton line direction. The sum of the relevant contributions provides the convergence of the β_1 and α_2 integrations. After a standard calculation, we obtain

$$R_{1234} + R_{2134} + R_{1243} + R_{2143} = = \frac{\beta_- \hat{p}_1 (\hat{q}_- - \hat{q}_1)_\perp}{\beta_+ \mathbf{q}_-^2 + \beta_- (\mathbf{q}_- - \mathbf{q}_1)^2} \frac{\hat{p}_2}{s} = -B \frac{\hat{p}_2}{s}, \quad B = \frac{\hat{q}_{-\perp} (\hat{q}_- - \hat{q}_1)_\perp}{\beta_+ \mathbf{q}_-^2 + \beta_- (\mathbf{q}_- - \mathbf{q}_1)^2}. \tag{10}$$

The last equality in Eq. (10) is the result of the Dirac equation for massless particles,

$$\bar{u}(q_-) \beta_- \hat{p}_1 \hat{p}_2 = -\bar{u}(q_-) \hat{q}_{-\perp} \hat{p}_2. \tag{11}$$

A result similar to Eq. (10) is obtained for the set of the crossing diagrams (Fig. 4*b*) corresponding to the R_{3412}, R_{3421}, R_{4312}, and R_{4321} terms in the amplitude, with only the replacement $B \to \tilde{B}$, where

$$\tilde{B} = \frac{(-\hat{q}_+ + \hat{q}_1)_\perp \hat{q}_{+\perp}}{\beta_- \mathbf{q}_+^2 + \beta_+ (\mathbf{q}_1 - \mathbf{q}_+)^2}. \tag{12}$$

b) We next consider the set of the diagrams R_{1342}, R_{1432}, R_{2341}, R_{2431} (Fig. 4*c*) and R_{3124}, R_{3214}, R_{4123}, R_{4213} (Fig. 4*d*), where exchanges with ion B (A) are attached to the lepton line between the interactions with ion A (B).

For definiteness, we consider the sum $R_{1342} + R_{1432}$. Using the relevant denominators of the lepton line, we obtain the following integrals over β_1 and α_2:

$$\int \frac{d\beta_1}{2\pi i} \frac{1}{s\alpha_-(\beta_- - \beta_1) - (\mathbf{q}_- - \mathbf{k}_1)^2 + i0} \times \times \frac{1}{-s\alpha_+(\beta_- - \beta_1) - (-\mathbf{q}_+ + \mathbf{q}_1 - \mathbf{k}_1)^2 + i0} \times \times \int \frac{d\alpha_2}{2\pi i} \left[\frac{s(\beta_- - \beta_1)}{s(\beta_- - \beta_1)(\alpha_- - \alpha_2) - (\mathbf{q}_- - \mathbf{k}_1)^2 + i0} + + \frac{s(\beta_- - \beta_1)}{s(\beta_- - \beta_1)(-\alpha_+ + \alpha_2) - (-\mathbf{q}_+ + \mathbf{q}_1 - \mathbf{k}_1)^2 + i0} \right]. \tag{13}$$

The second integral after closing the integration contour in the lower half-plane gives the function $\mathrm{sign}(\beta_- - \beta_1)$, and hence Eq. (13) becomes

$$\int \frac{d\beta_1}{2\pi i} \frac{\mathrm{sign}(\beta_1 - \beta_-)}{s\alpha_-(\beta_- - \beta_1) - (\mathbf{q}_- - \mathbf{k}_1)^2 + i0} \times \times \frac{1}{-s\alpha_+(\beta_- - \beta_1) - (-\mathbf{q}_+ + \mathbf{q}_1 - \mathbf{k}_1)^2 + i0}. \tag{14}$$

Using the relation

$$\int_{-\infty}^{\infty} \frac{dx}{2\pi i} \frac{\mathrm{sign}\, x}{(-ax - b + i0)(cx - d + i0)} = = \frac{1}{\pi i (ad + bc)} \ln \frac{ad}{bc}, \tag{15}$$

