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ON THE NEED FOR PHENOMENOLOGICAL THEORY OF P-VORTICES, OR DOES SPAGHETTI CONFINEMENT PATTERN ADMIT CONDENSED-MATTER ANALOGUES?

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The intuition from condensed matter physics is commonly used to provide the ideas for possible confinement mechanisms in gauge theories. Today, with a clear but puzzling «spaghetti» confinement pattern arising from a decade of lattice computer experiments and implying formation of a fluctuating net of peculiar magnetic vortices rather than condensation of homogeneously distributed magnetic monopoles, the time is coming to reverse the logic and search for similar patterns in condensed matter systems. The main effect to be sought in a condensed matter setup is the simultaneous existence of narrow tubes (P-vortices or 1-branes) of the direction-changing electric field and broader tubes (Abrikosov lines) of the magnetic field, a pattern dual to the one presumably underlying the confinement in gluodynamics. As one possible place for this search, we suggest the systems with coexisting charge-density waves and superconductivity.

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1. INTRODUCTION

A possible resolution of the confinement problem [1–12] should answer questions at two related but somewhat different levels[1)].

(i) It should allow a reliable evaluation of various quantities, such as a gap in the spectrum of perturbations around the true vacuum, the string tensions in the area laws for the Wilson loops in different representations, as well as the masses of glueballs and other hadrons (when light quarks are taken into consideration).

(ii) It should provide a simple qualitative «picture» of how the vacuum is formed, how the linear potential arises between remote sources with nonvanishing N-alities in the absence of light quarks, and how the massive colorless hadrons are formed in the absence as well as in the presence of light quarks.

Of principal importance for development of theoretical (not computer-experimental) quantitative methods at level (i) would be identification of the true vacuum $|vac\rangle$ — a functional of fields at a given moment of time, which is the lowest eigenstate of the nonpertur-

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1) We discuss confinement as a pure gluodynamical problem and ignore all issues related to fermion condensates and chiral symmetry breaking. In the real-world QCD, the effects related to light quarks can be more important for a large part of hadron physics and even the dominant confinement mechanism may be different [9]. Therefore, in the study of confinement in gluodynamics, one should rely more upon computer than accelerator experiments.

We also do not dwell upon the promising «holistic» approaches to confinement, exploiting various general properties of gluodynamics [10] or building one or another kind of self-consistent approximation to correlation functions [11, 12]. Instead, we discuss the lattice experiment results providing a microscopic description of relevant field configurations and their common properties and address the question of whether this mysterious pattern has ever been observed in other types of physical systems.

bative Yang – Mills Hamiltonian, — with all the other eigenstates presumably separated from $|vac\rangle$ by a nonvanishing gap.

The relevant approach to (ii) would rather identify a relatively small subspace in the space of all field configurations (labeled by a sort of collective coordinates) and substitute the original problem of Yang – Mills dynamics by that of a more or less familiar medium, the QCD aether (like a gas of monopoles or P-vortices, a dual superconductor or something else). The underlying belief here is that the original functional integral at low energies receives a dominant contribution from a restricted set of field configurations, and can therefore be substituted by some more familiar effective theory, describing (at least qualitatively) the low-energy quantities as averages over this auxiliary medium and expressing the problems of low-energy quantum Yang – Mills theory through those of the medium dynamics.

Understanding of confinement requires certain achievements at both levels (i) and (ii): the existence of a «picture» is what distinguishes «understanding» from just «calculability», while the possibility to make calculations or at least estimates is a criterion for selection of the correct «picture» among the alternative ones.

The problem of confinement consists of two parts: one should explain why

(α) all gauge fields are screened (i.e., all gluons, electric and magnetic, acquire effective masses $\sim \Lambda_{QCD}$) and

(β) there nevertheless exists a peculiar long-range color-electric interaction described by a narrow tube where electric force lines (carrying a flux with nonvanishing N-ality, i.e., in the representation that cannot be obtained in a product of adjoints, such that the tube is stable against string-breaking caused by creation of a set of gluons) are collimated and give rise to the linear interaction potential $V(R) \sim \sigma R$ at $R \gg \Lambda_{QCD}^{-1}$, with the string tension $\sigma \sim \Lambda_{QCD}^2$ and the string width $r_e \sim \Lambda_{QCD}^{-1} \log(R\Lambda_{QCD})$.

We call this double-face situation the dual Meissner – Abrikosov (MA) effect.

The spaghetti vacuum pattern [6], to be discussed below, implies that, in addition to (α) and (β),

(γ) one more long-range interaction survives, described by a very narrow tube (P-vortex or 1-brane), with collimated color-magnetic force lines, populated by 0-branes, looking in certain aspects like magnetic monopoles and antimonopoles, with the direction of the magnetic field reversed at the locations of the 0-branes,

(δ) the P-vortices can merge and split, they form a dense net percolating through the whole volume.

Thus, in some sense, the dual MA effect is complemented by a kind of the ordinary MA effect, although the magnetic Abrikosov tubes carry an essential additional structure (moreover, as we discuss below, the oversimplified description of this structure given in (γ) is not gauge-invariant and hence is not fully adequate).

