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THE SYMMETRY RELATING THE PROCESSES IN 2- AND 4-DIMENSIONAL SPACE–TIMES, AND THE VALUE $\alpha_0 = 1/4\pi$ OF THE BARE FINE STRUCTURE CONSTANT

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The symmetry manifests itself in exact mathematical relations between the Bogoliubov coefficients for the processes induced by an accelerated point mirror in $1+1$-dimensional space and the current (charge) densities for the processes caused by an accelerated point charge in $3+1$-dimensional space. The spectra of pairs of Bose (Fermi) massless quanta emitted by the mirror coincide with the spectra of photons (scalar quanta) emitted by the electric (scalar) charge up to the factor $e^2/\hbar c$. The integral relation between the propagator of a pair of oppositely directed massless particles in $1+1$-dimensional space and the propagator of a single particle in $3+1$-dimensional space leads to the equality of the vacuum–vacuum amplitudes for the charge and the mirror if the mean number of created particles is small and the charge $e = \sqrt{\hbar c}$. Due to the symmetry, the mass shifts of electric and scalar charges (the sources of Bose fields with spin 1 and 0 in $3+1$-dimensional space) for the trajectories with a subluminal relative velocity β_{12} of the ends and the maximum proper acceleration w_0 are expressed in terms of the heat capacity (or energy) spectral densities of Bose and Fermi gases of massless particles with the temperature $w_0/2\pi$ in $1+1$-dimensional space. Thus, the acceleration excites the 1-dimensional oscillations in the proper field of charges and the energy of oscillations is partly deexcited in the form of real quanta and partly remains in the field. As a result, the mass shift of an accelerated electric charge is nonzero and negative, while that of a scalar charge is zero. The symmetry is extended to the mirror and charge interactions with the fields carrying space-like momenta and defining the Bogoliubov coefficients $\alpha^{B,F}$. The traces $\mathrm{tr}\,\alpha^{B,F}$, which describe the vector and scalar interactions of the accelerated mirror with a uniformly moving detector, were found in analytic form for two mirror's trajectories with subluminal velocities of the ends. The symmetry predicts one and the same value $e_0 = \sqrt{\hbar c}$ for the electric and scalar charges in $3+1$-dimensional space. The arguments are adduced in favor of the conclusion that this value and the corresponding value $\alpha_0 = 1/4\pi$ of the fine structure constant are the bare, nonrenormalized values.

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1. INTRODUCTION

The Hawking mechanism for particle production at the black hole formation is analogous to the emission from an ideal mirror accelerated in the vacuum [1]. In its turn, there is a close analogy between the radiation of pairs of scalar (spinor) quanta from an accelerated mirror in $1+1$-dimensional space and the radiation of photons (scalar quanta) by an accelerated electric (scalar) charge in $3+1$-dimensional space [2, 3]. All these processes turn out to be mutually related. In problems with moving mirrors, the in-set $\phi_{in\,\omega'}$, $\phi^*_{in\,\omega'}$ and the out-set $\phi_{out\,\omega}$, $\phi^*_{out\,\omega}$ of the wave equation solutions are frequently used. For a massless scalar field, they are given by

$$\begin{aligned} &\phi_{in\,\omega'}(u,v) = \\ &= \frac{1}{\sqrt{2\omega'}}\left[\exp(-i\omega' v) - \exp(-i\omega' f(u))\right], \\ &\phi_{out\,\omega}(u,v) = \\ &= \frac{1}{\sqrt{2\omega}}\left[\exp(-i\omega g(v)) - \exp(-i\omega u)\right], \end{aligned} \qquad (1)$$

with zero boundary condition

$$\phi|_{traj} = 0$$

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on the mirror's trajectory. Here, the variables

$$u = t - x, \quad v = t + x$$

are used and the mirror (or charge) trajectory in the u, v plane is given by any of the two mutually inverse functions

$$v^{mir} = f(u), \quad u^{mir} = g(v).$$

We refer the reader to [3] for the in- and out-sets of massless Dirac equation solutions. Dirac solutions differ from Eqs. (1) by the presence of bispinor coefficients at the u- and v-plane waves. The current densities corresponding to these solutions have only tangential components at the boundary. Therefore, the boundary condition for both scalar and spinor field is purely geometrical, it does not contain any dimensional parameters.

The Bogoliubov coefficients $\alpha_{\omega'\omega}$ and $\beta_{\omega'\omega}$ appear as the coefficients of the expansion of the out-set solutions in the in-set solutions; the coefficients $\alpha^*_{\omega'\omega}$, $\mp\beta_{\omega'\omega}$ arise as the coefficients of the inverse expansion. The upper and lower signs correspond to the scalar (Bose) and spinor (Fermi) fields. The explicit form of the Bogoliubov coefficients is very simple:

$$\alpha^B_{\omega'\omega},\ \beta^{B*}_{\omega'\omega} = \sqrt{\frac{\omega'}{\omega}} \int\limits_{-\infty}^{\infty} dv \exp(i\omega' v \mp i\omega g(v)) = \\ = \pm\sqrt{\frac{\omega}{\omega'}} \int\limits_{-\infty}^{\infty} du \exp(\mp i\omega u + i\omega' f(u)). \quad (2)$$

The coefficients $\alpha^F_{\omega'\omega}$ and $\beta^{F*}_{\omega'\omega}$ for the Fermi field differ from these representations by the substitutions

$$\sqrt{\omega'/\omega} \to \sqrt{g'(v)}, \quad \pm\sqrt{\omega/\omega'} \to \sqrt{f'(u)}$$

in the integrands.

Then the mean number $d\bar{n}_\omega$ of quanta radiated by the accelerated mirror to the right half-space with a frequency ω and wave vector $\omega > 0$, and the total mean number $\bar{N}$ of quanta are given by the integrals

$$\begin{aligned} d\bar{n}^{B,F}_\omega &= \frac{d\omega}{2\pi} \int\limits_0^\infty \frac{d\omega'}{2\pi} |\beta^{B,F}_{\omega'\omega}|^2, \\ \bar{N}^{B,F} &= \iint\limits_0^\infty \frac{d\omega d\omega'}{(2\pi)^2} |\beta^{B,F}_{\omega'\omega}|^2. \end{aligned} \quad (3)$$

These expressions do not contain $\hbar$, but their interpretation as mean numbers of quanta follows from the secondary-quantized theory. The secondary-quantized theory allows constructing all possible amplitudes of many-particle creation, annihilation, and scattering via Bogoliubov coefficients [4–6].

At the same time, the spectra of photons and scalar quanta emitted by electric and scalar charges moving along the trajectory $x_\alpha(\tau)$ in $3+1$-dimensional space are defined by the Fourier transforms of the electric current density 4-vector $j_\alpha(x)$ and the scalar charge density $\rho(x)$,

$$\begin{aligned} j_\alpha(k),\ \rho(k) &= e \int d\tau \, \{\dot{x}_\alpha(\tau),\ 1\} \exp(-ik^\alpha x_\alpha(\tau)), \\ j_\alpha(x),\ \rho(x) &= e \int d\tau \, \{\dot{x}_\alpha(\tau),\ 1\} \delta_4(x - x(\tau)), \end{aligned} \quad (4)$$

and are given by the formulas

$$\begin{aligned} d\bar{n}^{(1,0)}_k &= \frac{1}{\hbar c} \{|j_\alpha(k)|^2,\ |\rho(k)|^2\} \frac{dk_+ dk_-}{(4\pi)^2}, \\ \bar{N}^{(1,0)} &= \frac{1}{\hbar c} \iint\limits_0^\infty \frac{dk_+ dk_-}{(4\pi)^2} \{|j_\alpha(k)|^2,\ |\rho(k)|^2\}, \end{aligned} \quad (5)$$

where the superscripts in $d\bar{n}^{(s)}_k$, $\bar{N}^{(s)}$, and k^α denote the spin and 4-momentum of quanta,

$$k^2 = k_1^2 + k_\perp^2 - k_0^2 = 0, \quad k_\perp^2 = k_0^2 - k_1^2 = k_+ k_-,$$

$$k_\pm = k^0 \pm k^1,$$

and it is supposed in Eqs. (5) that the trajectory $x^\alpha(\tau)$ has only the x^0 and x^1 nontrivial components, as the mirror's trajectory.

In contrast to the quantities in Eqs. (3), the $d\bar{n}^{(s)}_k$ and $\bar{N}^{(s)}$ in Eqs. (5) contain $\hbar$ because the charge entering the current and charge densities is considered a classical quantity. In essence, $d\bar{n}^{(s)}_k$ and $\bar{N}^{(s)}$ can be considered classical quantities because they are obtained from a purely classical radiation energy spectrum $d\bar{\mathcal{E}}^{(s)}_k$ divided by the energy $\hbar k^0$ of a single quantum, such that

$$\begin{aligned} d\bar{n}^{(s)}_k &= \frac{d\bar{\mathcal{E}}^{(s)}_k}{\hbar k^0}, \\ \bar{N}^{(s)} &= \int \frac{d\bar{\mathcal{E}}^{(s)}_k}{\hbar k^0}, \quad k^0 = \frac{1}{2}(k_+ + k_-). \end{aligned} \quad (6)$$

The symmetry between the creation of Bose or Fermi pairs by an accelerated mirror in $1+1$-dimensional space and the emission of single photons or scalar quanta by an electric or scalar charge in $3+1$-dimensional space consists, first of all, in the coincidence of the spectra. If we set $2\omega = k_+$ and $2\omega' = k_-$, then

$$|\beta^B_{\omega'\omega}|^2 = \frac{1}{e^2}|j_\alpha(k_+,k_-)|^2, \qquad |\beta^F_{\omega'\omega}|^2 = \frac{1}{e^2}|\rho(k_+,k_-)|^2. \tag{7}$$

Therefore, the spectra coincide as functions of two variables and functionals of the common trajectory of the mirror and the charge. The distinction in the factor $e^2/\hbar c$ can be removed if we set $e^2 = \hbar c$.

The symmetry under discussion connecting the classical and quantum theories in Minkowski spaces of 4 and 2 dimensions in some sense resembles the duality of classical and quantum descriptions in spaces of neighbor dimensions proposed by 't Hooft [7] and Susskind [8]. Such a duality was actually discovered by Gubser, Klebanov, and Polyakov [9] and by Maldacena [10] for different types of semiclassical supergravity in anti-de Sitter space and quantum conformal theories on the boundary of this space. It seems plausible that the general reason for such dualities consists in the correspondence between a single particle in the space of the higher dimension and a pair of particles in the space of the lower dimension. The description of a larger number of particles in the space of the lower dimension is needed in accounting for the quantum mechanical interference effects.