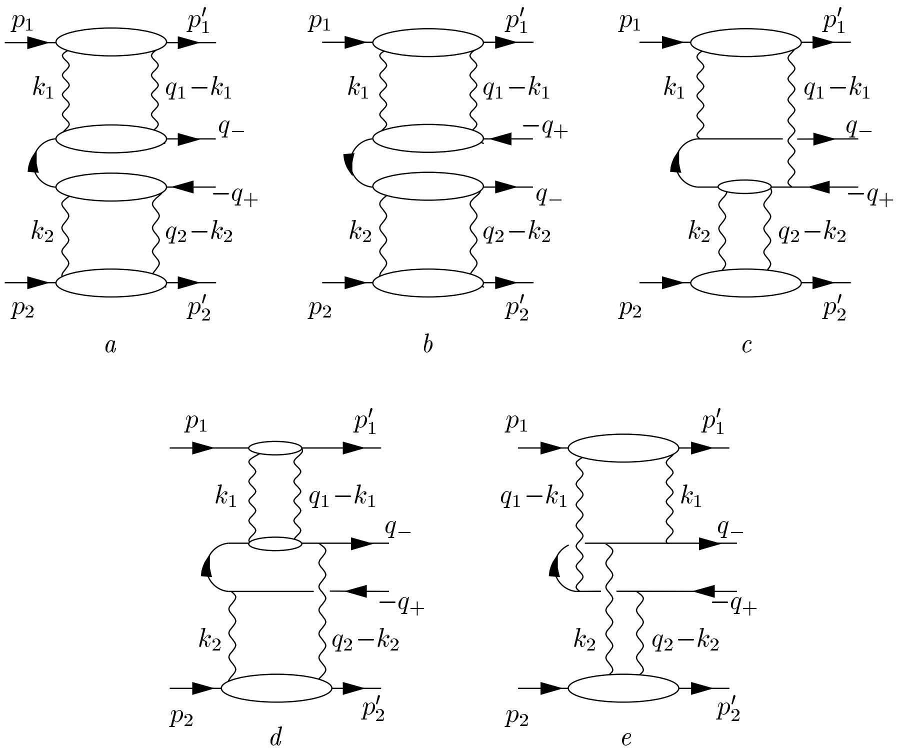

Fig. 4. The Feynman diagrams for the amplitude $M_{(2)}^{(2)}$

we obtain the result

$$
\begin{gathered}
R_{1342}+R_{1432}+R_{2341}+R_{2431}= \\
=\frac{\hat{p}_1}{i\pi s}\left[\frac{(\hat{q}_- - \hat{k}_1)_\perp(-\hat{q}_+ + \hat{q}_1 - \hat{k}_1)_\perp}{\alpha_+(\mathbf{q}_- - \mathbf{k}_1)^2+\alpha_-(-\mathbf{q}_+ + \mathbf{q}_1 - \mathbf{k}_1)^2}\times\right. \\
\times\ln\frac{\alpha_+(\mathbf{q}_- - \mathbf{k}_1)^2}{\alpha_-(-\mathbf{q}_+ + \mathbf{q}_1 - \mathbf{k}_1)^2}+ \\
+\frac{(\hat{q}_- - \hat{q}_1 + \hat{k}_1)_\perp(-\hat{q}_+ + \hat{k}_1)_\perp}{\alpha_+(\mathbf{q}_- - \mathbf{q}_1 + \mathbf{k}_1)^2+\alpha_-(-\mathbf{q}_+ + \mathbf{k}_1)^2}\times \\
\left.\times\ln\frac{\alpha_+(\mathbf{q}_- - \mathbf{q}_1 + \mathbf{k}_1)^2}{\alpha_-(-\mathbf{q}_+ + \mathbf{k}_1)^2}\right], \\
R_{3124}+R_{3214}+R_{4123}+R_{4213}= \\
=\frac{\hat{p}_2}{i\pi s}\left[\frac{(\hat{q}_- - \hat{k}_2)_\perp(-\hat{q}_+ + \hat{q}_2 - \hat{k}_2)_\perp}{\beta_+(\mathbf{q}_- - \mathbf{k}_2)^2+\beta_-(-\mathbf{q}_+ + \mathbf{q}_2 - \mathbf{k}_2)^2}\times\right. \\
\times\ln\frac{\beta_-(-\mathbf{q}_+ + \mathbf{q}_2 - \mathbf{k}_2)^2}{\beta_+(\mathbf{q}_- - \mathbf{k}_2)^2}+ \\
+\frac{(\hat{q}_- - \hat{q}_2 + \hat{k}_2)_\perp(-\hat{q}_+ + \hat{k}_2)_\perp}{\beta_+(\mathbf{q}_- - \mathbf{q}_2 + \mathbf{k}_2)^2+\beta_-(-\mathbf{q}_+ + \mathbf{k}_2)^2}\times \\
\left.\times\ln\frac{\beta_-(-\mathbf{q}_+ + \mathbf{k}_2)^2}{\beta_+(\mathbf{q}_- - \mathbf{q}_2 + \mathbf{k}_2)^2}\right].
\end{gathered}
\tag{16}
$$

We note that expressions (16) are purely imaginary, and therefore their interference with the Born term in the cross section is zero.

c) We now consider the case of interactions with different nuclei alternating along the lepton line, for instance, the amplitude R_{1324} (Fig. 4*e*). After some algebra, we obtain the relevant numerator

$$
\begin{gathered}
N_{1324}=s\hat{p}_1\hat{p}_2(\hat{q}_- - \hat{k}_1)_\perp\times \\
\times(\hat{q}_- - \hat{k}_1 - \hat{k}_2)_\perp(\hat{q}_- - \hat{q}_1 - \hat{k}_2)_\perp,
\end{gathered}
\tag{17}
$$

which is very different from the numerators of Born-like amplitudes. Specifically, it is a higher-order term in the running transverse momenta $\mathbf{k}_i$.

The relevant denominators are given by

$$
\begin{gathered}
\{1\}\equiv(q_- - k_1)^2+i0= \\
=s(\beta_- - \beta_1)\alpha_- - (\mathbf{q}_- - \mathbf{k}_1)^2+i0, \\
\{2\}\equiv(q_- - k_1 - k_2)^2+i0= \\
=s(\beta_- - \beta_1)(\alpha_- - \alpha_2)-(\mathbf{q}_- - \mathbf{k}_1 - \mathbf{k}_2)^2+i0, \\
\{3\}\equiv(-q_+ + q_2 - k_2)^2+i0= \\
=s(-\beta_+)(\alpha_- - \alpha_2)-(-\mathbf{q}_+ + \mathbf{q}_2 - \mathbf{k}_2)^2+i0.
\end{gathered}
\tag{18}
$$

The nonvanishing contribution only emerges if the poles are located in different α_2 half-planes, which takes place only if $\beta_1 < \beta_-$ ($\beta_\pm > 0$). Taking the residue at pole $\{2\}$, we find

$$\int \frac{s\,d\alpha_2}{2\pi i}\,\frac{1}{\{2\}\{3\}} = -\frac{\theta(\beta_- - \beta_1)}{(\beta_1 - \beta_-)(-\mathbf{q}_+ + \mathbf{q}_2 - \mathbf{k}_2)^2 - \beta_+(\mathbf{q}_- - \mathbf{k}_1 - \mathbf{k}_2)^2}. \tag{19}$$

Further integration over β_1 can be done using the relation

$$\int\limits_{-\infty}^{\infty} \frac{dx}{2\pi i}\,\frac{\theta(x)}{(ax - b + i0)(cx + d + i0)} = -\frac{1}{2(ad+bc)}\left(1 + \frac{i}{\pi}\ln\frac{ad}{bc}\right), \tag{20}$$

with the result

$$R_{1324} = -\frac{\beta_- N_{1324}}{2sD_{1324}}\left(1 + \frac{i}{\pi}\ln\frac{ad}{bc}\right), \tag{21}$$

$$D_{1324} = \beta_-(\mathbf{q}_- - \mathbf{k}_1)^2(-\mathbf{q}_+ + \mathbf{q}_2 - \mathbf{k}_2)^2 + +\beta_+\mathbf{q}_-^2(\mathbf{q}_- - \mathbf{k}_1 - \mathbf{k}_2)^2 = ad + bc.$$

The highly nonlinear denominator given by Eq. (21) makes the contribution of the considered case dramatically different from the Born amplitude and corrections to it from the higher-order processes in which only one photon is emitted by one of the ions [12]. Technically, the nonlinearity is not surprising because of the related nonlinearity of the numerator. The principal difference from the Born-like amplitude is that with the alternating ordering of interactions, we have the situation in which the p_+ component of the lightcone momentum is conserved in the scattering on one ion but is not conserved in the scattering on the second ion. Depending on the ordering of interaction vertices and the order of integrations, we encounter a sequence of vertices with conservation and nonconservation of the p_--component of the lightcone momentum.