2. SCREENING IN THE ABELIAN THEORY

It is well known that the MA effect per se does not require a non-Abelian gauge theory for its manifestation. It can already be discussed at the Abelian level.

There are many ways to obtain one or another kind of the screening effect (α), and many of them allow one or another kind of long-range interactions to survive.

Massive photon. Complete screening with no long-range interactions is described by the effective Lagrangian of the type

$$\frac{1}{e^2}F_{\mu\nu}^2 + m^2 A_\mu^2. \tag{1}$$

It explicitly breaks gauge invariance and contains nonpropagating degrees of freedom A_0, giving rise to an instantaneous, but still screened, interaction.

Debye screening. It occurs in ordinary conductors, electrolytes, and some phases of plasma and is described by the effective Largangian

$$\frac{1}{e^2}F_{\mu\nu}^2 - E_i \frac{m^2}{\partial^2} E_i. \tag{2}$$

It explicitly breaks the Lorentz invariance and completely screens static electric fields, while magnetic and time-oscillating electric fields remain long-range. The massive term is usually produced by the process shown

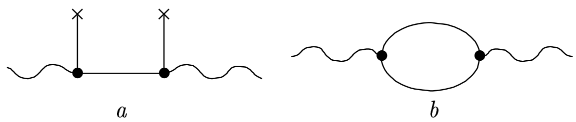

Fig. 1. The origin of the gauge field mass in the Debye screening mechanism. a) The case where charged paticles are originally in the medium. The entire diagram is proportional to the concentration n_0 of these particles in the medium. For nonvanishing temperatures (unavoidable in any lattice calculations), n_0 is never zero (but can be exponentially small). b) The case where the charged pairs are greated in the medium (including the physical vacuum) by the gauge field itself. In this case, the screening is usually much softer and can result in a slow running of the coupling constant rather than in exponential screening

in Fig. 1, with m^2 being proportinal to the concentration n_0 of the electric charges in the medium. If these charges are not originally present, then $m^2 \propto n_0$ either due to nonvanishing temperature, or, if the temperature is zero, to the probability of charge–anticharge creation by an imposed external electric field. This probability, and hence m^2, normally contains extra powers of 4-momenta, such that the screening mechanism becomes essentially softened and leads, for example, to the slow running coupling phenomenon in QED and QCD, described (in these Lorentz-invariant cases) by the effective Lagrangian

$$F_{\mu\nu}\frac{1}{e^2(\Delta)}F_{\mu\nu}. \tag{3}$$

In $3+1$ dimensions, the Δ-dependence is just logarithmic, at least in the leading approximation, and hence no real screening occurs, gauge fields remain massless. In non-Abelian theories, magnetic interactions also enter the game, producing the anti-screening effect in (3), overweighting the screening one [13]. It is not quite clear whether just this anti-screening could lead to the confinement effect beyond the leading-logarithm approximation (see, e.g., [11]).

To be more precise, in realistic systems, the effective Lagrangian (in the case of a linear response, i.e., weak fields) is expressed in terms of the dielectric constant[2)]

$$\epsilon_{ij} \equiv \left(\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2}\right)\epsilon_\perp(\omega,\mathbf{k}) + \frac{k_i k_j}{\mathbf{k}^2}\epsilon_\parallel(\omega,\mathbf{k}),$$

$$\mathcal{L} = \frac{1}{e^2}\left[F_{\mu\nu}^2 + (\epsilon_\perp - 1)\mathbf{E}^2 + (\epsilon_\perp + \epsilon_\parallel) \times \right.$$
$$\left. \times \left(\operatorname{div}\mathbf{E}\frac{1}{\boldsymbol{\partial}^2}\operatorname{div}\mathbf{E}\right)\right], \tag{4}$$

and is not universal, because the frequency and momentum dependence of $\epsilon_\parallel$ and $\epsilon_\perp$ can be very different in different regimes. Important for the Debye screening (long-distance exponential decay of the field correlator) is the presence of a singularity in the longitudinal dielectric constant at large distances (small $\mathbf{k}^2$) [15]:

$$\epsilon_\parallel = 1 + \frac{e^2 m^2}{\mathbf{k}^2} + O(\omega),$$

where the omitted terms describe a highly nontrivial frequency dependence. Indeed, the static correlator is

$$\langle E_i E_j\rangle \sim \frac{k_i k_j}{\epsilon_\parallel \mathbf{k}^2} = \frac{k_i k_j}{\mathbf{k}^2 + P_{00}}, \tag{5}$$

where $P_{00} = (\epsilon_\parallel - 1)\mathbf{k}^2$ is the static value of the component of the photon polarization operator $P_{\mu\nu}$ (the «electric» mass [16]).