2. SYMMETRY AND PHYSICAL CONTENT AND THE DISTINCTION BETWEEN $\beta^*_{\omega'\omega}$ AND $\alpha_{\omega'\omega}$

It follows from the secondary-quantized theory that the absolute pair production amplitude and the single-particle scattering amplitude are related by

$$\langle \text{out}\omega''\omega|\text{in}\rangle = -\sum_{\omega'}\langle \text{out}\omega''|\omega'\text{in}\rangle\, \beta^*_{\omega'\omega}. \tag{8}$$

This formula allows interpreting $\beta^*_{\omega'\omega}$ as the amplitude of a source of a pair of massless particles potentially emitted to the right and to the left with the respective frequences ω and ω' [6]. While the particle with the frequency ω actually escapes to the right, the particle with the frequency ω' propagates for some time to the left and is then reflected by the mirror and is actually emitted to the right with an altered frequency ω''. Then, in the time interval between pair creation and reflection of the left particle, we have a virtual pair with the energy k^0, momentum k^1, and mass m:

$$k^0 = \omega + \omega', \quad k^1 = \omega - \omega', \qquad m = \sqrt{-k^2} = 2\sqrt{\omega\omega'}. \tag{9}$$

Apart from this polar time-like 2-vector k^α, very important is the axial space-like 2-vector q^α,

$$q_\alpha = \varepsilon_{\alpha\beta}k^\beta, \quad q^0 = -k^1 = -\omega + \omega', \qquad q^1 = -k^0 = -\omega - \omega' < 0. \tag{10}$$

In terms of k^α and q^α, the symmetry between the α and β coefficients becomes expressed clearly:

$$s = 1, \quad e\beta^{B*}_{\omega'\omega} = -\frac{q_\alpha j^\alpha(k)}{\sqrt{k_+k_-}}, \qquad e\alpha^B_{\omega'\omega} = -\frac{k_\alpha j^\alpha(q)}{\sqrt{k_+k_-}}, \tag{11}$$

$$s = 0, \quad e\beta^{F*}_{\omega'\omega} = \rho(k), \quad e\alpha^F_{\omega'\omega} = \rho(q). \tag{12}$$

We note that Eqs. (4) define the current density $j^\alpha(k)$ and the charge density $\rho(k)$ as functionals of the trajectory $x^\alpha(\tau)$ and functions of any 2- or 4-vector k^α. It can be shown that in $1+1$-dimensional space, $j^\alpha(k)$ and $j^\alpha(q)$ are space-like and time-like polar vectors if k^α and q^α are time-like and space-like vectors correspondingly.

In the vacuum of a massless scalar or spinor field, the boundary condition at the mirror evokes the appearance of vector or scalar disturbance waves bilinear in the massless fields. There are two types of these waves:

1) the waves with the amplitude $\alpha_{\omega'\omega}$ ($\alpha^*_{\omega'\omega}$) that carry a space-like momentum directed to the left (right), and

2) the waves with the amplitude $\beta^*_{\omega'\omega}$ ($\beta_{\omega'\omega}$) that carry a time-like momentum with a positive (negative) frequency.

The waves with space-like momenta appear even if the mirror is at rest or moves uniformly (Casimir effect), while the waves with time-like momenta appear only in the case of an accelerated mirror.

The pair of Bose (Fermi) particles has spin 1 (0) because its source is the current density vector (charge density scalar), see [11] or problem 12.15 in [12].

3. VACUUM–VACUUM AMPLITUDE $\langle\text{out}|\text{in}\rangle = e^{iW}$, SELF-ACTION, AND MASS SHIFTS

It follows from the secondary quantized theory that in the vacuum–vacuum amplitude

$$\langle\text{out}|\text{in}\rangle = e^{iW},$$

the expression $\mathrm{Im}\, W^{B,F}$ is well-defined. According to DeWitt [4], Wald [5], and others (including myself [6]),

$$2\,\mathrm{Im}\, W^{B,F} = \pm\frac{1}{2}\mathrm{tr}\,\ln(1\pm\beta^+\beta) \quad \text{or} \quad \pm\,\mathrm{tr}\,\ln(1\pm\beta^+\beta) \tag{13}$$

in the respective cases where the particle is identical or nonidentical to the antiparticle. We confine ourselves by the last case and by the smallness condition

$$\mathrm{tr}\,\beta^+\beta \ll 1.$$

Then

$$2\,\mathrm{Im}\, W^{B,F} \approx \mathrm{tr}\,(\beta^+\beta)^{B,F} \equiv \equiv \iint\limits_0^\infty \frac{d\omega d\omega'}{(2\pi)^2}\,|\beta^{B,F}_{\omega'\omega}|^2 = \bar{N}^{B,F}. \tag{14}$$

In the integrand of $\bar{N}^{B,F}$, we use representations (2) for $\beta^{B,F}$, the variables $x_\mp(\tau)$ and $x_\pm(\tau')$ instead of u, $f(u)$ and v, $g(v)$, and hyperbolic variables ρ and ϑ instead of ω and ω',

$$d\omega d\omega' = \frac{1}{2}\rho d\rho d\vartheta, \quad \omega = \frac{1}{2}\rho e^{\vartheta}, \quad \omega' = \frac{1}{2}\rho e^{-\vartheta}, \quad \rho = 2\sqrt{\omega\omega'}, \quad \vartheta = \ln\sqrt{\frac{\omega}{\omega'}}, \tag{15}$$

to obtain the imaginary part of the causal function in $1+1$-dimensional space, $\mathrm{Im}\,\Delta^f_2(z,\rho)$, after integration over ϑ, and then the imaginary part of the causal function in $3+1$-dimensional space, $\mathrm{Im}\,\Delta^f_4(z,\mu)$, after integration over $\rho = m$, the variable that coincides with the mass of the virtual pair according to Eqs. (9). This result is a special case of the very important integral relation between the causal functions of wave equations for d- and $d+2$-dimensional space–times [13],

$$\Delta^f_{d+2}(z,\mu) = \frac{1}{4\pi}\int\limits_{\mu^2}^\infty dm^2\,\Delta^f_d(z,m). \tag{16}$$

The small mass parameter

$$\mu = 2\sqrt{\omega\omega'}|_{min} \neq 0$$

is introduced instead of zero to avoid the infrared divergence in what follows. We thus obtain

$$2\,\mathrm{Im}\, W^{B,F} = = \mathrm{Im}\iint d\tau\, d\tau' \left\{ \begin{array}{c} \dot{x}_\alpha(\tau)\dot{x}^\alpha(\tau') \\ 1 \end{array} \right\} \Delta^f_4(z,\mu), \quad z_\alpha = x_\alpha(\tau) - x_\alpha(\tau'). \tag{17}$$

We can omit the Im symbols on both sides of this equation and define the actions for Bose and Fermi mirrors in $1+1$-dimensional space as

$$W^{B,F} = = \frac{1}{2}\iint d\tau\, d\tau' \left\{ \begin{array}{c} \dot{x}_\alpha(\tau)\dot{x}^\alpha(\tau') \\ 1 \end{array} \right\} \Delta^f_4(z,\mu). \tag{18}$$

This is to be compared with the well-known actions for electric and scalar charges in $3+1$-dimensional space:

$$W_{1,0} = = \frac{1}{2}e^2\iint d\tau\, d\tau' \left\{ \begin{array}{c} \dot{x}_\alpha(\tau)\dot{x}^\alpha(\tau') \\ 1 \end{array} \right\} \Delta^f_4(z,\mu). \tag{19}$$

The symmetry would be complete if $e^2 = 1$, i.e., if the fine structure constant were $\alpha = 1/4\pi$. This «ideal» value of fine structure constant for the charges would correspond to the ideal, geometrical boundary condition at the mirror.

The appearance of the causal function $\Delta^f_4(z,\mu)$ in the action has lucid physical grounds.

1. The action must represent not only the radiation of real quanta but also the self-energy and polarization effects. While the former effects are described by the solutions of the homogeneous wave equation, the latter ones require the inhomogeneous wave equation solutions that contain information about the proper field of a source. Such solutions of the homogeneous and inhomogeneous wave equations are the functions

$$(1/2)\Delta^1 = \mathrm{Im}\,\Delta^f, \quad \bar{\Delta} = \mathrm{Re}\,\Delta^f.$$

2. While the appearance of $\mathrm{Im}\,\Delta^f$ in the imaginary part of action (17) is a consequence of a mathematical transformation of the integral $\bar{N}^{B,F}$ (similar to the Plancherel theorem), the function

$$\bar{\Delta} \equiv \mathrm{Re}\,\Delta^f$$

in the real part of the action is unique if it appears as the real part of the analytic continuation of the function $i\,\mathrm{Im}\,\Delta^f(z,\mu)$ to negative z^2 that is even in z as $\mathrm{Im}\,\Delta^f$ itself.

Both the propagator $\Delta^f_2(z,m)$ of a virtual pair with the mass

$$m = \rho = 2\sqrt{\omega\omega'}$$

in 2-dimensional space–time and the mass spectrum of these pairs arise owing to the transition from the variables ω and ω' to the hyperbolic variables ρ and ϑ, which reflect the Lorentz symmetry of the problem.

Further integration over the mass leads to the propagator $\Delta_4^f(z,\mu)$ of a particle moving in 4-dimensional space–time with the mass μ equal to the least mass of virtual pairs. Thus, relation (16) is immanent to the Lorentz symmetry and the symmetry connecting the processes in 2- and 4-dimensional space–times.

For point-like charges, the $W_{1,0}$ contain ultraviolet divergences, which must be eliminated. The removal of ultraviolet divergences in the self-actions $W_{1,0}|^F$ of accelerated charges (with the force $F \neq 0$) consists in the subtraction of the corresponding self-actions $W_{1,0}|^{F=0}$ of uniformly moving charges; as a result, the changes

$$\mathit{\Delta} W_{1,0} = W_{1,0}|_0^F = W_{1,0}|^F - W_{1,0}|^{F=0}$$

of the self-actions owing to acceleration do not contain ultraviolet singularities, have a positive imaginary part,

$$\operatorname{Im} \mathit{\Delta} W_{1,0} > 0,$$

and vanish together with the acceleration.