Similar results can be obtained for other contributions of these types.

d) The final result is given by (see Table)

$$M_{(2)}^{(2)} = \frac{is}{(2!)^2}(16\pi\alpha^2 Z_1 Z_2)^2 N_1 N_2 \times \times \int \frac{d^2k_1}{\pi}\,\frac{d^2k_2}{\pi}\,\frac{\bar{u}(q_-)R_{(2)}^{(2)}\hat{p}_2 v(q_+)}{s\mathbf{k}_1^2\mathbf{k}_2^2(\mathbf{q}_1 - \mathbf{k}_1)^2(\mathbf{q}_2 - \mathbf{k}_2)^2}, \tag{22}$$

$$R_{(2)}^{(2)} = \sum_{n=1}^{2}\frac{[\hat{a}_n\hat{b}_n]_\perp}{\beta_-\mathbf{b}_n^2 + \beta_+\mathbf{a}_n^2} - \sum_{n=3}^{10}\frac{[\hat{a}_n\hat{b}_n\hat{c}_n\hat{d}_n]_\perp}{2[\beta_-\mathbf{b}_n^2\mathbf{d}_n^2 + \beta_+\mathbf{a}_n^2\mathbf{c}_n^2]} \times \times\left(1 + i\frac{(-1)^{n+1}}{\pi}\ln\frac{\beta_-\mathbf{b}_n^2\mathbf{d}_n^2}{\beta_+\mathbf{a}_n^2\mathbf{c}_n^2}\right) + + \sum_{n=11}^{12} i\frac{(-1)^{n+1}}{\pi}\,\frac{[\hat{a}_n\hat{b}_n]_\perp}{\beta_-\mathbf{b}_n^2 + \beta_+\mathbf{a}_n^2}\ln\frac{\beta_-\mathbf{b}_n^2}{\beta_+\mathbf{a}_n^2}. \tag{23}$$

To verify gauge invariance, we give the explicit form for the real part of the amplitude:

The coefficients in formula (23). The brackets denote index permutation, e.g., $(12) \equiv 12+21$

n	R_{ijkl}	a_n	b_n	c_n	d_n
1	$R_{(12)(34)}$	q_-	$q_- - q_1$	–	–
2	$R_{(34)(12)}$	$q_1 - q_+$	q_+	–	–
3	R_{1324}	q_-	$q_- - k_1$	$q_- - k_1 - k_2$	$q_- - q_1 - k_2$
4	R_{1423}	q_-	$q_- - k_1$	$q_- - q_2 + k_2 - k_1$	$-q_+ + k_2$
5	R_{2314}	q_-	$q_- - q_1 + k_1$	$q_- - q_1 + k_1 - k_2$	$-q_+ + q_2 - k_2$
6	R_{2413}	q_-	$q_- - q_1 + k_1$	$-q_+ + k_1 + k_2$	$-q_+ + k_2$
7	R_{4231}	$q_- - q_2 + k_2$	$-q_+ + k_1 + k_2$	$-q_+ + k_1$	q_+
8	R_{3241}	$q_- - k_2$	$q_- - q_1 + k_1 - k_2$	$-q_+ + k_1$	q_+
9	R_{4132}	$q_- - q_2 + k_2$	$q_- - q_2 + k_2 - k_1$	$-q_+ + q_1 - k_1$	q_+
10	R_{3142}	$q_- - k_2$	$q_- - k_1 - k_2$	$-q_+ + q_1 - k_1$	q_+
11	$R_{3(12)4}$	$q_- - k_2$	$-q_+ + q_2 - k_2$	–	–
12	$R_{4(12)3}$	$q_- - q_2 + k_2$	$-q_+ + k_2$	–	–