Dual Debye screening. It is described by a dual effective Largangian of the type

$$\frac{1}{e^2}F_{\mu\nu}^2 + H_i\frac{m^2}{\partial^2}H_i \tag{6}$$

and imply screening of static magnetic fields. It is unclear whether any condensed-matter systems with this type of behavior have already been discovered. In ordinary electrodynamics without magnetic charges, we have a counterpart of (5),

$$\langle H_i H_j\rangle = \frac{\mathbf{k}^2\delta_{ij} - k_i k_j}{\mathbf{k}^2 + P}, \tag{7}$$

where the «magnetic» mass P is given by the static value of the spatial components of the photon polarization operator ($P_{ij} \stackrel{\omega=0}{=} \left(\delta_{ij} - k_i k_j/\mathbf{k}^2\right) P$ due to the gauge invariance). In a gas of magnetic monopoles, it becomes (see Polyakov's book in [1])

$$\langle H_i H_j\rangle = \delta_{ij} - \frac{k_i k_j}{k^2 + M^2}. \tag{8}$$

Chern–Simons screening. It is described by the peculiar gauge-invariant Lagrangian,

$$\frac{1}{e^2}F_{\mu\nu}^2 + m^{\alpha\ldots\beta}\epsilon_{\mu\nu\lambda\alpha\ldots\beta}A_\lambda F_{\mu\nu}. \tag{9}$$

It describes aspects of the Hall effect and related phenomena, is Lorentz invariant (m is a scalar) only in 2+1 dimensions, and — only in this dimension — makes the photon massive, but still the long-range Aharonov–Bohm interaction survives [17].

Abelian Higgs model. The ordinary (not the dual) Meissner–Abrikosov effect is modeled by the Abelian Higgs (Landau–Ginzburg) effective Lagrangian

$$\frac{1}{e^2}F_{\mu\nu}^2 + |D_\mu\phi|^2 + \lambda(|\phi|^2 - m^2)^2. \tag{10}$$

After ϕ condenses, $\langle\phi\rangle = me^{i\theta}$, the gauge fields become massive, thus giving rise to effect (α): the Meissner effect for magnetic and electric fields. However, the mass is actually acquired not by the A_μ field but by the gauge-invariant combination $\hat{A}_\mu = A_\mu - \partial_\mu\theta$, and

2) We note that the formulation in terms of the dielectric constant and magnetic permeability μ can be useful in the search for solid-state counterparts of the confinement phenomenon (see, e.g., [14]): the electric confinement (similarly to that in QCD) can be described by $\epsilon = 0$, while the magnetic confinement (similarly to the Meissner effect in superconductors) is attributed to $\mu = 0$.

hence the mode $\hat{A}_\mu = 0$ can still propagate through large distances, which explains effect (β): emergence of Abrikosov tubes. $\hat{A}_\mu = 0$ does not imply that $A_\mu = \partial_\mu \theta$ is a pure gauge if $\theta(x)$ is singular and $\oint_C A_\mu \, dx^\mu \neq 0$ for some contours C. In an Abrikosov tube stretched along the z axis, $\theta = \mathrm{arctg}(y/x)$ is the angle in the xy plane and C is any contour in this plane encircling the origin. Because θ is the phase of the smooth field ϕ, the modulus $|\phi|$ should vanish on the z axis, where θ is not well defined, i.e., the condition $|\langle\phi\rangle| = m$ is destroyed in the vicinity of z axis, in a tube with the cross section $\Sigma = \pi r_m^2$. This leads to the energy $\lambda m^4 \Sigma$ per unit length of the tube, while the energy of the magnetic flux Φ in the tube is $\sim (\Phi/\Sigma)^2 \Sigma = \Phi^2/\Sigma$. Minimization of the sum of these terms with respect to Σ defines the characteristic width of the tube

$$\Sigma_m = \pi r_m^2 \sim \frac{\Phi}{\sqrt{\lambda} a^2}.$$

If electric charges q smaller than that of the Higgs field ϕ are present in the theory, then $q\Phi$ can be smaller than 1 and the Aharonov–Bohm effect is observed when such charges travel around the Abrikosov tube at any distance: thus, even though all gauge fields are massive, the Aharonov–Bohm interaction also remains long-range (unscreened) [18].

The technical reason allowing magnetic Abrikosov lines to exist is that the equation $F_{xy} = \delta(x)\delta(y)$ can be easily solved:

$$A_x = \partial_x \,\mathrm{arctg}\, \frac{y}{x}, \quad A_y = \partial_y \,\mathrm{arctg}\, \frac{y}{x},$$

and the Higgs field just provides a source of the needed form, with the electric current

$$J_x = \partial_y F_{xy} = \delta(x)\delta'(y), \quad J_y = -\partial_x F_{xy} = -\delta'(x)\delta(y)$$

rotating around the z axis.

To obtain an electric Abrikosov line, we need to solve the equation $F_{0z} = \delta(x)\delta(y)$, which violates Bianchi identity and requires the existence of a magnetic current (rotating around the z axis) and hence, in a Lorentz-invariant setting, of magnetic charges (monopoles)[3]. Therefore, in order to describe confinement with properties (α) and (β), where the dual MA effect is needed, the dual Abelian Higgs model (the dual superconductor model), is often used, where the Higgs field $\tilde{\phi}$ is magnetically charged, i.e., interacts with the dual field $\tilde{A}_\mu$, such that

$$\tilde{F}_{\mu\nu} = \partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta} F_{\alpha\beta} = \epsilon_{\mu\nu\alpha\beta}\partial_\alpha A_\beta.$$

In this type of scenarios, the role of non-Abelian degrees of freedom is thought to be the imitation of Higgs degrees of freedom (see, e.g., $W^\pm$ in Eq. (20) below and Ref. [19]) and the problem is to find a mechanism leading to their appropriate condensation.