The following representations for the self-actions of uniformly moving electric and scalar charges are very instructive:

$$\begin{aligned} W_{1,0}|^{F=0} &= \\ &= \frac{1}{2}e^2 \iint d\tau\, d\tau' \{\dot{x}_\alpha(\tau)\dot{x}^\alpha(\tau'),\ 1\}\, \Delta_4^f(z,\mu)|^{F=0} = \\ &= \mp \frac{e^2}{4\pi}\,\frac{1-i}{2\sqrt{2\varepsilon}}\tau. \end{aligned} \quad (20)$$

They arise if we introduce the integration variable $x = \tau' - \tau$ instead of τ', with $z^2 = -x^2$, set $\mu = 0$, and use the representation

$$\begin{aligned} \Delta_4^f(z,\mu)|_{\mu=0} &= -\frac{1}{4\pi^2}\,\frac{i}{x^2 - i\varepsilon} = \\ &= \frac{1}{4\pi^2}\left(\frac{\varepsilon}{x^4+\varepsilon^2} - i\frac{x^2}{x^4+\varepsilon^2}\right), \quad \varepsilon \to 0. \end{aligned}$$

The opposite signs of the self-actions are due to the repulsion of electric charges of the same sign and to the attraction of scalar ones. The coefficients before τ are the classical proper energies $-\delta m_{1,0}$ of the charges taken with the minus sign, and $\sqrt{2\varepsilon}$ characterizes the charge dimension. Different signs of $\operatorname{Im} W_{1,0}|^{F=0}$ lead, in accordance with the amplitudes $\exp(iW_{1,0}|^{F=0})$, to the disappearance (screening) of the electric charge and to an unlimited growth (antiscreening) of the scalar charge.

These extraordinary properties of the self-actions occur because the charges are point-like. For the vector- and scalar-field sources $j^\alpha(x)$ and $\rho(x)$ distributed in space and slowly varying in time, the self-actions are free from singularities and have no imaginary parts [11]:

$$\begin{aligned} W_{1,0} &= \int dt \times \\ &\times \int \frac{d^3x\, d^3x'}{4\pi|\mathbf{x}-\mathbf{x}'|}\{j_\alpha(x)\, j^\alpha(x'), \rho(x)\,\rho(x')\}_{t'=t}. \end{aligned} \quad (21)$$

In this form, the self-actions contain the Ampere and Coulomb laws for current and charge interactions and the law of attraction of scalar charges of the same sign. Self-actions (20) and (21) are in accordance with the general assertion that the interaction of charges of the same sign transferred by odd-spin quanta leads to repulsion and by even-spin quanta to attraction.

We give an example of the self-action changes $\mathit{\Delta} W_{1,0}$ of electric and scalar charges in the case of accelerated motion along the very important quasihyperbolic trajectory

$$\begin{aligned} x(t) &= \frac{\beta_1^2}{w_0} - \beta_1\sqrt{\frac{\beta_1^2}{w_0^2} + t^2}, \quad \beta_{1,2} = \pm \operatorname{th}\frac{\theta}{2}, \\ \beta_{12} &= \frac{\beta_1 - \beta_2}{1 - \beta_1\beta_2} = \operatorname{th}\theta, \end{aligned} \quad (22)$$

with the initial β_1 and final β_2 velocities at $t = \mp\infty$ and proper acceleration $-w_0$ at $t = 0$. The proper acceleration at any moment is given by the formula

$$a(t) = -\frac{w_0}{(1+t^2/t_c^2)^{3/2}}, \quad t_c = \frac{\beta_1}{w_0\sqrt{1-\beta_1^2}}. \quad (23)$$

Therefore, the quasihyperbolic motion is close to the hyperbolic one on the time interval $|t| < t_c$.

The self-action changes $\mathit{\Delta} W_{1,0}(\theta,\lambda)$ are Lorentz-invariant functions of the two variables

$$\theta = \operatorname{Arth}\beta_{12}$$

and

$$\lambda = \mu^2/w_0^2$$

with singularities at $\lambda = 0$ and $\theta = \pm\infty$.

The case where $\lambda \to 0$ and θ is arbitrary was considered by the author in [13]:

$$\begin{aligned} \mathit{\Delta} W_1 &= \frac{e^2}{8\pi^2}\left\{\pi\left(\frac{\theta}{\operatorname{th}\theta} - 1\right) + i\left[\left(\frac{\theta}{\operatorname{th}\theta} - 1\right) \times \right.\right. \\ &\left.\left. \times \ln\frac{4(\operatorname{ch}\theta + 1)^2}{\gamma^2\lambda(\operatorname{ch}\theta - 1)} + 2 - \ln 2 - \operatorname{ch}\theta\, R(\theta)\right]\right\}, \end{aligned} \quad (24)$$

$$\Delta W_0 = \frac{e^2}{8\pi^2}\left\{\pi\left(1-\frac{\theta}{\operatorname{sh}\theta}\right) + i\left[\left(1-\frac{\theta}{\operatorname{sh}\theta}\right) \times \right.\right.$$
$$\left.\left. \times \ \ln\frac{4(\operatorname{ch}\theta+1)^2}{\gamma^2\lambda(\operatorname{ch}\theta-1)} - 2 + \ln 2 + R(\theta)\right]\right\}, \quad (25)$$

where $\gamma = 1.781$ and $R(\theta)$ is an even function of θ related to the Euler dilogarithm $L_2(z)$ [14]:

$$R(\theta) = \int\limits_0^\infty d\alpha \, \frac{\ln(\operatorname{ch}\theta + \operatorname{ch}\alpha)}{\operatorname{ch}\theta + \operatorname{ch}\alpha} = $$
$$= \frac{L_2(1-e^{-2\theta}) + \theta^2 - \ln 2\cdot\theta}{\operatorname{sh}\theta}. \quad (26)$$

In the case where $\theta \to \pm\infty$ and λ is arbitrary, considered in [15, 16],

$$\Delta W_{1,0} = -|\theta|\frac{e^2}{8\pi^2} S_{1,0}(\lambda),$$
$$S_n(\lambda) = (-1)^{n+1}\int\limits_0^\infty dz \exp\left(-\frac{i\lambda}{2z}\right) \times \quad (27)$$
$$\times \left[\exp(iz)K_n(iz) - \sqrt{\frac{\pi}{2iz}}\right],$$

where $K_n(iz)$ is the Macdonald function. As $\lambda \to 0$,

$$S_1(\lambda) = -\pi - i\left(\ln\frac{4}{\gamma^2\lambda} - 1\right), \quad S_0(\lambda) = -i. \quad (28)$$

For the trajectory with a subluminal relative velocity β_{12} of the ends, $\operatorname{Re}\Delta W_{1,0}$ are given by the unique formulas independent of the trajectory details:

$$\operatorname{Re}\Delta W_1 = \frac{e^2}{8\pi}\left(\frac{\theta}{\operatorname{th}\theta} - 1\right),$$
$$\operatorname{Re}\Delta W_0 = \frac{e^2}{8\pi}\left(1 - \frac{\theta}{\operatorname{sh}\theta}\right). \quad (29)$$

As $\beta_{12} \to 1$, the trajectory actually becomes hyperbolic with the charge velocity

$$\beta(\tau) = -\operatorname{th} w_0\tau$$

at the proper time τ, and

$$\theta = w_0(\tau_2 - \tau_1) \to \infty.$$

Then

$$\operatorname{Re}\Delta W_1 = \frac{e^2 w_0}{8\pi}(\tau_2 - \tau_1), \quad \operatorname{Re}\Delta W_0 = \frac{e^2}{8\pi}, \quad (30)$$

while the mass shifts of the uniformly accelerated charges are

$$\Delta m = -\frac{\partial \Delta W}{\partial\tau_2} = \frac{e^2 w_0}{8\pi^2} S(\lambda); \quad (31)$$

$$\text{at } \lambda \to 0, \quad \operatorname{Re}\Delta m_1 = -\frac{e^2 w_0}{8\pi}, \quad \operatorname{Re}\Delta m_0 = 0.$$

The real parts of the action changes in (29) have interesting integral representations ascending to Legendre [17],

$$\operatorname{Re}\Delta W_{1,0} = \frac{e^2}{4\pi}\int\limits_0^\infty dx \, \frac{\sin x}{e^{\pi x/\theta} \mp 1}, \quad \theta = \operatorname{Arth}\beta_{12}. \quad (32)$$

If β_{12} is close to 1, then on a large interval of the quasi-hyperbolic trajectory, the velocity

$$\beta(\tau) \approx -\operatorname{th} w_0\tau,$$

i.e., is the same as for the hyperbolic trajectory, and the parameter

$$\theta \approx w_0(\tau_2 - \tau_1),$$

where

$$\Delta\tau = \tau_2 - \tau_1$$

is the proper time interval within which the charge moves with the acceleration w_0 and outside with the constant initial and final velocities β_1 and β_2.

In the acceleration interval, the mass shift of a charge can be defined by one of the two relations

$$\operatorname{Re}\Delta m = -\frac{\partial \operatorname{Re}\Delta W}{\partial\tau_2}, \quad \operatorname{Re}\widetilde{\Delta m} = -\frac{\operatorname{Re}\Delta W}{\Delta\tau}. \quad (33)$$

In accordance with the first definition, using the Legendre representation and the formula

$$\theta = w_0(\tau_2 - \tau_1),$$

we obtain

$$\operatorname{Re}\Delta m_1 = -\frac{e^2 w_0}{8\pi}\left(\operatorname{cth}\theta - \frac{\theta}{\operatorname{sh}^2\theta}\right),$$
$$\operatorname{Re}\Delta m_0 = -\frac{e^2 w_0}{8\pi}\,\frac{\theta\operatorname{cth}\theta - 1}{\operatorname{sh}\theta},$$
$$\operatorname{Re}\Delta m_{1,0} = -e^2\,T\int\limits_0^\infty \frac{d\omega}{2\pi}\,\frac{\sin 2\omega\Delta\tau}{\omega}\,c^{B,F}(\omega/T), \quad (34)$$
$$c^{B,F}(z) = \frac{z^2 e^z}{(e^z \mp 1)^2},$$

where $T = w_0/2\pi$ is the Davies–Unruh «temperature» [18, 19] and $c^{B,F}(\omega/T)$ are the heat capacity spectral densities of Bose and Fermi gases of massless particles in one-dimensional space, see Secs. 49 and 105 in [20].

We have $\operatorname{Re}\Delta m_{1,0} \leqslant 0$ for all finite $\theta \geqslant 0$; for $\theta \ll 1$,

$$\operatorname{Re}\Delta m_1 = 2\operatorname{Re}\Delta m_0 = -\frac{e^2 w_0}{8\pi}\,\frac{2}{3}\,\theta, \quad (35)$$

and for $\theta \to \infty$,

$$\operatorname{Re} \Delta m_1 = -\frac{e^2 w_0}{8\pi}, \quad \operatorname{Re} \Delta m_0 = 0. \qquad (36)$$

We note that as the acceleration duration $\Delta\tau \to \infty$,

$$\left.\frac{\sin 2\omega\Delta\tau}{\omega}\right|_{\Delta\tau\to\infty} = \pi\,\delta(\omega). \qquad (37)$$

The function in the left-hand side is the Fourier transform of the acceleration switching function. The acceleration interval can be regulated by rescaling the «temperature» parameter and the frequency, $T \to kT$, $\omega \to k\omega$, at a constant ratio ω/T. Thus, the temperature $T = 2w_0/\pi$ can also be used [16].