$$
\begin{aligned}
\operatorname{Re} R^{(2)}_{(2)} = & \frac{[\hat q_-(\hat q_- - \hat q_1)]_\perp}{\beta_+\mathbf{q}_-^2 + \beta_-(\mathbf{q}_- - \mathbf{q}_1)^2} + \frac{[(-\hat q_+ + \hat q_1)\hat q_+]_\perp}{\beta_-\mathbf{q}_+^2 + \beta_+(\mathbf{q}_+ - \mathbf{q}_1)^2} - \\
& - \frac{[\hat q_-(\hat q_- - \hat k_1)(\hat q_- - \hat k_1 - \hat k_2)(\hat q_- - \hat q_1 - \hat k_2)]_\perp}{2[\beta_-(\mathbf{q}_- - \mathbf{k}_1)^2(-\mathbf{q}_+ + \mathbf{q}_2 - \mathbf{k}_2)^2 + \beta_+\mathbf{q}_-^2(\mathbf{q}_- - \mathbf{k}_1 - \mathbf{k}_2)^2]} - \\
& - \frac{[\hat q_-(\hat q_- - \hat k_1)(\hat q_- - \hat q_2 + \hat k_2 - \hat k_1)(-\hat q_+ + \hat k_2)]_\perp}{2[\beta_-(\mathbf{q}_- - \mathbf{k}_1)^2(-\mathbf{q}_+ + \mathbf{k}_2)^2 + \beta_+\mathbf{q}_-^2(\mathbf{q}_- - \mathbf{q}_2 + \mathbf{k}_2 - \mathbf{k}_1)^2]} - \\
& - \frac{[\hat q_-(\hat q_- - \hat q_1 + \hat k_1)(\hat q_- - \hat q_1 + \hat k_1 - \hat k_2)(-\hat q_+ + \hat q_2 - \hat k_2)]_\perp}{2[\beta_-(\mathbf{q}_- - \mathbf{q}_1 + \mathbf{k}_1)^2(-\mathbf{q}_+ + \mathbf{q}_2 - \mathbf{k}_2)^2 + \beta_+\mathbf{q}_-^2(\mathbf{q}_- - \mathbf{q}_1 + \mathbf{k}_1 - \mathbf{k}_2)^2]} - \\
& - \frac{[\hat q_-(\hat q_- - \hat q_1 + \hat k_1)(-\hat q_+ + \hat k_1 + \hat k_2)(-\hat q_+ + \hat k_2)]_\perp}{2[\beta_-(\mathbf{q}_- - \mathbf{q}_1 + \mathbf{k}_1)^2(-\mathbf{q}_+ + \mathbf{k}_2)^2 + \beta_+\mathbf{q}_-^2(-\mathbf{q}_+ + \mathbf{k}_1 + \mathbf{k}_2)^2]} - \\
& - \frac{[(\hat q_- - \hat q_2 + \hat k_2)(-\hat q_+ + \hat k_1 + \hat k_2)(-\hat q_+ + \hat k_1)\hat q_+]_\perp}{2[\beta_-\mathbf{q}_+^2(-\mathbf{q}_+ + \mathbf{k}_1 + \mathbf{k}_2)^2 + \beta_+(-\mathbf{q}_+ + \mathbf{k}_1)^2(\mathbf{q}_- - \mathbf{q}_2 + \mathbf{k}_2)^2]} - \\
& - \frac{[(\hat q_- - \hat k_2)(\hat q_- - \hat q_1 + \hat k_1 - \hat k_2)(-\hat q_+ + \hat k_1)\hat q_+]_\perp}{2[\beta_-\mathbf{q}_+^2(\mathbf{q}_- - \mathbf{q}_1 + \mathbf{k}_1 - \mathbf{k}_2)^2 + \beta_+(-\mathbf{q}_+ + \mathbf{k}_1)^2(\mathbf{q}_- - \mathbf{k}_2)^2]} - \\
& - \frac{[(\hat q_- - \hat q_2 + \hat k_2)(\hat q_- - \hat q_2 + \hat k_2 - \hat k_1)(-\hat q_+ + \hat q_1 - \hat k_1)\hat q_+]_\perp}{2[\beta_-\mathbf{q}_+^2(\mathbf{q}_- - \mathbf{q}_2 + \mathbf{k}_2 - \mathbf{k}_1)^2 + \beta_+(\mathbf{q}_- - \mathbf{q}_2 + \mathbf{k}_2)^2(-\mathbf{q}_+ + \mathbf{q}_1 - \mathbf{k}_1)^2]} - \\
& - \frac{[(\hat q_- - \hat k_2)(\hat q_- - \hat k_1 - \hat k_2)(-\hat q_+ + \hat q_1 - \hat k_1)\hat q_+]_\perp}{2[\beta_-\mathbf{q}_+^2(\mathbf{q}_- - \mathbf{k}_1 - \mathbf{k}_2)^2 + \beta_+(-\mathbf{q}_+ + \mathbf{q}_1 - \mathbf{k}_1)^2(\mathbf{q}_- - \mathbf{k}_2)^2]}.
\end{aligned}
$$

We can then verify that the following condition is satisfied:

$$
\operatorname{Re} R^{(2)}_{(2)} = 0 \quad \text{if} \quad \mathbf{k}_1 = 0 \quad \text{or} \quad \mathbf{k}_2 = 0 \quad \text{or} \quad \mathbf{k}_1 = \mathbf{q}_1 \quad \text{or} \quad \mathbf{k}_2 = \mathbf{q}_2. \tag{24}
$$

This fact is also correct for the whole amplitude (23). This property (24) is crucial for the gauge invariance and infrared convergence of integrations over d^2k_i.

In the loop integration, we can shift the integration variable as $\mathbf{k}_i \to \mathbf{q}_i - \mathbf{k}_i$. Then expression (23) for $\operatorname{Re} R^{(2)}_{(2)}$ can be simplified to

$$\operatorname{Re} R_{(2)}^{(2)} = \frac{\hat{q}_{-\perp}(\hat{q}_- - \hat{q}_1)_\perp}{\beta_+\mathbf{q}_-^2 + \beta_-(\mathbf{q}_- - \mathbf{q}_1)^2} + \frac{(-\hat{q}_+ + \hat{q}_1)_\perp \hat{q}_{+\perp}}{\beta_-\mathbf{q}_+^2 + \beta_+(\mathbf{q}_1 - \mathbf{q}_+)^2} - \\ - 2\frac{[\hat{q}_-(\hat{q}_- - \hat{k}_1)(\hat{q}_- - \hat{k}_1 - \hat{k}_2)(\hat{q}_- - \hat{q}_1 - \hat{k}_2)]_\perp}{\beta_-(\mathbf{q}_- - \mathbf{k}_1)^2(-\mathbf{q}_+ + \mathbf{q}_2 - \mathbf{k}_2)^2 + \beta_+\mathbf{q}_-^2(\mathbf{q}_- - \mathbf{k}_1 - \mathbf{k}_2)^2} - \\ - 2\frac{[(-\hat{q}_+ + \hat{q}_1 + \hat{k}_2)(-\hat{q}_+ + \hat{k}_1 + \hat{k}_2)(-\hat{q}_+ + \hat{k}_1)\hat{q}_+]_\perp}{\beta_-\mathbf{q}_+^2(-\mathbf{q}_+ + \mathbf{k}_1 + \mathbf{k}_2)^2 + \beta_+(-\mathbf{q}_+ + \mathbf{k}_1)^2(\mathbf{q}_- - \mathbf{q}_2 + \mathbf{k}_2)^2}. \quad (25)$$

Although the gauge invariance property is not manifested here, as in the previous case, the final results after integration over k_i coincide.

3. THE WIDE-ANGLE LIMIT OF THE $M_{(2)}^{(2)}$ AMPLITUDE

We consider the behavior of expression (25) in the case where the transverse component of lepton momenta is large compared to the momenta transferred to the ions,

$$\mathbf{q}_- \approx -\mathbf{q}_+ = \mathbf{q}, \quad |\mathbf{q}| \gg |\mathbf{q}_{1,2}|. \quad (26)$$

The main contribution to the matrix element is then given by the region

$$|\mathbf{q}_i| \ll |\mathbf{k}_i| \ll |\mathbf{q}|. \quad (27)$$

The amplitude $M_{(1)}^{(1)}$ is

$$M_{(1)}^{(1)} = -is\frac{(8\pi\alpha)^2 N_1 N_2 Z_1 Z_2}{\mathbf{q}_1^2\mathbf{q}_2^2}\bar{u}(q_-)\frac{R_{(1)}^{(1)}}{s}v(q_+),$$
$$R_{(1)}^{(1)} = \hat{p}_1\frac{\hat{q}_- - \hat{q}_1}{(q_- - q_1)^2}\hat{p}_2 + \hat{p}_2\frac{\hat{q}_1 - \hat{q}_+}{(q_1 - q_+)^2}\hat{p}_1 = \\ = (B - \tilde{B})\hat{p}_2. \quad (28)$$