As already mentioned, the lattice experiments (see Sec. 4 below) imply that the real pattern (and, perhaps, the mechanism) of confinement can be more sophisticated and may imply the coexistence of (β) electric and (γ) structured magnetic tubes. Therefore, it is important to note that no Abelian model is known that allows the coexistence of magnetic and electric MA effects, e.g., no effective Lagrangian of the form

$$\frac{1}{e^2} F_{\mu\nu}^2 + m_m^2 \hat{A}_\mu^2 + m_e^2 \hat{\tilde{A}}_\mu^{\,2} \tag{11}$$

is allowed. Therefore, if such coexistence is not an artifact of lattice experiments (which is not considered too probable nowadays), it requires construction of more sophisticated models. A natural hope is that such models can be straightforwardly built in modern string theory (involving branes) and realized in condensed-matter systems.

We note that some kind of restoration, at least partial, of electro–magnetic duality present in Abelian photodynamics is needed. This duality is usually broken by all known relevant modifications: by the introduction of electric charges (without adding their magnetic counterparts), by embedding into a non-Abelian theory (where electric and magnetic interactions of gluons are different), by the addition of a Chern–Simons term, or by coupling to Higgs scalars and going to a superconducting phase. Lattice experiments strongly suggest the need for some — yet unstudied (topological, i.e., with the field content of a field, not string, theory) — stringy phases with both «fundamental» and $D1$ strings present, where screening and MA phenomena do not contradict the electromagnetic duality.

3. 3*d* COMPACT QED

The sample example [2] of the confinement proof in the Abelian (2+1)-dimensional compact electrodynamics (embedded into the non-Abelian Georgi–Glashow model to justify compactness and provide ultraviolet

[3] Similarly, in order to have a magnetic tube, where the field is not constant along the line (in particular, changes direction at some points z_a), we must solve the equation $F_{xy} = \frac{1}{2}\delta(x)\delta(y)\prod_a \mathrm{sign}(z - z_a)$, which violates Bianchi identity at $x = y = 0$, $z = z_a$ and therefore requires magnetic charges (monopoles) at these points.

regularization, rendering the instanton action finite) actually deals with random confinement [6, 12] and with Wilson's confinement criterion [1]: not fluxes but their squares acquire vacuum averages, $\langle\Phi\rangle = 0$, $\langle\Phi^2\rangle \neq 0$, and this suffices to provide the area-law behavior for the Wilson loop averages. In this example, the relevant medium in 2 space dimensions is obtained as a time slice of an instanton gas with Debye screening. Instantons in the Abelian $(2+1)$-dimensional theory are just the ordinary 3-dimensional monopoles and antimonopoles with the magnetic fields

$$H_\mu = \epsilon_{\mu\nu\lambda}F^{\nu\lambda} = \pm g\frac{r_\mu}{r^3}, \tag{12}$$

or

$$H_\mu = \pm g\frac{r_\mu}{(r^2+\varepsilon^2)}e^{-r/\xi}, \tag{13}$$

where ϵ and ξ provide the respective ultraviolet (from the underlying non-Abelian theory) and infrared (from the Debye screening in the monopole–antimonopole gas) regularizations; g is the monopole charge, normalized such that $2eg$ = integer. Thus, the medium looks like a set of appearing and disappearing vortex–antivortex pairs with the pseudoscalar $2d$ magnetic and vector $2d$ electric fields

$$\begin{aligned} B &= \epsilon_{ij}F^{ij} = \pm\frac{gt}{(\mathbf{x}^2+t^2)^{3/2}}, \\ E_i &= F_{0i} = \pm g\frac{\epsilon_{ij}x^j}{(\mathbf{x}^2+t^2)^{3/2}}. \end{aligned} \tag{14}$$

The field E_i produced by the time variation of B has nontrivial vorticity and hence contributes to the rectangular Wilson average over this medium,

$$\left\langle \exp\left(ie\oint_C (A_0\,dt + A_i\,dx^i)\right)\right\rangle = \left\langle \exp\left(ie\int_S \mathbf{E}\cdot d\mathbf{x}\,dt\right)\right\rangle, \tag{15}$$

where the contour C lies in the xt plane and S intersects the xy plane by a segment $\tilde{C}$. The contribution of a vortex to the integral $\int_{\tilde{C}} \mathbf{E}\cdot d\mathbf{x}$ is equal to

$$\pm\int_{-L}^{L}\frac{y\,dx}{(x^2+y^2+t^2)^{3/2}} \sim \pm\frac{2y}{y^2+t^2} \tag{16}$$

for $L \gg \sqrt{y^2+t^2}$ (with the distance $\sqrt{y^2+t^2}$ actually bounded from above by the Debye radius ξ) and further integration over t gives

$$\pm 4\pi g\Phi = \pm 2\pi g\frac{y}{|y|} = \pm 2\pi g\,\mathrm{sign}\,y \tag{17}$$