In accordance with the second definition,

$$\begin{aligned} \operatorname{Re} \widetilde{\Delta m_1} &= -\frac{e^2 w_0}{8\pi}\left(\operatorname{cth}\theta - \frac{1}{\theta}\right), \\ \operatorname{Re} \widetilde{\Delta m_0} &= -\frac{e^2 w_0}{8\pi}\left(\frac{1}{\theta} - \frac{1}{\operatorname{sh}\theta}\right), \\ \operatorname{Re} \widetilde{\Delta m_{1,0}} &= -e^2 \int_0^\infty \frac{d\omega}{2\pi}\,\frac{\sin 2\omega\Delta\tau}{\omega}\,\frac{\omega}{e^{\omega/T} \mp 1}. \end{aligned} \qquad (38)$$

In this case, the spectral representation contains the energy spectral density of a Bose or Fermi gas of massless particles in 1-dimensional space. The quantities in both representations are related by

$$\operatorname{Re} \Delta m_{1,0} = T\frac{\partial}{\partial T}\operatorname{Re} \widetilde{\Delta m_{1,0}}, \qquad (39)$$

which follows from the standard relation

$$c^{B,F}(\omega/T) = \frac{\partial}{\partial T}\left(\frac{\omega}{e^{\omega/T} \mp 1}\right) \qquad (40)$$

between heat capacity and energy, see Secs. 14 and 42 in [20].

As functions of θ, the shifts $\operatorname{Re}\Delta m_{1,0}$ and $\operatorname{Re}\widetilde{\Delta m_{1,0}}$ differ in magnitude at $\theta \lesssim 1$, for example,

$$\operatorname{Re} \Delta m_{1,0} = 2\operatorname{Re}\widetilde{\Delta m_{1,0}}, \quad \theta \ll 1, \qquad (41)$$

but have the same limit values (36) as $\theta \to \infty$. Evidently, the mass shift formation requires proper time not less than the inverse acceleration w_0^{-1}.

Thus, according to the spectral representations in (34) and (38), the symmetry being discussed also reveals itself in the formation of the mass shifts of electric and scalar charges at acceleration. The vector and scalar massless Bose fields of the charges in $3+1$-dimensional space again turn out to be related to the massless scalar (Bose) and spinor (Fermi) fields in $1+1$-dimensional space. The symmetry explaines why the Legendre representations of the self-action changes and mass shifts of the electric and scalar charges, the sources of Bose fields, contain the spectral distributions characteristic of the Bose and Fermi fields in 1-dimensional space.

The symmetry explaines the limit values (36) of the mass shifts $\operatorname{Re}\Delta m_{1,0}$ for uniformly accelerated electric and scalar charges in $3+1$-dimensional space in terms of the nonzero and zero low-frequency limits of the heat capacity (or energy) spectral densities for Bose and Fermi gases in $1+1$-dimensional space:

$$\begin{aligned} &c^{B,F}(\omega/T)\big|_{\omega=0} = 1,\ 0; \\ &u^{B,F}(\omega) = \left.\frac{\omega}{e^{\omega/T} \mp 1}\right|_{\omega=0} = T,\ 0. \end{aligned} \qquad (42)$$

The appearance of the heat quantum mechanical distributions in the spectral representations of the dynamical mass shifts $\Delta m_{1,0}$ is no less intriguing than their appearance in the Hawking effect [1], especially when the absence of the horizons for the quasihyperbolic trajectory is taken into account.

According to the spectral formulas for Δm and $\widetilde{\Delta m}$, the proper field energy of the charges decreases at acceleration due to radiation at the frequencies

$$\omega_n = \frac{\pi(n+1/2)}{2\Delta\tau} \qquad (43)$$

with even $n = 0, 2, \ldots$, and increases due to excitation at the frequencies ω_n with odd $n = 1, 3, \ldots$ We can say that the proper field releases (deconfines) the excitations with even n and confines those with odd n. The rescaling of T and ω does not change this assertion. Eventually, for every finite $\theta > 0$, the radiation–excitation balance leads to

$$\operatorname{Re}\Delta m < 0, \quad \operatorname{Re}\Delta W > 0, \quad \operatorname{Im}\Delta W > 0.$$

Simultaneous radiation and excitation of the proper field of a charge at acceleration is supported by the positive and negative contributions with even- and odd-n frequencies ω_n to the imaginary part of the self-action change,

$$\operatorname{Im}\Delta W = \frac{1}{\pi}\operatorname{Re}\Delta W \cdot \ln\frac{1}{\lambda} + \cdots, \qquad (44)$$

more precisely, to its leading, infrared part, see (24) and (25).

Due to the symmetry, the quantities $\Delta W^{B,F}$ and $\Delta m^{B,F}$ for the mirror interacting with massless Bose or Fermi field can be obtained from $\Delta W_{1,0}$ and $\Delta m_{1,0}$ by the substitution $e^2 \to \hbar c$.

4. ARGUMENTS IN FAVOR OF THE VALUE $\alpha_0 = 1/4\pi$ OF THE BARE FINE STRUCTURE CONSTANT

At the collisions of charged particles (two electrons, for example), emission of soft photons occurs, which does not affect the motion of the colliding charges. As a result, the cross section of the particle scattering with the emission of n soft photons is given by formula (98.21) in [23],

$$d\sigma = d\sigma_{scat}\, w(n), \quad w(n) = \frac{\bar{n}^n}{n!}\, e^{-\bar{n}}, \qquad (45)$$

where $w(n)$ is the probability of the emission of n soft photons in the appropriate frequency interval (ω_1, ω_2) and $\bar{n}$ is their mean number, which can be found from classical electrodynamics. In this paper, the vacuum–vacuum amplitude is considered whose modulus squared is equal to $w(0) = e^{-\bar{n}}$.

It is important that the leading, logarithmic term of $\bar{n}$,

$$\bar{n} = \alpha\, \frac{2}{\pi}\, (\theta \operatorname{cth} \theta - 1) \left(\ln \frac{\omega_2}{\omega_1} + f(\theta) \right), \qquad (46)$$

(see Secs. 98, 120 in [23] and formula (24) in this paper, where $2\,\mathrm{Im}\, \Delta W_1 = \bar{n}$, $\mu = \omega_1$, and $w_0 = \omega_2$), is independent of the details of the charge motion and is determined by the invariant momentum transfer

$$\xi = \frac{q}{2m} = \frac{\sqrt{t}}{2m} = \operatorname{sh} \frac{\theta}{2},$$

which together with the total energy $\sqrt{-s}$ defines the main hard process. Thus, independently of the charge motion («trajectory») inside the forming region of the hard process, the mean number of photons emitted by the charge is defined by the global parameter — the momentum transfer or the Lorentz-invariant velocity change β_{12} of the charge in the above region, $\theta = \mathrm{Arth}\, \beta_{12}$.

The quantity $w(0) = e^{-\bar{n}}$ with the leading, logarithmic term for $\bar{n}$ is given by Abrikosov formula (136.11) in [23] for high energy and momentum transfer. It coincides with (46) where

$$\omega_1 = \omega_m, \quad \omega_2 = \varepsilon.$$

In the Abrikosov approximation, the effective (running) fine structure constant $\alpha_{eff}(q^2)$ [22, 23] does not differ from α,

$$\alpha_{eff}(q^2) = \frac{\alpha}{1 - (\alpha/3\pi)\, N_i \ln(q^2/m_i^2)}. \qquad (47)$$

Here, m_i and N_i are the masses and numbers of different-type vacuum charges screening the bare charge, $m_i < q$. For ultrahigh momentum transfers, this formula does not work.

If variant (b) of the Gell-Mann and Low paper [21] is realized in quantum electrodynamics, then on the distances less than some ultrashort $\Lambda^{-1} \ll m^{-1}$, QED is characterized by a finite point bare charge e_0 and the charge density $e_0\, \delta(\mathbf{x})$. In more detail, if the bare fine structure constant

$$\alpha_0 = \frac{e_0^2}{4\pi\hbar c}$$

is finite, then [21]

1) it is independent of the value of the fine structure constant α;

2) α must be less than α_0;

3) the charge density at very short distances reduces to the delta-function $e_0 \delta(\mathbf{x})$.

Therefore, at a collision of charges with the total energy

$$\sqrt{-s} = 2E$$

and momentum transfer

$$\sqrt{t} \approx 2E \gg \Lambda,$$

the cross section $d\sigma_{scat}$ is defined by the bare charge e_0, and $\bar{n}$ is given by the formula

$$\bar{n} = \alpha_0 \frac{2}{\pi} (\theta \operatorname{cth} \theta - 1) \left(\ln \frac{\omega_2}{\omega_1} + f_0(\theta) \right) \qquad (48)$$

if the frequencies ω_1 and ω_2 satisfy the condition

$$\Lambda \lesssim \omega_1 \ll \omega_2 \ll E.$$

In this case, although the photon emission comes from the ultrashort region of the order Λ^{-1} where the charge is point-like and equals e_0, it no longer affects the dynamics of the hard process. Under these conditions, the motion of each colliding charge is 1-dimensional and can be approximated by the classical trajectory with fixed $\theta = \mathrm{Arth}\, \beta_{12}$ related to this ultrashort region.

The symmetry discussed consists in the coincidence of the number spectrum of pairs of Bose (Fermi) massless quanta emitted by a point mirror in $1+1$-dimensional space with the number spectrum of photons (massless scalar quanta) emitted by a point electric (scalar) charge. The first one is obtained via quantum field theory with the corresponding zero boundary condition at the mirror, while the second is obtained by dividing the classical energy spectrum by $\hbar\omega$. The corresponding spectra coincide as functions of two variables and functionals of any common trajectory of the mirror and the charge. The only distinction in the factor $e^2/\hbar c$ can be removed if we set $e^2 = \hbar c$.

This symmetry is a consequence of

1) the invariant structure of scalar products in quantum theory of scalar and spinor fields;

2) the point-like structure of the mirror and the charge;

3) the fact that quantum emission does not affect the mirror and charge motions;

4) the space 1-dimensionality of the motion.

The 2-dimensional model of quantum field theory with a point-like mirror interacting with the secondary-quantized Bose (Fermi) massless field [4] is purely geometrical: it has no mass-dimension parameters and its Planck constant is dimensionless and equals 1. The usual Planck constant appears in the comparison of this quantum field theory model results with the results of QED, which involves charge, mass, and momenta and energies instead of wave vectors and frequencies, or with the results of classical electrodynamics, which involves charge, mass, and the energy of radiation.