For wide-angle kinematics, we have

$$\frac{1}{s}R_{(1)}^{(1)} = \frac{\hat{p}_2}{s}\frac{1}{b_1^2(\mathbf{q}^2)^2}\left[2\mathbf{q}\cdot\mathbf{q}_2(b_1\hat{q}\hat{q}_1 + 2\beta_-\mathbf{q}\cdot\mathbf{q}_1) + \right. \\ \left. + \mathbf{q}^2(b_1\hat{q}_1\hat{q}_2 + 2\beta_+\mathbf{q}_1\cdot\mathbf{q}_2)\right], \quad (29)$$

where $b_1 = \beta_- + \beta_+$, $\mathbf{q} = \mathbf{q}_- \approx -\mathbf{q}_+$, and $\mathbf{q}_{1,2}$ are the momenta transferred to ions.

For the matrix element $M_{(2)}^{(1)}$ we have (in agreement with the result obtained in paper [16])

$$M_{(2)}^{(1)} = -s\frac{2^7\pi^2\alpha^3 Z_1 Z_2^2 N_1 N_2}{\mathbf{q}_1^2} \times \\ \times \int\frac{d^2k_2}{\pi}\frac{\bar{u}(q_-)R_{(2)}^{(1)}\hat{p}_2 v(q_+)}{s\mathbf{k}_2^2(\mathbf{q}_2 - \mathbf{k}_2)^2}, \quad (30)$$

where

$$R_{(2)}^{(1)} = B + \tilde{B} - \\ - \frac{(\hat{q}_- - \hat{k}_2)_\perp(\hat{q}_- - \hat{q}_1 - \hat{k}_2)_\perp}{\beta_-(\mathbf{q}_- - \mathbf{q}_1 - \mathbf{k}_2)^2 + \beta_+(\mathbf{q}_- - \mathbf{k}_2)^2} - \\ - \frac{(\hat{q}_+ - \hat{k}_2 - \hat{q}_1)_\perp(\hat{q}_+ - \hat{k}_2)_\perp}{\beta_+(\mathbf{q}_- - \mathbf{q}_2 + \mathbf{k}_2)^2 + \beta_-(\mathbf{q}_+ - \mathbf{k}_2)^2}. \quad (31)$$

In the considered limit, this expression becomes

$$R_{(2)}^{(1)} \sim \frac{1}{b_1\mathbf{q}^2}\Big[(2\beta_-\mathbf{q}_-\cdot\mathbf{q}_1 + \hat{q}_-\hat{q}_1) \times \\ \times \left(\frac{4(\mathbf{q}_-\cdot\mathbf{k}_2)^2}{(\mathbf{q}^2)^2} - \frac{\mathbf{k}_2^2}{\mathbf{q}^2}\right) - \\ - \frac{2\mathbf{q}_-\cdot\mathbf{k}_2}{\mathbf{q}^2}(\hat{k}_2\hat{q}_1 + 2\beta_-\mathbf{k}_2\cdot\mathbf{q}_1)\Big] + (\beta_- \to \beta_+), \\ |\mathbf{k}_2| \gg |\mathbf{q}_2|. \quad (32)$$

This expression vanishes after angular averaging. It can be shown that the quantity $M_{(3)}^{(1)}$ also vanishes in the limit of wide-angle pair production and is proportional to $|\mathbf{q}_2|/|\mathbf{q}| \ll 1$, which is in agreement with [3].

For the amplitude $M_{(2)}^{(2)}$ considered in Eq. (22), the quantity $R_{(2)}^{(2)}$ plays the role of a cut-off parameter in the region $|\mathbf{k}_i| > |\mathbf{q}|$. From very general arguments, it can be written in the form

$$\operatorname{Re} R_{(2)}^{(2)} \approx \frac{[k_1^\mu(q_1 - k_1)^\nu k_2^\alpha(q_2 - k_2)^\beta]_\perp}{(\mathbf{q}^2)^2}R_{\mu\nu\alpha\beta}, \quad (33)$$

with some dimensionless tensor matrix $R_{\mu\nu\alpha\beta}$ independent of $\mathbf{k}_i$ and $\mathbf{q}_i$. Expanding expression (25), we obtain

$$\int\frac{d^2k_1 d^2k_2}{\pi^2}\frac{\operatorname{Re} R_{(2)}^{(2)}}{\mathbf{k}_1^2\mathbf{k}_2^2(\mathbf{q}_1 - \mathbf{k}_1)^2 q(\mathbf{q}_2 - \mathbf{k}_2)^2} \approx \\ \approx \frac{I}{(\mathbf{q}^2)^2}\frac{4(\beta_+ - \beta_-)}{(\beta_- + \beta_+)^2}\ln\frac{\mathbf{q}_{max}^2}{\mathbf{q}_1^2}\ln\frac{\mathbf{q}_{max}^2}{\mathbf{q}_2^2}, \quad (34)$$

where I is the unit matrix and $q_{max} \approx 1/R$ is the upper integration limit, with R being the nucleus radius. Such a logarithmic enhancement is absent if the number of the exchanged photons from each ion exceeds two (Fig. 5). In fact, the amplitudes $M_{(n)}^{(2)}$, $M_{(2)}^{(n)}$, $n > 2$ contain only the first power of the large logarithm, whereas $M_{(n)}^{(m)}$, $m, n > 2$ do not contain such a factor at all because the corresponding loop momenta integrals are convergent in both infrared and ultraviolet regions and we can safely put $|\mathbf{q}_{1(2)}| = 0$ in loop integrations.