for the contribution of a vortex provided the vortex lies in a slice of width $\xi \ll L$ around the surface S. This flux is one-half of the full flux $4\pi g$ of the charge-g monopole. The factor $1/2$ appears here because only half of the vorticity of $\mathbf{E}$ contributes to the integral. Because contributions of vortices and antivortices have opposite signs, the average of $\int \mathbf{E}\,d\mathbf{x}\,dt$ itself is of course vanishing, but the even powers of this integral, and hence the Wilson exponential, can have nonvanishing averages. The simplest estimate with the help of Poisson distributions gives [20]

$$\begin{aligned} &\langle\exp\left(ie\oint_C (A_0\,dt + A_i\,dx^i)\right)\rangle = \\ &= \sum_{n_+,n_-=0}^{\infty}\left[e^{-\bar{n}}\frac{\bar{n}^{n_+}}{n_+!}\right]\left[e^{-\bar{n}}\frac{\bar{n}^{n_-}}{n_-!}\right]\times \\ &\times\exp(4\pi ieg(n_+-n_-)\Phi) = \\ &= \exp(-2\bar{n}(1-\cos(4\pi eg\Phi))). \end{aligned} \tag{18}$$

Because the average number of contributing vortices and antivortices is $\bar{n} = \xi A_S n_0$, where A_S is the area of the surface S and n_0 is the concentration of vortices (depending primarily on the instanton action, which is in turn defined by the ultraviolet regularization), we obtain the area law for the Wilson loop, at least for the minimal value $eg = 1/2$ allowed by the Dirac quantization condition[4]. The average

$$\left\langle \exp\left(ie\oint_C (A_0\,dt + A_i\,dx^i)\right)\right\rangle = \left\langle \exp\left(ie\int_S B\,dx\,dy\right)\right\rangle \tag{19}$$

of a space-like Wilson loop with S lying in the xy plane and bounded by the curve C can be calculated similarly. This average is given by the same formula (18).

Another interpretation of the same calculation [3] implies that the distribution of vortices is affected by the presence of the loop, such that the vortices and antivortices are concentrated around the surface S and screen it.

4. CONFINEMENT IN 4*d*

In 3+1 dimensions, no such simple calculation from first principles is yet known. The main difference is that

[4] There are corrections to this oversimplified calculation [20, 21], which can in particular destroy the prediction in (18), that confinement disappears for even magnetic charges (when the relevant flux Φ is integer).

ordinary instantons in $3+1$ dimensions are no longer charged: their field vanishes too fast at infinity and, therefore, the confinement mechanism should involve an additional dissociation of instantons into something like magnetically charged merons [3, 22]. Time slices of instantons are now 3-dimensional objects, namely monopole-antimonopole pairs (if viewed in a special gauge), and the instanton describes the process of their spontaneous creation and annihilation.

The expectation is that in the dense instanton gas (or liquid), recombination takes place between monopoles and antimonopoles from different pairs, thus picking up a chain of instantons from the liquid (see Fig. 2).

The spaghetti vacuum pattern implies that such chains are actually spread out through the entire volume and form a «percolating cluster» [20, 23].

As in the $(2+1)$-dimensional case, the electric fields with nonvanishing vorticities, caused by the moving monopoles and antimonopoles, contribute to the Wilson averages in $3+1$ dimensions and give rise to the area laws.

At the moment, there is no absolutely convincing theoretical argument in favor of this kind of ideas; instead, they received considerable support from computer experiments.

«Experimental» lattice results. Lattice computer simulations are primarily targeted at producing qualitative results in the spirit of (i) and thus at providing a proof that the Yang–Mills functional integral indeed describes a theory with a mass gap, a linear potential, a realistic hadronic spectrum, and realistic hadron interactions. Remarkably enough, these experiments could also be used for research in direction (ii) and they indeed produced very inspiring results. But up to now, the simulations are not very detailed and the functional integral is actually replaced by a sum over a rather small random subset of field configurations that are believed to give the dominant contribution. According to (ii), one can hope that most of these dominant configurations have something in common — and this is what actually happens — providing a clear description of the medium required in (ii).

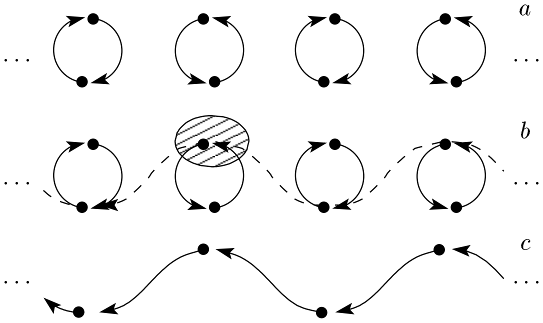

Fig. 2. Possible phases of the recombinant plasma of the instanton gas: *a*) Recombinant phase (ordinary instanton gas in $3+1$ dimensions): each instanton is the process of creation and annihilation of a monopole–antimonopole pair. *b*) Transition to the jumping recombinant phase (instantons dissociated into merons): created pairs do not coincide with annihilating pairs. The dashed domain corresponds to a meron. *c*) Jumping recombinant phase: a chain is naturally formed