The dimensionless factor $e^2/\hbar c$, whereby the number spectrum of soft photons ($\hbar\omega \ll mc^2$ in the proper system of a charge) in QED differs from the number spectrum of Bose pairs in the 2-dimensional quantum field theory model, is less than 1 because the charge in QED has a finite size of the order of $\hbar/mc$ due to the screening, while the mirror (the source of Bose pairs) is point-like.

If QED has a finite charge e_0 of a vanishingly small size for the ultrahigh energy and momentum transfer, then this size cannot be defined better than by setting it equal to the inverse energy $\hbar c/\sqrt{-s}$ of two head-on colliding charges. Therefore, it is reasonable to assume that as

$$\sqrt{-s} \approx \sqrt{t} \to \infty,$$

the spectrum of photons with frequencies

$$\Lambda \lesssim \hbar\omega \ll \sqrt{-s}$$

emitted by the bare charge e_0 does not differ from the spectrum of Bose pairs radiated by a point mirror. Then $e_0^2 = \hbar c$ and $\alpha_0 = 1/4\pi$. The Gell-Mann–Low properties of α_0 are fulfilled.

We consider the head-on collision of two electrons with mass m, charge e, and a very high energy E at infinity. The elastic scattering cross section depends on two invariants, s and t, which in the center-of-mass system are equal to

$$s = -4E^2, \quad t = 2p^2(1-\cos\vartheta),$$

where $E = \sqrt{p^2+m^2}$, p, and ϑ are the electron energy, momentum, and scattering angle in the center-of-mass system. At a fixed energy E, the smallest distance between the charges is attained at the largest momentum transfer, i.e., at $\vartheta = \pi$, when the charges move along the same straight line. In this case, each of them most deeply penetrates the screening coat of the other. If we suppose that the total energy is sufficiently high to penetrate the region where the electron charges become bare, then the minimal distance between them is equal to

$$r^c_{min} = \frac{\alpha_0}{2E} \tag{49}$$

in accordance with the classical theory.

But according to quantum mechanics, at a distance r between the charges, the uncertainty in their momentum is not less than $\Delta p \approx 1/r$. It may be thought that the charges cannot be separated by the distance less than r^q_{min}, at which the momentum uncertainty gives the energy greater than $2E$. Then

$$2E = 2\sqrt{M^2+\Delta p^2}, \quad \Delta p = \sqrt{E^2-M^2},$$

where M is the mass of the bare charge. The Gell-Mann–Low point-like nature of the bare charge forces one to assume that $M \sim E$.

For r^q_{min}, we have

$$r^q_{min} = \frac{1}{\Delta p} = \frac{1}{\sqrt{E^2-M^2}} = \frac{2E}{\alpha_0\sqrt{E^2-M^2}} r^c_{min}. \tag{50}$$

Because the minimal quantum distance is distinctly larger than the classical one, the turning point can be considered to be defined just by r^q_{min}. Then the proper acceleration of the charge at the turning point can be found from the equation

$$M\,w_0 = \frac{e_0^2}{4\pi\, r^{q\,2}_{min}} = \alpha_0\,(E^2-M^2). \tag{51}$$

The quantum motion of the charges is little affected by the emission of photons with frequencies not greater than w_0 because the ratio

$$\frac{w_0}{E} = \alpha_0\,\frac{E^2-M^2}{EM} \tag{52}$$

is small if α_0 is small and $M \sim E$. Therefore, for the calculation of soft photon emission, such motion can be approximated by the classical trajectory with the acceleration w_0 at the turning point. As a result, for $\bar{n}$, we obtain formula (48) with $\omega_2 = w_0$ and the parameter θ given by

$$\theta = 2\operatorname{Arsh}\frac{q}{2M}, \quad q = \Delta p = \sqrt{E^2-M^2}. \tag{53}$$

At $M \sim E$, the force acting on the charge e_0 according to (51) is of the order of $\alpha_0 M^2$ and is small in comparison both with the classical force M^2/α_0, when classical electrodynamics becomes inconsistent, and with the quantum force M^2, when QED requires quantum corrections, see Sec. 75 in [24].

About 45 years ago, Wigner remarked that special relativity is the physics of Lorentz transformations, and quantum mechanics is the physics of Fourier transformations. Processes induced by a point mirror in $1+1$-dimensional space are described by the simplest relativistic quantum theory, which is incarnated in the Bogoliubov coefficients. They are Lorentz-invariant scalar products reduced to Fourier transforms of massless scalar and spinor wave equation solutions. They can be considered a concentrate of genetic information about processes in $3+1$-dimensional space.

5. SELF-ACTION CHANGES $\Delta W_{1,0}$ AND THE TRACES $\mathrm{tr}\,\alpha^{B,F}$

The basis for the symmetry between the processes induced by the mirror in 2-dimensional and by the charge in 4-dimensional space–time is relations (11) and (12) between the Bogoliubov coefficients $\beta^{B,F}_{\omega'\omega}$ and the current density $j^\alpha(k)$ or charge density $\rho(k)$ depending on a time-like momentum k^α. The squares of these quantities represent the spectra of real pairs and particles radiated by the accelerated mirror and charge.

The symmetry is extended to the self-actions of the mirror and the charge and to the corresponding vacuum–vacuum amplitudes, cf. (18) and (19). In essence, it is embodied in the integral relation (16) between propagators of a massive pair in 2-dimensional space and of a single particle in 4-dimensional space.

Formula (18) for $W^{B,F}$ was obtained under the condition that the mean number $\bar{N}^{B,F}$ of pairs created is small and the interference of two or more pairs is negligible. In the general case, $W^{B,F}$ is given by formula (13), which can also be written as

$$2\,\mathrm{Im}\,W^{B,F} = \pm\mathrm{tr}\,\ln(\alpha^+\alpha)^{B,F}, \tag{54}$$

because

$$\alpha^+\alpha \mp \beta^+\beta = 1,$$

see [4, 6]. As can be seen from (13), the imaginary part of the action differs from zero and is then positive only if $\beta \neq 0$, i.e., if the radiation of real particles has indeed occurred.

For $W^{B,F}$, formula (54) allows choosing the expression

$$W^{B,F} = \pm i\,\mathrm{tr}\,\ln\,\alpha^{B,F}, \tag{55}$$

which was called natural by DeWitt [4]. However, this expression is by no means unique, the expressions involving $\alpha e^{i\gamma}$ or α^+ have the same imaginary part. Nevertheless, formula (55) is interesting as the definition of both the real and imaginary parts of the self-actions $W^{B,F}$ in terms of the Bogoliubov coefficients $\alpha^{B,F}_{\omega'\omega}$ only, which, according to formulas (11) and (12), reduce to the current density $j^\alpha(q)$ or to the charge density $\rho(q)$ dependent on the space-like momentum q^α. This means that the field of the corresponding perturbations propagates in the vacuum together with the mirror, comoves it, and, at the same time, contains the information about the radiation of real quanta.

Unfortunately, the author failed to find a simple integral representation for the matrix $\ln\alpha$. Nevertheless, if we again assume that the mean number of emitted particles is small, we can consider α, or $i\alpha$, or $\pm i\alpha^{B,F}$ close to 1. The last phase factor is most acceptable, as we see in what follows. Then, expanding $\ln(\pm i\alpha^{B,F})$ near $\pm i\alpha^{B,F} = 1$ and confining ourselves to the first term, we obtain

$$W^{B,F} = \pm i\,\mathrm{tr}\,\ln\,(\pm i\alpha^{B,F}) \approx \pm i\,\mathrm{tr}\,(\pm i\alpha^{B,F} - 1) = \\ = -\mathrm{tr}\,\alpha^{B,F} + \ldots \tag{56}$$

These qualitative arguments allow us to state that the functionals $\mathrm{tr}\,\alpha^{B,F}$ are similar to the corresponding self-actions with the opposite sign and must therefore have negative imaginary parts. This is confirmed by the general examples considered below, in which at least the initial or the final velocity of the mirror is subluminal.

However, as is shown in the next section, the above reasoning is very crude. The exact physical meaning of $\mathrm{tr}\,\alpha^{B,F}$ is conveyed by formula (99) or (102). As a result, each of the traces represents the mass shift of a field, entrained by an accelerated mirror, multiplied by the effective proper time of shift formation. This time is of the order of w_0^{-1}.

6. INVARIANT STRUCTURE OF THE BOGOLIUBOV COEFFICIENTS

Here, using the Bogoliubov coefficients for hyperbolic motion of the mirror [25, 26]

$$\alpha^{B,F}_{\omega'\omega} = \\ = \frac{2}{\sqrt{\varkappa\varkappa'}}\exp\left[i\left(\frac{\omega}{\varkappa} + \frac{\omega'}{\varkappa'}\right)\right] K_{1,0}\left(2i\sqrt{\frac{\omega\omega'}{\varkappa\varkappa'}}\right) = \\ = \frac{2}{w_0}\exp\left[i\frac{\rho}{w_0}\mathrm{ch}\,(\vartheta - \alpha)\right] K_{1,0}\left(\frac{i\rho}{w_0}\right), \tag{57}$$

$$\beta^{B,F*}_{\omega'\omega} = (-i)^{1,0}\frac{2}{\sqrt{\varkappa\varkappa'}} \times$$
$$\times \exp\left[i\left(-\frac{\omega}{\varkappa}+\frac{\omega'}{\varkappa'}\right)\right] K_{1,0}\left(2\sqrt{\frac{\omega\omega'}{\varkappa\varkappa'}}\right) =$$
$$= (-i)^{1,0}\frac{2}{w_0}\exp\left[-i\frac{\rho}{w_0}\mathrm{sh}\,(\vartheta-\alpha)\right] K_{1,0}\left(\frac{\rho}{w_0}\right) \quad (58)$$

as an example, we consider the invariant properties of the coefficients with respect to Lorentz transformations and the transformation properties with respect to transfer of the origin from one point on the trajectory to another.