Thus, the differential cross section for the considered kinematics is determined by the interference term $\left(M_{(1)}^{(1)}\right)^* M_{(2)}^{(2)}$, which has the form (for comparison, we also present the Born term)

$$\frac{d\sigma_0}{db_1 dx} = \frac{16(Z_1 Z_2 \alpha^2)^2}{\pi^4} \times \frac{x^2+(1-x)^2}{\mathbf{q}_1^2 \mathbf{q}_2^2 (\mathbf{q}^2)^2 b_1} d^2 q_1 d^2 q_2 d^2 q, \quad (35)$$

$$\frac{d\sigma_{int}}{db_1 dx} = \frac{16(Z_1 Z_2 \alpha^2)^3}{\mathbf{q}_1^2 \mathbf{q}_2^2 \mathbf{q}_+^2 \mathbf{q}_-^2} \frac{1-2x}{b_1} \times \ln \frac{\mathbf{q}_{max}^2}{\mathbf{q}_1^2} \ln \frac{\mathbf{q}_{max}^2}{\mathbf{q}_2^2} Q \, d^2 q_1 d^2 q_2 d^2 q_-, \quad (36)$$

where

$$Q = \frac{\mathbf{q}_- \cdot (\mathbf{q}_1 - \mathbf{q}_-)}{(1-x)\mathbf{q}_-^2 + x(\mathbf{q}_- - \mathbf{q}_1)^2} + \frac{\mathbf{q}_+ \cdot (\mathbf{q}_+ - \mathbf{q}_1)}{x\mathbf{q}_+^2 + (1-x)(\mathbf{q}_1 - \mathbf{q}_+)^2},$$

$$x = \frac{\beta_-}{b_1}, \quad \epsilon < x, \quad b_1 < 1-\epsilon, \quad \epsilon = \frac{4m^2 x(1-x)}{\mathbf{q}_\pm^2}.$$

We note that expression (36) is symmetric under simultaneous substitutions $\mathbf{q}_+ \leftrightarrow \mathbf{q}_-$ and $\beta_+ \leftrightarrow \beta_-$ due to the C-even nature of the interference.

Finally, from a very straightforward generalization of (33), it can be shown that the set of amplitudes with an odd number of exchanges with one or both nuclei is suppressed in the limit of wide-angle production:

$$M_{(2n+1)}^{(2m)} \sim O\Big(\frac{|\mathbf{q}_1|}{|\mathbf{q}|}\Big), \quad M_{(2n)}^{(2m+1)} \sim O\Big(\frac{|\mathbf{q}_2|}{|\mathbf{q}|}\Big), \quad M_{(2n+1)}^{(2m+1)} \sim O\Big(\frac{|\mathbf{q}_1||\mathbf{q}_2|}{|\mathbf{q}^2|}\Big). \quad (37)$$

4. MULTIPHOTON EXCHANGE

We generalize the above picture to the case of multiple photon exchanges ($m, n > 2$). Using the relation

$$I_n = \frac{1}{\pi^{n-1}} \times \int \frac{d^2 k_1 \ldots d^2 k_{n-1}}{(\mathbf{k}_1^2+\lambda^2)\ldots(\mathbf{k}_{n-1}^2+\lambda^2)[(\mathbf{q}-\mathbf{k}_1-\ldots-\mathbf{k}_{n-1})^2+\lambda^2]} = \frac{n \ln^{n-1}(\mathbf{q}^2/\lambda^2)}{\mathbf{q}^2} \quad (38)$$

and taking the combinatorial factor $1/n!$ coming from the symmetric integration over α_i and β_i into account, we have to replace any single photon exchange with an infinite set of photons by multiplying the amplitude by eikonal factors of the type $\exp\{i\varphi_i(\mathbf{q}^2)\}$ with the phase $\varphi_i(\mathbf{q}^2) = \pm\alpha Z_i \ln(\mathbf{q}^2/\lambda^2)$. The scattering amplitudes of an electron and a positron differ only by the sign of the phase (which is positive for electrons) [9]. This replacement is shown in Fig. 6, where the double photon line corresponds to infinitely many photons.

Using the same technique as in [17], we can see that the amplitude relevant to Fig. 7a and Fig. 7b can be written as

$$\tilde{R}_{(1)}^{(1)} = B \exp\left\{-i[\varphi_1(\mathbf{q}_1^2) - \varphi_2(\mathbf{q}_2^2)]\right\} + \tilde{B} \exp\left\{i[\varphi_1(\mathbf{q}_1^2) - \varphi_2(\mathbf{q}_2^2)]\right\}. \quad (39)$$

The interactions of the electron and the positron with the Coulomb field differ only by signs. Although this expression is infrared-unstable in the case where $Z_1 \neq Z_2$, the regularization parameter λ enters it in a standard way.

We now consider the class of diagrams shown in Fig. 7c. In subsection 2.2, we obtained expressions (16) in the case where $m = n = 2$, with $\mathrm{Re}\, R_{1(34)2} = 0$. It can be shown that higher-order terms with any even number of photons from the same nucleus attached to the lepton world line between two photons from other nuclei do not contribute to the amplitude of the process under consideration. This follows from the relation $(\mathrm{sign}\,\alpha)^{2k+1} = \mathrm{sign}\,\alpha$.

The general structure of the amplitude corresponding to Fig. 7c can be constructed using the lowest-order truncated amplitude (without single-photon propagators) $R_{(2)}^{(1)}$,

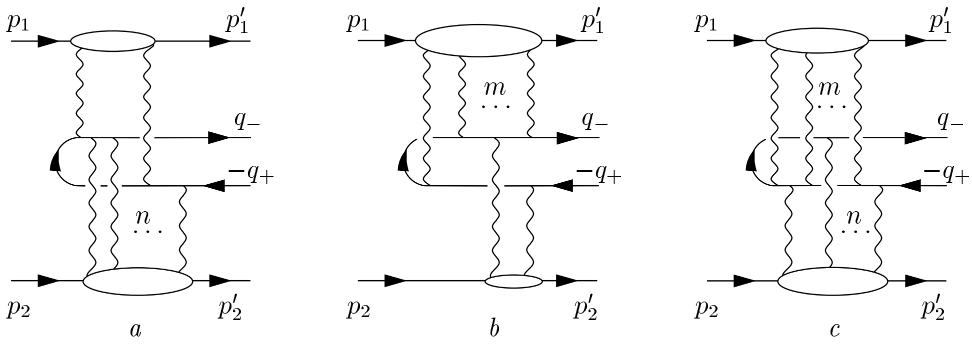

Fig. 5. Some Feynman diagrams for amplitudes of the type $M^{(2)}_{(n)}$ (a), $M^{(n)}_{(2)}$ (b), and $M^{(m)}_{(n)}$ (c) with $m, n \geq 2$