This experimentally discovered [24, 25] medium appears to be somewhat unexpected (see [6] for the original suggestion of this «Copenhagen spaghetti vacuum» and [26] for comprehensive modern reviews and references): it turns out to be filled with peculiar one-dimensional objects (with two-dimensional world surfaces) — P-vortices — which in a certain Abelian approximation (see the next subsection) look like narrow (of width $r_m \ll \Lambda_{QCD}^{-1}$) tubes of magnetic field, directed along the tube and changing direction to the opposite at locations of monopoles and antimonopoles, which form a 1-dimensional gas inside the tube[5)]. Such objects are obviously stable against the creation of monopole–antimonopole pairs: such processes cannot break the tube into two, because the magnetic flux through any section outside the monopole cores is $1/2$[6)]. The net of these direction-changing color-magnetic tubes fills the entire space [20] (forming a «percolating cluster» [23])[7)], and in this medium the force lines of color-electric fields (emitted by sources of nonvanishing N-ality) also form tubes (of width

[5)] In contrast to the P-vortices themselves, the monopoles and antimonopoles inside them are difficult to define in a gauge-invariant way. Even the direction of the tentative Abelian magnetic field and hence the exact positions of monopoles and antimonopoles inside the P-vortex are unphysical: they can be changed by gauge transformations. Indeed, to change the direction of an Abelian field strength $F^3_{\mu\nu}$ at a given point, it suffices to make a singular gauge transformation conjugating the fields by a unitary matrix like σ^1 at this point (although it is not absolutely clear how to make such operation consistent with the maximal Abelian projection described in the next subsection). There is still a controversy in the literature (see, e.g., [27] for different points of view) about the actual internal structure of the P-vortices and the (dis)advantages of visualizing them in terms of monopoles and antimonopoles.

[6)] This does not contradict the possibility that isolated monopoles are screened [28].

[7)] In addition to the precolating cluster, there also exists a variety of nonpercolating ones, also populated by monopoles. There is no agreement in the literature on whether these nonpercolating clusters are lattice UV-artifacts or they actually contribute in the continuum limit.

$r_e \sim \Lambda_{QCD}^{-1}$), thus giving rise the to the confinement phenomenon. In lattice experiments, the area laws for approriate Wilson-loop averages are explicitly verified and the P-vortices from percolating cluster are shown to give the dominant contribution to the string tensions. Theoretically, the contribution of P-vortices to the string tension depends on their abundance, and one of the tasks of the theory is to explain the origin of the medium of P-vortices and ensure its consistency with Lorentz invariance.

So far, there is no clear theoretical explanation of why and how such a medium is formed in non-Abelian gauge theories and why — once formed — it can give rise to a dual Meissner effect and lead to confinement, although the (lattice) experimental evidence in favor of this pattern is rapidly growing.

A serious drawback of the published results of lattice experiments is that they do not provide the essential information about instanton-like and meron-like configurations and their probable association with the localized P-vortex clusters; furthermore, they do not explicitly study the configurations of collimated color-electric force lines between sources with nonvanishing N-ality (which do not need to be fermions). Information about these color-electric tubes is extracted indirectly from the study of Wilson averages. This is not enough to understand what happens to these tubes, for example, after the maximal Abelian projection, and whether their content indeed looks like an Abelian electric field exactly in the same projection where the P-vortices look like the tubes of a direction-changing Abelian magnetic field. Any data touching upon this issue would be extremely useful for further clarification of the situation.

Maximal Abelian Projection. The «P» in «P-vortices» comes from the word «projection» [29]. It is inspired from the way they are often sought and studied, which is not gauge invariant, even though the P-vortices themselves are in fact gauge invariant (see Fig. 3).

A procedure called the maximal Abelian projection (MAP) is commonly used[8]. It splits into two steps. First, for every configuration of the fields $A_\mu^a(x)$, taken with the weight dictated by the true non-Abelian action, the «maximal Abelian gauge» is chosen by minimizing the lattice counterpart of $\int W_\mu^+ W_\mu^-(x)\, d^4x$

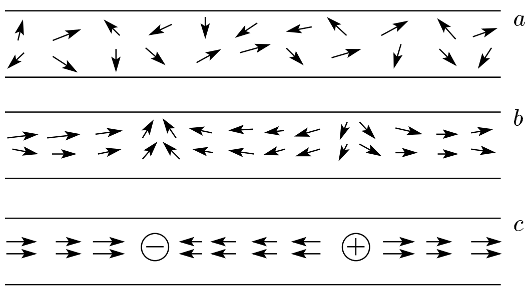

Fig. 3. This figure borrowed from the seminal paper [25] is the best existing illustration of what P-vortices are and what the maximal Abelian projection does. a) A fragment of the distribution on field strength in an original configuration of fields $A_\mu^a(x)$, from the set of the those fields that give a dominant contribution to the non-Abelian functional integral. The strenghts are nonvanishing within a narrow tube, the P-vortex. Actually, the entire configuration looks like a net of P-vortices, containing the «percolating cluster», which has proper scaling properties and survives in the continuum limit. The arrows indicate directions in color space. b) The maximal Abelian gauge is chosen, which minimizes $\int W_\mu^+ W_\mu^-(x)\, d^4x$. It is just a choice of gauge (field strenths are rotated), no approximation is involved. Certain structures are clearly seen in the distribution of field strenghts inside the tube. c) Maximal Abelian projection is performed: $W_\mu^\pm(x)$ are set equal to zero. The structures seen in Fig. b turn into a clear (but approximate) pattern of collimated magnetic force lines, changing direction at the location of monopoles and antimonopoles. No peaks of magnetic energy occur at these locations