The Bogoliubov coefficients are functionals of the trajectory and functions of the frequencies ω, ω' and parameters $\varkappa$, $\varkappa'$. The latter characterize the mirror trajectory

$$u^{mir} = g(v)$$

near the coordinate origin $u = v = 0$ chosen on the trajectory:

$$u^{mir} = g(v) =$$
$$= \frac{1}{\varkappa}\left(\varkappa' v + b(\varkappa' v)^2 + \frac{1}{3}c(\varkappa' v)^3 + \ldots\right). \quad (59)$$

The velocity and proper acceleration of the mirror at the point $u = v = 0$ are equal to

$$\beta_0 = \frac{1-\varkappa'/\varkappa}{1+\varkappa'/\varkappa}, \quad a_0 = -b\sqrt{\varkappa\varkappa'}. \quad (60)$$

Under the Lorentz transformation with the velocity $\beta = \mathrm{th}\,\delta$, the parameters $\varkappa$ and $\varkappa'$ are transformed just as the frequencies ω and ω',

$$\tilde{\omega} = \frac{\omega-\beta\omega}{\sqrt{1-\beta^2}} = \omega\, e^{-\delta}, \quad \tilde{\omega}' = \frac{\omega'+\beta\omega'}{\sqrt{1-\beta^2}} = \omega'\, e^{\delta}, \quad (61)$$

and the product

$$\tilde{\omega}\tilde{\omega}' = \omega\omega'$$

is invariant. Therefore, the frequencies ω, ω' and parameters $\varkappa$, $\varkappa'$ can be represented as

$$\omega = \sqrt{\omega\omega'}\, e^{\vartheta},\ \omega' = \sqrt{\omega\omega'}\, e^{-\vartheta}; \quad \varkappa = \sqrt{\varkappa\varkappa'}\, e^{\alpha},\ \varkappa' = \sqrt{\varkappa\varkappa'}\, e^{-\alpha}. \quad (62)$$

In the coordinate system moving with the velocity β_C relative to the laboratory system,

$$\beta = \beta_C = \frac{\omega-\omega'}{\omega+\omega'} = \mathrm{th}\,\vartheta, \quad \vartheta = \ln\sqrt{\frac{\omega}{\omega'}}, \quad (63)$$

the frequencies ω and ω' of the reflected and incident waves coincide and are equal to the invariant $\sqrt{\omega\omega'}$, while the vectors

$$k^\alpha = (k^1,\, k^0) = (\omega-\omega',\, \omega+\omega'), \quad q^\alpha = (q^1,\, q^0) = (-\omega-\omega',\, -\omega+\omega'), \quad (64)$$

have only temporal and only spatial components correspondingly:

$$k^\alpha_C = \left(0,\, 2\sqrt{\omega\omega'}\right), \quad q^\alpha_C = \left(-2\sqrt{\omega\omega'},\, 0\right). \quad (65)$$

These formulas were used in coefficients (57) and (58) for the hyperbolic trajectory

$$t(\tau) = \frac{\mathrm{sh}\,(w_0\tau-\alpha)+\mathrm{sh}\,\alpha}{w_0}, \quad x(\tau) = \frac{\mathrm{ch}\,\alpha-\mathrm{ch}\,(w_0\tau-\alpha)}{w_0}, \quad (66)$$

for which the proper acceleration is equal to

$$a_0 = -\sqrt{\varkappa\varkappa'} = -w_0.$$

The velocity of the mirror on this trajectory at the instant $w_0\tau$ is equal to

$$\beta(w_0\tau) = \frac{\dot{x}(w_0\tau)}{\dot{t}(w_0\tau)} = -\mathrm{th}\,(w_0\tau-\alpha). \quad (67)$$

The mirror passes the coordinate origin with the velocity

$$\beta_0 = \beta(0) = \mathrm{th}\,\alpha$$

at the instant $w_0\tau = 0$, passes the turning point at the instant $w_0\tau = \alpha$, $\beta(\alpha) = 0$, and at the instants $w_0\tau_{1,2} = \alpha \mp (\vartheta-\alpha)$ before and after the turn, its velocities are equal to

$$\beta(w_0\tau_{1,2}) = \pm\mathrm{th}\,(\vartheta-\alpha) = \pm\frac{\beta_C-\beta_0}{1-\beta_C\beta_0} = \pm\beta_{C0}. \quad (68)$$

The velocities β_C and β_{C0} are the velocities of the pair of waves ω and ω' in the laboratory system and in the system moving relative to the laboratory system with the velocity β_0. This last system is called the system of a detector that moves with the constant velocity β_0 and touches the mirror at the point $t = x = 0$.

Thus, the laboratory time intervals

$$\Delta t_{1,2} = t(w_0\tau_{1,2}) - t(\alpha) = \mp\frac{\mathrm{sh}\,(\vartheta-\alpha)}{w_0} \quad (69)$$

and the laboratory space intervals

$$\Delta x_{1,2} = x(w_0\tau_{1,2}) - x(\alpha) = -\frac{\mathrm{ch}\,(\vartheta-\alpha)-1}{w_0} \quad (70)$$

counted from the turning point define the time and length of the deceleration,

$$w_0\tau_1 \leqslant w_0\tau \leqslant \alpha,$$

and acceleration,

$$\alpha \leqslant w_0\tau \leqslant w_0\tau_2,$$

intervals on the world trajectory of the mirror where its velocity changes monotonically in the interval

$$-\beta_{C0} \leqslant \beta \leqslant \beta_{C0} \tag{71}$$

between the values opposite in sign to (68) and takes zero value at the turning point. It is assumed that $\vartheta > \alpha$. In the case where $\vartheta < \alpha$, the instant $\tau_2 < \tau_1$ and the deceleration and acceleration intervals are given by $w_0\tau_2 \leqslant w_0\tau \leqslant \alpha$ and $\alpha \leqslant w_0\tau \leqslant w_0\tau_1$ correspondingly.

We now show that the intervals $\Delta t_{1,2}$ and $\Delta x_{1,2}$ are Lorentz-invariant, i.e., are unchanged under the transition to another Lorentz coordinate system. Let the system $\tilde{K}$ move with the velocity $\beta = \operatorname{th}\delta$ relative to the laboratory system K. Then the mirror motion equations in the system $\tilde{K}$ become

$$\tilde{t}(w_0\tau) = \frac{t(w_0\tau) - \beta\, x(w_0\tau)}{\sqrt{1-\beta^2}} = \frac{\operatorname{sh}(\alpha-\delta) + \operatorname{sh}(w_0\tau - \alpha + \delta)}{w_0}, \tag{72}$$

$$\tilde{x}(w_0\tau) = \frac{x(w_0\tau) - \beta\, t(w_0\tau)}{\sqrt{1-\beta^2}} = \frac{\operatorname{ch}(\alpha-\delta) + \operatorname{ch}(w_0\tau - \alpha + \delta)}{w_0}, \tag{73}$$

differing from the nontransformed ones by the shift

$$\alpha \to \tilde{\alpha} = \alpha - \delta$$

of the parameter α.

The velocity of the mirror in the new system is

$$\tilde{\beta}(w_0\tau) = \frac{\dot{\tilde{x}}(w_0\tau)}{\dot{\tilde{t}}(w_0\tau)} = -\operatorname{th}(w_0\tau - \alpha + \delta). \tag{74}$$

At the instant $w_0\tau = 0$ of passage through the origin, the velocity is equal to

$$\tilde{\beta}_0 = \tilde{\beta}(0) = \operatorname{th}(\alpha - \delta);$$

the turning point is passed at the instant $w_0\tau = \alpha - \delta$.

Because the frequencies ω and ω' go over into the frequencies $\tilde{\omega}$ and $\tilde{\omega}'$ under the Lorentz transformation with the velocity $\beta = \operatorname{th}\delta$, with

$$\tilde{\omega} = \sqrt{\omega\omega'}\, e^{\vartheta-\delta}, \quad \tilde{\omega}' = \sqrt{\omega\omega'}\, e^{-\vartheta+\delta}, \tag{75}$$

and differ from the nontransformed ones by the shift

$$\vartheta \to \tilde{\vartheta} = \vartheta - \delta$$

of the parameter ϑ, the velocity $\beta_C = \operatorname{th}\vartheta$ of the pair of waves ω and ω' goes into the velocity

$$\tilde{\beta}_C = \operatorname{th}(\vartheta - \delta)$$

of the Lorentz-transformed pair of waves $\tilde{\omega}$ and $\tilde{\omega}'$. But the relative velocity of this pair of waves and the detector,

$$\tilde{\beta}_{C0} = \frac{\tilde{\beta}_C - \tilde{\beta}_0}{1 - \tilde{\beta}_C\tilde{\beta}_0} = \operatorname{th}(\vartheta - \alpha) = \beta_{C0}, \tag{76}$$

remains unchanges because

$$\tilde{\vartheta} - \tilde{\alpha} = \vartheta - \alpha,$$

see (62).

In the new system, the time and length of the intervals of deceleration,

$$w_0\tilde{\tau}_1 = 2\alpha - \vartheta - \delta \leqslant w_0\tau \leqslant \alpha - \delta,$$

and acceleration,

$$\alpha - \delta \leqslant w_0\tau \leqslant w_0\tilde{\tau}_2 = \vartheta - \delta,$$

from the same initial velocity

$$\tilde{\beta}(w_0\tilde{\tau}_1) = \operatorname{th}(\vartheta - \alpha)$$

to the same final velocity

$$\tilde{\beta}(w_0\tilde{\tau}_2) = -\operatorname{th}(\vartheta - \alpha)$$

are independent of the parameter δ and remain the previous functions of the Lorentz-invariant difference $\vartheta - \alpha = \tilde{\vartheta} - \tilde{\alpha}$:

$$\Delta\tilde{t}_{1,2} = \tilde{t}(w_0\tilde{\tau}_{1,2}) - \tilde{t}(\alpha - \delta) = \mp\frac{\operatorname{sh}(\vartheta - \alpha)}{w_0} = \Delta t_{1,2}, \tag{77}$$

$$\Delta\tilde{x}_{1,2} = \tilde{x}(w_0\tilde{\tau}_{1,2}) - \tilde{x}(\alpha - \delta) = -\frac{\operatorname{ch}(\vartheta - \alpha) - 1}{w_0} = \Delta x_{1,2}. \tag{78}$$

This difference is nothing but the proper time (multiplied by w_0) of the deceleration or acceleration.

For the parameter $\delta = \alpha$, the system $\tilde{K}$ moves with the velocity β_0 relative to the laboratory system and coincides with the proper system of the detector, touched by the mirror at its turning point $\tilde{t} = \tilde{x} = 0$. The frequencies $\tilde{\omega}$ and $\tilde{\omega}'$ of the waves of a pair in the detector system are denoted by Ω and Ω':

$$\Omega = \omega\sqrt{\frac{\varkappa'}{\varkappa}}, \quad \Omega' = \omega'\sqrt{\frac{\varkappa}{\varkappa'}}. \tag{79}$$

Evidently, they are Lorentz-invariant quantities.