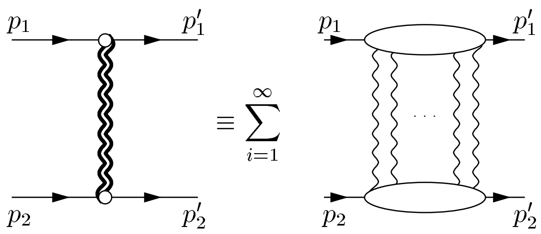

Fig. 6. The representation of all eikonal exchanges

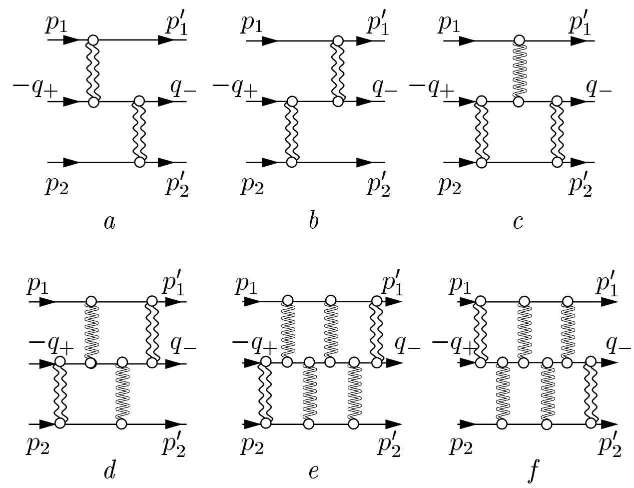

Fig. 7. The Feynman diagrams for the amplitudes with many photon exchanges. The double photon line represents any number of exchanged photons, the double zigzag line represents only an odd number of exchanged photons

$$\tilde{R}^{(1)}_{(2)} = \frac{\cos(\varphi_1(\mathbf{q}_1^2))}{q_1^2} R^{(1)}_{(2)} \times \exp\left\{i[\varphi_2(\mathbf{k}^2) - \varphi_2((\mathbf{q}_2 - \mathbf{k})^2)]\right\},$$

$$R^{(1)}_{(2)} = \frac{1}{i\pi}\,\frac{(\hat{q}_- - \hat{q}_2 + \hat{k})_\perp(-\hat{q}_+ + \hat{k})_\perp}{\beta_-(\mathbf{q}_+ - \mathbf{k})^2 + \beta_+(\mathbf{q}_- - \mathbf{q}_2 + \mathbf{k})^2} \times \ln\frac{\beta_+(\mathbf{q}_- - \mathbf{q}_2 + \mathbf{k})^2}{\beta_-(\mathbf{q}_+ - \mathbf{k})^2}. \qquad (40)$$

The further generalization is obvious. For instance, we give the expression corresponding to the diagram in Fig. 7d,

$$\tilde{R}^{(2)}_{(2)} = \cos(\varphi_1(\mathbf{k}_1^2)) \exp\left[-i\varphi_1((\mathbf{q}_1 - \mathbf{k}_1)^2)\right] \times \cos(\varphi_2(\mathbf{k}_2^2)) \exp\left[i\varphi_2((\mathbf{q}_2 - \mathbf{k}_2)^2)\right] R_{1324}. \qquad (41)$$

From the above consideration, we conclude that the general structure of the matrix element $M^{(m)}_{(n)}$ corresponding to m photon exchanges from one ion (with 4-momenta k_i) and n exchanges from the other (with 4-momenta κ_i) can be schematically written as

$$M^{(m)}_{(n)} = isN_1N_2(Z_1\alpha)^m(Z_2\alpha)^n\,\frac{\pi^2}{16n!m!} \times \int \frac{d^2k_1}{\pi}\cdots\frac{d^2k_{m-1}}{\pi}\,\frac{d^2\kappa_1}{\pi}\cdots\frac{d^2\kappa_{n-1}}{\pi}\,\frac{1}{\mathbf{k}_1^2\ldots\mathbf{k}_m^2} \times \frac{1}{\boldsymbol{\kappa}_1^2\ldots\boldsymbol{\kappa}_n^2}\bar{u}(q_-)\bar{R}^{(m)}_{(n)}\frac{\hat{p}_2}{s}\,v(q_+), \qquad (42)$$

where m and n satisfy the condition $|m - n| \leq 1$. At this stage, we omit phase factors in the structure $R^{(m)}_{(n)}$ (in order to understand the problem clearly), and it can therefore be written as

$$\bar{R}^{(m)}_{(n)} = \bar{R}^{(1)}_{(1)} + \bar{R}^{(1)}_{(2)} + \bar{R}^{(2)}_{(1)} + \bar{R}^{(2)}_{(2)} + \bar{R}^{(2)}_{(3)} + \bar{R}^{(3)}_{(2)} + \bar{R}^{(3)R}_{(3)} + \bar{R}^{(3)L}_{(3)} \ldots, \qquad (43)$$

where

$$\bar{R}^{(2)}_{(1)} = \frac{1}{i\pi}\,\frac{(\hat{q}_- - \hat{k})_\perp(-\hat{q}_+ + \hat{q}_1 - \hat{k})_\perp}{\alpha_-(-\mathbf{q}_+ + \mathbf{q}_1 - \mathbf{k})^2 + \alpha_+(\mathbf{q}_- - \mathbf{k})^2} \times \times \ln\frac{\alpha_+(\mathbf{q}_- - \mathbf{k})^2}{\alpha_-(-\mathbf{q}_+ + \mathbf{q}_1 - \mathbf{k})^2},$$

$$\bar{R}^{(2)}_{(3)} = \bar{R}^{(3)}_{(2)} = 0,$$

$$\bar{R}^{(3)R}_{(3)} = \frac{1}{c_1 + c_2}\left[\frac{\pi^2}{2} + \frac{1}{2}\ln^2\frac{c_1}{c_2}\right],$$

$$c_1 = \beta_-(\mathbf{q}_- - \mathbf{k}_1)^2(\mathbf{q}_- - \mathbf{k}_1 - \mathbf{k}_2 - \boldsymbol{\kappa}_1)^2 \times \times (-\mathbf{q}_+ + \mathbf{q}_2 - \boldsymbol{\kappa}_1 - \boldsymbol{\kappa}_2)^2,$$