along the gauge orbit. This first step is absolutely justified (although technically it suffers from ambiguities caused by the existence of Gribov copies).

This allows introducing the induced effective action $\tilde{S}(A)$, obtained after integration over the other components ($W_\mu^\pm \equiv A_\mu^1 \pm iA_\mu^2$, $D_\mu(A) \equiv \partial_\mu + ieA_\mu^3$),

$$\exp\left(-\tilde{S}(A)\right) = \int DW^+ DW^- \delta\left(\left|D_\mu(A) W_\mu^+\right|^2\right) \times \\ \times \det{}_{FP}^2 \left(\partial_\mu D_\mu(A)\right) \times \\ \times \exp\left(-\frac{1}{g^2}\left[\left(F_{\mu\nu} + (W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+)\right)^2 + \\ + |D_\mu(A) W_\nu^+|^2\right]\right). \quad (20)$$

At the second step, $\tilde{S}(A)$ is used to define Abelian correlation functions

[8] Comparison with the results of lattice experiments in other Abelian approximations usually demonstrates that the (gauge noninvariant and necessarily approximate) language of monopoles is most reliable in the MAP, the use of this language in other calculational schemes can often be misleading [30].

$$\langle\prod_i O(A_\mu^a)\rangle_{MAP} \equiv \langle\prod_i O(W_\mu^\pm = 0, A_\mu^3)\rangle = \\ = \int DA_\mu^3 e^{-\tilde{S}(A_\mu^3)} \prod_i O(W_\mu^\pm = 0, A_\mu^3). \quad (21)$$

This step implies that the true non-Abelian action is used, i.e., contributions from the virtual $W^\pm$-bosons in loops are included, although omitted from external lines. Therefore, the second step — the projection itself — is an approximation:

$$\langle\prod_i O(W_\mu^\pm, A_\mu^3)\rangle \neq \\ \neq \int DA_\mu^3 e^{-\tilde{S}(A_\mu^3)} \prod_i O(W_\mu^\pm = 0, A_\mu^3). \quad (22)$$

Its experimentally discovered [31] surprising efficiency (as compared with the complete answer including non-Abelian fields) is often called the hypothesis of Abelian dominance. Although theoretically so far unjustified and uncontrollable, it provides a convenient language for description (visualization) of the confinement phase: it is at this level that monopoles and antimonopoles appear. Figure 3 can serve as an illustration of how the MAP works.

The theoretical problem of evaluation of $\tilde{S}(A)$ remains open. We refer to [19] for interesting attempts to identify condensating modes and vortex-like structures in functional integral (20) and to [32] for a supersymmetric model with BPS configurations that look like magnetic P-vortices populated by monopoles.

5. ARE THERE CONDENSED-MATTER ANALOGUES OF CONFINEMENT?

Returning to the lattice results above, a natural question to ask is whether anything similar can be found in other avatars of gauge theories, for example, in condensed matter or plasma physics. There, one would rather expect to encounter a dual type of medium: electric P-vortices formed by chains of positive and negative electric charges, connected by narrow tubes of electric fields with fluxes $\pm 1/2$, and the ordinary (magnetic) MA effect, implying formation of magnetic-field tubes with a constant unit flux (and confinement of hypothetical magnetic charges), caused by or at least consistent with the existence of such electric P-vortices. In condensed matter analogues, the underlying non-Abelian Yang – Mills dynamics responsible for the formation of P-vortices should presumably be replaced by some other dynamics (additional forces), allowed in condensed matter systems. The whole situation (the coexistence and even mutual influence of electric P-vortices and magnetic MA effect) is already exotic enough to make one wonder if anything like this can occur in any kind of natural matter systems.

The main effect to be sought in a condensed matter setup is the simultaneous existence of narrow tubes (P-vortices) of direction-changing electric field and broader tubes (Abrikosov lines) of magnetic field — a dual pattern to the one underlying the spaghetti confinement mechanism of gluodynamics. This clearly implies that superconductivity (from the dual superconductor scenario), if relevant at all, should be of a more sophisticated nature than just the single-field condensation (monopole condensation), the superconducting order should be caused by or at least coexist with an order of some other type (responsible for the formation of P-vortices). This looks almost like the requirement that the Meissner – Abrikosov effect (for the magnetic field) coexists with (or, perhaps, is even implied by) the dual Meissner – Abrikosov effect (for the electric field), but actually the tubes of the electric field should be different: they should have an internal structure, namely a one-dimensional gas of positive and negative electric charges, the electric field along the tube that changes direction at the locations of these charges and be stable against possible «string breaking» caused by creation or annihilation of charge–hole pairs. Moreover, the width of electric tubes should/can be different (much smaller?) than that of magnetic tubes.