In this system, $\tilde{\beta}_0 = 0$ and the invariant relative velocity

$$\beta_{C0} = \tilde{\beta}_{C0} = \tilde{\beta}_C = \frac{\Omega - \Omega'}{\Omega + \Omega'} = \operatorname{th}\Theta = \operatorname{th}(\vartheta - \alpha),$$
$$\Theta = \ln\sqrt{\frac{\Omega}{\Omega'}} = \ln\sqrt{\frac{\omega\varkappa'}{\omega'\varkappa}} = \vartheta - \alpha, \tag{80}$$

coincides with the velocity $\tilde{\beta}_C$ of the pair of waves Ω and Ω' and is defined by the ratio Ω/Ω' of the transformed frequencies only.

The intervals $\Delta\tilde{t}_{1,2}$ and $\Delta\tilde{x}_{1,2}$ are given by formulas (77) and (78), where

$$\delta = \alpha, \quad w_0\tilde{\tau}_{1,2} = \mp(\vartheta - \alpha), \quad \tilde{t}(0) = \tilde{x}(0) = 0.$$

Therefore,

$$\Delta\tilde{t}_{1,2} = \tilde{t}(w_0\tilde{\tau}_{1,2}) = \mp\frac{\operatorname{sh}(\vartheta - \alpha)}{w_0} = \Delta t_{1,2}, \tag{81}$$

$$\Delta\tilde{x}_{1,2} = \tilde{x}(w_0\tilde{\tau}_{1,2}) = -\frac{\operatorname{ch}(\vartheta - \alpha) - 1}{w_0} = \Delta x_{1,2}. \tag{82}$$

At switching off the acceleration, the trajectory of the mirror coincides with the trajectory of the detector, and $\alpha_{\omega'\omega}$ becomes the matrix diagonal in frequencies (79):

$$\alpha^{B,F}_{\omega'\omega} = 2\pi\,\delta(\Omega - \Omega'). \tag{83}$$

Its functional dependence on the trajectory then reduces to the dependence on the parameter

$$\beta_0 = \operatorname{th}\alpha = \operatorname{th}\left(\ln\sqrt{\varkappa/\varkappa'}\right)$$

or the Doppler factor $\sqrt{\varkappa/\varkappa'}$ entering Ω and Ω'.

In the absence of acceleration, the frequencies ω and ω' satisfy the condition $\Omega = \Omega'$, and the velocities β_C and β_0 coincide. Acceleration leads to nonzero Bogoliubov coefficients $\beta_{\omega'\omega} \neq 0$ and to the absence of the relation $\Omega = \Omega'$ or $\beta_C = \beta_0$. The distinction between the frequencies Ω and Ω' or the velocities β_C and β_0 can be described by the invariant relative velocity β_{C0}, see (68) and (76), and leads to the appearance of invariant phases of the Bogoliubov coefficients defined by this parameter.

With intervals (69) and (70), the Bogoliubov coefficients can be written as

$$\alpha^{B,F^*}_{\omega'\omega} = \frac{2}{w_0}\exp\left(-i\rho\Delta x_2 + \frac{i\rho}{w_0}\right)K_{1,0}\left(\frac{i\rho}{w_0}\right),$$
$$\beta^{B,F^*}_{\omega'\omega} = \frac{2(-i)^{1,0}}{w_0}\exp(-i\rho\Delta t_2)K_{1,0}\left(\frac{\rho}{w_0}\right), \tag{84}$$

i.e., in the form of eigenfunctions of the invariant operators $-i\partial/\partial\Delta x_2$ and $i\partial/\partial\Delta t_2$:

$$-i\frac{\partial\alpha}{\partial\Delta x_2} = -\rho\,\alpha, \quad i\frac{\partial\beta^*}{\partial\Delta t_2} = \rho\,\beta^*, \tag{85}$$

with invariant eigenvalues of the momentum transfer $-\rho$ and mass ρ correspondingly.

Thus, the phases of coefficients (84) are defined by the length $\Delta x_{1,2}$ or the time $\Delta t_{1,2}$ of motion of the mirror near the turning point, where the velocity of the mirror changes its sign and does not exceed in magnitude the velocity of a pair created with a time-like momentum.

In one and the same laboratory system, two coordinate systems K and K' can be introduced that are related by a parallel shift of space–time coordinates

$$x = x_1 + x', \quad t = t_1 + t'. \tag{86}$$

Monochromatic in- and out-waves in the K and K' systems differ only by phase factors

$$\exp(-i\omega' v) = \exp(-i\omega' v_1)\exp(-i\omega' v'),$$
$$\exp(-i\omega u) = \exp(-i\omega u_1)\exp(-i\omega u'). \tag{87}$$

Therefore, the Bogoliubov coefficients in the systems K and K' also differ by phase factors:

$$\alpha_{\omega'\omega} = \exp(-i(q\Delta))\alpha'_{\omega'\omega},$$
$$-(q\Delta) = \omega' v_1 - \omega u_1,$$
$$\beta^*_{\omega'\omega} = \exp(-i(k\Delta))\beta'^*_{\omega'\omega},$$
$$-(k\Delta) = \omega' v_1 + \omega u_1, \tag{88}$$

where $\Delta^\alpha = (x_1, t_1)$ is the 2-vector of the shift, and k^α and q^α are the wave 2-vectors (9) and (10).

In particular, the origin $x = t = 0$ of the coordinate system K can be chosen at the point of the trajectory where the mirror has a nonzero velocity β_0, and the origin $x' = t' = 0$ of the coordinate system K' at the turning point, where $\beta_1 = 0$. Then x_1 and t_1 are the coordinates of the turning point in the K-system. In this case, for the hyperbolic trajectory, we have

$$u_1 = \frac{1}{w_0} - \frac{1}{\varkappa}, \quad v_1 = \frac{1}{\varkappa'} - \frac{1}{w_0},$$
$$w_0 = \sqrt{\varkappa\varkappa'}, \quad \beta_0 = \operatorname{th}\alpha, \quad \alpha = \ln\sqrt{\frac{\varkappa}{\varkappa'}}. \tag{89}$$

The phases of the corresponding factors in (88) are equal to the differences of phases of Bogoliubov coefficients (57) and (58) with nonzero and zero values of the parameter α:

$$\begin{aligned} -(q\Delta) &= \frac{\omega}{\varkappa} + \frac{\omega'}{\varkappa'} - \frac{\omega+\omega'}{w_0} = \\ &= \frac{\rho}{w_0}\operatorname{ch}(\vartheta-\alpha) - \frac{\rho}{w_0}\operatorname{ch}\vartheta, \\ -(k\Delta) &= -\frac{\omega}{\varkappa} + \frac{\omega'}{\varkappa'} - \frac{-\omega+\omega'}{w_0} = \\ &= -\frac{\rho}{w_0}\operatorname{sh}(\vartheta-\alpha) + \frac{\rho}{w_0}\operatorname{sh}\vartheta. \end{aligned} \tag{90}$$

The phases of the Bogoliubov coefficients can be written as the scalar products

$$\frac{\omega}{\varkappa} + \frac{\omega'}{\varkappa'} = -(q\Delta x), \quad -\frac{\omega}{\varkappa} + \frac{\omega'}{\varkappa'} = -(k\Delta x) \tag{91}$$

of 2-vectors q^α and k^α defined only by the frequencies ω and ω' and a space-like 2-vector Δx^α defined only by the parameters $\varkappa$ and $\varkappa'$:

$$\Delta x^1 = \frac{1}{2\varkappa} + \frac{1}{2\varkappa'}, \quad \Delta x^0 = \frac{1}{2\varkappa'} - \frac{1}{2\varkappa}. \tag{92}$$

The length of Δx^α is given by $1/\sqrt{\varkappa\varkappa'}$, which is equal to $1/w_0$ for the hyperbolic trajectory.

Consequently, we have the following forms for the phases:

$$\begin{aligned} -\rho\left(\Delta x_2 - \frac{1}{w_0}\right) &= -(q\Delta x) = \frac{\rho}{w_0}\operatorname{ch}(\vartheta-\alpha), \\ -\rho\,\Delta t_2 &= -(k\Delta x) = -\frac{\rho}{w_0}\operatorname{sh}(\vartheta-\alpha). \end{aligned} \tag{93}$$

The vector Δx^α is closely related to the acceleration 2-vector a^α that for the trajectory $u^{mir} = g(v)$ is given by the expression

$$a^\alpha = (a^1, a^0) = -\frac{g''}{4g'^2}(1+g', 1-g'), \quad g = g(v). \tag{94}$$

At the point $u = v = 0$, we obtain

$$a_0^\alpha = a_0\frac{\Delta x^\alpha}{\sqrt{\Delta x^2}}, \quad a_0 = -b\sqrt{\varkappa\varkappa'}, \tag{95}$$

where a_0 is the proper acceleration at zero point.

The Lorentz-invariant quantity $\operatorname{tr}\alpha$ was defined in [26] by the formula

$$\operatorname{tr}\alpha = = \iint\limits_0^\infty \frac{d\omega d\omega'}{(2\pi)^2}\,\alpha_{\omega'\omega}\,2\pi\,\delta\left(\sqrt{\frac{\varkappa'}{\varkappa}}\omega - \sqrt{\frac{\varkappa}{\varkappa'}}\omega'\right), \tag{96}$$
$$\Omega = \sqrt{\frac{\varkappa'}{\varkappa}}\omega, \quad \Omega' = \sqrt{\frac{\varkappa}{\varkappa'}}\omega',$$

in which the Lorentz-invariant argument of the δ-function is the difference of the frequencies Ω and Ω' of the reflected and incident waves in the proper system of the mirror at zero point $u = v = 0$, where the mirror has the velocity β_0 and acceleration $a_0 = -b\sqrt{\varkappa\varkappa'}$. The factors $\sqrt{\varkappa'/\varkappa}$ and $\sqrt{\varkappa/\varkappa'}$ are the Doppler factors relating the frequencies in the laboratory system and zero point proper system of the mirror (or the proper system of the detector).

Thus, in the trace formation of the matrix α, its elements diagonal in the invariant frequencies are involved, i.e., the elements $\alpha_{\omega'\omega}$ where $\omega/\varkappa = \omega'/\varkappa'$. We note that the matrix elements $\alpha_{\omega'\omega}$ and $\beta^*_{\omega'\omega}$, being scalar functions of the frequencies ω and ω', can be written in the detector system if we perform the changes

$$\begin{aligned} &\omega,\, \omega' \to \Omega,\, \Omega', \\ &u,\, v \to U = \sqrt{\frac{\varkappa}{\varkappa'}}u,\ V = \sqrt{\frac{\varkappa'}{\varkappa}}v, \\ &f(u),\, g(v) \to F(U) = \sqrt{\frac{\varkappa'}{\varkappa}}f(u), \\ &G(V) = \sqrt{\frac{\varkappa}{\varkappa'}}g(v), \end{aligned} \tag{97}$$

in their expressions (2). Then

$$\alpha_{\omega'\omega} = A_{\Omega'\Omega}, \quad \beta^*_{\omega'\omega} = B^*_{\Omega'\Omega}, \tag{98}$$

and the diagonal elements $A_{\Omega\Omega}$ with $\Omega = \Omega' = \sqrt{\omega\omega'}$ are involved in trace (96).