$$c_2 = \beta_+\mathbf{q}_-^2(\mathbf{q}_- - \mathbf{k}_1 - \boldsymbol{\kappa}_1)^2 \times \times (\mathbf{q}_- - \mathbf{k}_1 - \mathbf{k}_2 - \boldsymbol{\kappa}_1 - \boldsymbol{\kappa}_2)^2,$$

$$\bar{R}^{(3)}_{(4)} = \frac{1}{d_1 + d_2}\left[\frac{\pi^2}{2} + \frac{1}{2}\ln^2\frac{d_1}{d_2}\right],$$

$$d_1 = \beta_+(\mathbf{q}_- - \boldsymbol{\kappa}_1)^2(\mathbf{q}_- - \boldsymbol{\kappa}_1 - \boldsymbol{\kappa}_2 - \mathbf{k}_1)^2 \times \times (\mathbf{q}_- - \boldsymbol{\kappa}_1 - \boldsymbol{\kappa}_2 - \boldsymbol{\kappa}_3 - \mathbf{k}_1 - \mathbf{k}_2)^2,$$

$$d_2 = \beta_-(\mathbf{q}_- - \boldsymbol{\kappa}_1 - \mathbf{k}_1)^2 \times \times (\mathbf{q}_- - \boldsymbol{\kappa}_1 - \boldsymbol{\kappa}_2 - \mathbf{k}_1 - \mathbf{k}_2)^2 \times \times (-\mathbf{q}_+ + \mathbf{q}_2 - \boldsymbol{\kappa}_1 - \boldsymbol{\kappa}_2 - \boldsymbol{\kappa}_3)^2. \quad (44)$$

Here, $\bar{R}^{(2)}_{(2)}$ is only the second term in the right-hand side in Eq. (23) and the index $R(L)$ denotes two possible configurations of photons for $\bar{R}^{(3)R}_{(3)}$ (Fig. 7*e*) and $\bar{R}^{(3)L}_{(3)}$ (Fig. 7*f*).

Thus, the general algorithm for constructing an arbitrary term is transparent. Unfortunately, we cannot obtain a compact expression for the whole amplitude. The reason is the increasing nonlinearity of the propagators with the order of interaction. The behavior of the above denominators is very different from the Born-like case, where the simplicity of propagators allows one to obtain eikonal-like expressions.

The result of partial summation like (41) suffers from infrared divergences and cannot be considered final. On the other hand, the final result (44) implies the summation over the classes $R^{(m)}_{(n)}$ of FD and must contain all the dependence on the «photon mass» λ in the form of a general phase factor, proving the infrared stability of the cross section. We believe that this question will be the subject of a separate investigation.

5. CONCLUSIONS

The wide-angle lepton pair production in peripheral interactions of ultrarelativistic heavy ions is an archetype reaction for hard processes in central hadronic hard collisions of heavy nuclei. In the electromagnetic case, the expansion parameter $Z_{1,2}\,\alpha \sim 1$ makes the multiple photon collisions $m\gamma + n\gamma \to l^+l^-$ potentially important, and similarly, the effect of multiple gluon collisions in central collisions is enhanced by a large number of nucleons at the same impact parameter. The crucial issue is whether such multiple photon collisions can be described by the Born cross section in terms of the collective photon fields of colliding nuclei. We have obtained the expression for the amplitude for the $2\gamma + 2\gamma \to l^+l^-$ process and have shown that its contribution is dominant in the wide-angle limit. Our principal finding is that the amplitude is manifestly of a non-Born nature, which is suggestive of the complete failure of the linear $k_\perp$-factorization even in the Abelian case.

The leading term of the multiphoton collision contribution to the amplitude of the production of high transverse momentum leptons, $2\gamma + 2\gamma \to l^+l^-$, is found to have a logarithmic enhancement, while such an enhancement is absent in higher-order terms. We presented the algorithm that allows constructing the full amplitude in all orders. The obtained results can be useful in application to the QCD process of production of high-$k_\perp$ jets as well as the bound state creation (positronium, charmonium), the issue which will be investigated elsewhere.

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REFERENCES

1. H. Bethe and L. Maximon, Phys. Rev. **93**, 768 (1954); H. Davies, H. Bethe, and L. Maximon, ibid **93**, 788 (1954).
2. L. D. Landau and E. M. Lifshitz, *Quantum Mechanics*, Nauka, Moscow (1989).
3. D. Yu. Ivanov and K. Melnikov, Phys. Rev. D **57**, 4025 (1998).
4. S. R. Gevorkyan and S. S. Grigoryan, Phys. Rev. A **65**, 022505 (2002).
5. N. N. Nikolaev, W. Schafer, B. G. Zakharov, and V. R. Zoller, Pis'ma v Zh. Eksp. Teor. Fiz. [JETP Lett. **97**, 441 (2003)].

6. N. N. Nikolaev, W. Schafer, B. G. Zakharov, and V. R. Zoller, E-print Archive, hep-ph/0408054.

7. A. J. Baltz and L. McLerran, Phys. Rev. C **58**, (1998).

8. B. Segev and J. C. Wells, Phys. Rev. A **57**, 1849 (1998).

9. U. Eichmann, J. Reinhardt, and W. Greiner, Phys. Rev. A **61**, 062710 (2000).

10. D. Yu. Ivanov, A. Schiller, and V. G. Serbo, Phys. Lett. B **457**, 155 (1999).

11. R. N. Lee and A. I. Milstein, Phys. Rev. A **61**, 032103 (2000).

12. E. Bartoš, S. R. Gevorkyan, N. N. Nikolaev, and E. A. Kuraev, Phys. Rev. A **66**, 042720 (2002).

13. E. Bartoš, S. R. Gevorkyan, N. N. Nikolaev, and E. A. Kuraev, Phys. Lett. B **538**, 45 (2002).

14. S. R. Gevorkyan and E. A. Kuraev, J. Phys. G **29**, 1227 (2003).

15. V. N. Gribov, L. N. Lipatov, and G. V. Frolov, Sov. J. Nucl. Phys. **12**, 543 (1971) [Yad. Fiz. **12**, 994 (1970)].

16. I. F. Ginzburg, S. L. Panfil, and V. G. Serbo, Nucl. Phys. B **284**, 685 (1987).

17. E. Bartoš, S. R. Gevorkyan, and E. A. Kuraev, Yad. Phys. **67**, 1923 (2004).