The main goal of this paper is to bring these issues to the attention of experts in other fields, such as condensed matter and plasma physics and to emphasize the fact that the discovery of a similar picture arising under any circumstances would be of great help for the development of the confinement theory and in particular for the understanding of possible 2-dimensional vortex theories living on the world sheets of the relevant branes, as well as of the phase structure of these theories[9]. If, on the contrary, no such pattern exists in

[9] Among other things, it would be interesting to exploit the idea of the topological confinement, which, in different versions, often works in condensed matter physics. A characteristic feature of the topological confinement is that it depends on the dynamics of the theory only through the properties of particular excitations (quasiparticles), while their interactions do not matter. For example, one-dimensional objects can be tied and, therefore, be unseparable, and this can work for real one-dimensional excitations, like Abrikosov tubes, and for point-like magnetic monopoles and/or hedgehogs that have Dirac strings attached. In practice, topological confinement can look very similar to the mechanism we discuss throughout the paper. See [33, 34] for some examples, see also [35].

condensed matter physics, this would once again emphasize the pecularities of non-Abelian gauge theories (where elementary quanta carry more structure than just point-like charges and thus the naive screening behavior is from the very beginning substituted by antiscreening and further nonnaive phenomena are naturally expected to occur).

The rest of this paper is purely speculative, added for encouragement: in order to demonstrate that superconductivity (probably responsible for the magnetic Meissner – Abrikosov effect) can indeed coexist with some kind of the dual order at least (although the example below falls short from exhibiting narrow tubes of the direction-changing electric field).

Charge density waves. As a possible (but by no means unique) candidate analogue of the electric P-vortices, we suggest the charge density waves (CDW); the questions that then arise are:

(a) are there any tube-like CDWs with a charge density similar to $\rho(x,y,z) \sim \delta^{(2)}(x,y)\sin z$ and (perhaps, direction-changing) electric force lines collimated along the z axis?

(b) can the CDW coexist with superconductivity (SC), which would be a natural reason for the Meissner – Abrikosov effect?

(c) can the CDW cause or at least enhance superconductivity?

(d) can the widths of the CDW-like P-vortices be much smaller than those of Abrikosov lines (where the Cooper – Higgs-like condensate is broken)?

Remarkably, a very similar set of questions is currently under intense investigation in connection with high-T_c superconductivity (where the adequate theoretical pattern also remains unknown), and it looks like the above possibilities are indeed open, as can be seen in [36] and the references therein. Of course, the real media appearing in condensed matter examples have a lot of additional structure (primarily, the highly anisotropic crystal lattice in the background, playing a key role in the formation of realistic CDW), which one does not expect to find in gluodynamics. For closer analogues with gluodynamics, one can also look for phenomena in liquid He [33], dense relativistic plasma, segnetoelectrics [14] or even biological membranes [37]. Still, we want to emphasize once again that today, when the formulation of a phenomenological theory of P-vortices is so important, one needs to consider all examples where objects of this kind are presumably present, irrespective of the underlying microscopic structure, and the solid-state systems with the coexisting CDW and SC orders should not be neglected — especially because, along with the confinement in gauge theories, they are now under close scrunity, and considerable progress can result rather fast from comparison of ideas from the two fields.

The simplest facts and ideas about the CDW-SC systems, although not immediately coinciding with (a)–(d), do not seem to be in obvious contradiction. The relevant properties seem to include the following list:

• The CDW formation causes transition to an insulator phase (Peierls – Fröhlich – Mott transition), while the SC transition gives rise to a (super)conductor.

• Thus CDW and SC orders compete with each other, with CDW usually a stronger competitor than SC [38].

• The CDW and SC orders can nevertheless coexist [39, 40].

• Even if both CDW and SC orders are not established simultaneously at long distances, they interfere locally, one phase appears in the regions where the other is broken: SC appears in the vicinity of CDW vortices and CDW appear in the vicinity of Abrikosov lines [40]. This can be enough, for example, to obtain the SC phase when CDW dislocations percolate through the entire volume.

The phenomenological descriprion of the CDW is in terms of electron – phonon interactions [41]. We note that the vector nature of phonons makes them closer to the W-fields in (20) than to the scalar fields used in Abelian Higgs model (10).

6. CONCLUSION

The theory of the Copenhagen spaghetti vacuum should, of course, be developed in the context of string theory. The appropriate name for P-vortices is 1-branes. Monopoles living on these 1-branes are, naturally, 0-branes. The coexistence of electric and magnetic Abrikosov tubes should be modeled by that of coexisting «fundamental strings» and $D1$ branes. The problems raised in this paper are related to the lack of any «underlying model» for which the theory of strings and branes would be an effective model, the lack which seriously undermines the progress in modern string theory. We emphasize that the spaghetti vacuum in gluodynamics can by itself provide such a model and we also suggest to start a more extensive search for possible underlying models in modern condensed-matter physics.

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