For the trajectories in the Minkowski plane on the left of their tangent line $X^\alpha(\tau')$ at zero point, the coordinate

$$z^1 = X^1(\tau') - x^1(\tau) \geqslant 0.$$

For these trajectories, $\operatorname{tr}\alpha$ can be transformed to the form [26]

$$\begin{aligned} &\operatorname{tr}\alpha^{B,F} = \\ &= \pm i \iint d\tau d\tau' \left\{ \begin{array}{c} \dot{x}_\alpha(\tau)\dot{X}^\alpha(\tau') \\ 1 \end{array} \right\} \Delta_4^{LR}(z,\nu), \\ &z^\alpha = X^\alpha(\tau') - x^\alpha(\tau), \end{aligned} \tag{99}$$

where the singular function $\Delta_4^{LR}(z,\nu)$ differs from the causal function $\Delta_4^f(z,\mu)$ by complex conjugation and the replacement $\mu \to i\nu$ (or by the replacement $z^2 \to -z^2$, $\mu \to \nu$):

$$\begin{aligned} \Delta_4^{LR}(z,\nu) = \frac{1}{4\pi}\delta(z^2) - \frac{\nu}{8\pi\sqrt{z^2}}\theta(z^2)H_1^{(2)}\left(\nu\sqrt{z^2}\right) + \\ + i\frac{\nu}{4\pi^2\sqrt{-z^2}}\theta(-z^2)K_1\left(\nu\sqrt{-z^2}\right). \end{aligned} \tag{100}$$

The expression obtained allows interpreting $\operatorname{tr}\alpha^{B,F}$ as a functional describing the interaction of two vector or scalar sources by means of exchange by vector or scalar quanta with space-like momenta. At the same time, one of the sources moves along the trajectory of the mirror and the other moves along the line tangent to it at zero point. The last source can be considered a probe or detector of excitation created by the accelerated mirror in the vacuum.

Because the detector moves with a constant velocity β_0, its 2-velocity $\dot X^\alpha(\tau')$ is independent of τ'. Consequently,

$$\dot x_\alpha(\tau)\dot X^\alpha(\tau') = -\gamma_*(\tau)$$

is the relative Lorentz factor defined by the relative velocity $\beta_*(\tau)$ of the mirror and detector:

$$\gamma_*(\tau) = \frac{1-\beta(\tau)\,\beta_0}{\sqrt{1-\beta^2(\tau)}\,\sqrt{1-\beta_0^2}} = \frac{1}{\sqrt{1-\beta_*^2(\tau)}}, \quad \beta_*(\tau) = \frac{\beta(\tau)-\beta_0}{1-\beta(\tau)\,\beta_0}, \tag{101}$$

and is a Lorentz-invariant quantity for each τ. Then

$$\operatorname{tr}\alpha^{B,F} = -i\int d\tau \left\{ \begin{matrix} \gamma_*(\tau) \\ 1 \end{matrix} \right\} J(\tau,\nu), \quad J(\tau,\nu) = \int d\tau'\,\Delta_4^{LR}(z(\tau,\tau'),\nu). \tag{102}$$

It can be seen from this representation that at $\theta \neq \infty$, when the Lorentz factor $\gamma_*(\tau)$ is bounded on the whole trajectory, both traces have the same qualitative behavior as the parameter $\nu \to 0$. It is clear that their infrared (logarithmic) singularities in this parameter occur due to the behavior of the integral $J(\tau,\nu)$ as $\tau \to \pm\infty$. For the trajectories with subluminal relative velocities β_{10} and β_{20} of the ends, both $\operatorname{tr}\alpha^{B,F}$ have infrared singularities at $\nu = 0$. Besides, the singularities of $\operatorname{tr}\alpha^B$ differ from those of $\operatorname{tr}\alpha^F$ only by the values of the relative Lorentz factor $\gamma_*(\tau)$ for initial and final ends of the trajectory, i.e., by the factors $1/\sqrt{1-\beta_{10}^2}$ and $1/\sqrt{1-\beta_{20}^2}$. Because the infrared singularities from the initial and final ends occur in $\operatorname{tr}\alpha^F$ with the factors

$$\frac{\sqrt{1-\beta_{10}^2}}{2\beta_{10}}, \quad \frac{\sqrt{1-\beta_{20}^2}}{2|\beta_{20}|}, \tag{103}$$

they disappear in $\operatorname{tr}\alpha^F$ for the trajectories with luminal velocities of the ends, $\beta_{10} = 1$, $\beta_{20} = -1$, but remain in $\operatorname{tr}\alpha^B$. The disappearance of singularities in $\operatorname{tr}\alpha^F$ for such trajectories means that the function $J(\tau,\,\nu)$ is integrable in τ at $\tau \to \pm\infty$ even if $\nu = 0$. At the same time, the function $\gamma_*(\tau)\,J(\tau,\nu)$ is integrable in this region only at $\nu \neq 0$.

The weakening of interaction of scalar charges with increasing their relative velocity, contrary to the constancy of the interaction of electric charges, is related to a different geometrical structure of scalar and vector field sources $\rho(x)$ and $j^\alpha(x)$. They are given by Eq. (4) for point-like charges moving along the trajectory $x^\alpha(\tau)$.

The charges of the scalar and vector field sources are defined by the space integrals of their charge densities $\rho(\mathbf{x},t)$ and $j^0(\mathbf{x},t)$, and for point-like sources are equal to

$$Q_0,\ Q_1 = \int d^3x\,\{\rho(\mathbf{x},t),\ j^0(\mathbf{x},t)\} = e\int d\tau\{1,\ \dot x^0(\tau)\}\,\delta(t-x^0(\tau)) = e\{\gamma^{-1}(t),\ 1\}, \tag{104}$$

because

$$\frac{d\tau}{dt'} = \gamma^{-1}(t') \quad \text{if} \quad t' = x^0(\tau).$$

Obviously, the charge for the point-like source $T^{\alpha\beta}(x)$ of a tensor field with spin 2 increases as the particle energy,

$$Q_2 = e\gamma(t).$$

As can be seen from the regularized representation

$$\operatorname{tr}\alpha^{B,F}|_{reg} = \frac{1}{2\pi}\int_0^\infty ds \left[\int_{-\infty}^\infty dx \left\{1,\ \sqrt{G'(x)}\right\} \times \exp(-is(G(x)-x)) - \sqrt{\frac{\pi}{ibs}}\right], \quad s = \frac{\omega}{\varkappa}, \tag{105}$$

obtained in [26], the ultraviolet divergences in $\operatorname{tr}\alpha^{B,F}$ are removed by subtraction from the first integrand of its asymptotic expansion in s as $s \to \infty$. The invariant variable

$$s = \frac{\omega}{\varkappa} = \sqrt{\frac{\omega\omega'}{\varkappa\varkappa'}} = \frac{b\rho}{2w_0}$$

is proportional to the momentum transfer ρ in units of the proper acceleration w_0 of the mirror at the point of its tangency with the detector. The subtracted term, being integrated over ρ up to a large but finite ρ_{max},

$$\frac{1}{2\pi}\int_0^{s_{max}} ds\sqrt{\frac{\pi}{ibs}} = \frac{1}{2\pi}\sqrt{\frac{\pi\rho_{max}}{w_0}}(1-i), \tag{106}$$

is one and the same for Bose and Fermi cases and explicitly depends on the acceleration.

When the space interval Δx between the mirror and the detector becomes less than $\hbar/2\Delta p$, the uncontrolled momentum transfer between them becomes greater than Δp and leads to an ultraviolet divergence in nonregularized $\mathrm{tr}\,\alpha^{B,F}$. Because the mirror coordinate near the point of tangency with detector changes in time according to the law

$$x(t) = -\frac{w_0 t^2}{2},$$

the time interval τ necessary for the momentum transfer Δp is of the order of

$$\tau \approx 2\sqrt{\frac{\hbar}{\Delta p\, w_0}} = \frac{2}{\sqrt{w_0 \rho_{max}}}$$

if we set

$$\Delta p = \hbar \rho_{max}.$$

Then the subtracted term that regularizes $\mathrm{tr}\,\alpha^{B,F}$ acquires the form

$$\begin{gathered}\frac{1}{2\pi}\sqrt{\frac{\pi\rho_{max}}{w_0}}(1-i) = \frac{1}{4\pi}\sqrt{\pi}\rho_{max}(1-i)\tau,\\ \tau \approx \frac{2}{\sqrt{w_0\rho_{max}}}.\end{gathered} \tag{107}$$

As distinct from (20), this term has the same sign for Bose and Fermi cases. This can be understood as a consequence of a positive momentum transfer from the detector to the mirror in both cases. The differences in the meaning of

$$\rho_{max} \approx \frac{1}{\sqrt{2\varepsilon}}$$

and τ are more understandable.

Unlike $\Delta W_{1,0}$, describing the change of self-action of charges due to acceleration, the functionals $\mathrm{tr}\,\alpha^{B,F}$ describe the interaction of the accelerated mirror with the probe executing uniform motion along the tangent to the trajectory of the mirror at the point where it has the acceleration w_0. This interaction is transmitted by vector or scalar perturbations created by the mirror in the vacuum of the Bose or Fermi field and carrying a space-like momentum of the order of w_0. According to Eq. (100), at distances of the order of w_0^{-1} from the mirror, the field of these perturbations decreases exponentially in time-like directions and oscillates with damped amplitude in space-like directions. It can be said that such a field moves together with the mirror and is its «proper field». Hence, the probe interacts with the mirror for a time of the order of w_0^{-1}, while the charge constantly interacts with itself and feels the change of interaction over all the acceleration time. Therefore, it is not surprising that $-\mathrm{tr}\,\alpha^{B,F}$ coincide in essence with $\Delta W_{1,0}$ if in these latter we set

$$\tau_2 - \tau_1 = \frac{2\pi}{w_0}, \quad e^2 = 1.$$

In other words, $\mathrm{tr}\,\alpha^{B,F}$ are the mass shifts of the mirror proper field multiplied by a characteristic proper time of their formation.

7. INTERACTION WITH THE PROPER FIELD OF AN ACCELERATED MIRROR MOVING WITH SUBLUMINAL VELOCITY

For a trajectory with subluminal velocities of the ends, $\mathrm{tr}\,\alpha$ is an invariant function of the relative velocities β_{12}, β_{10}, and β_{20} connected by the relation

$$\beta_{12} = \frac{\beta_{10}-\beta_{20}}{1-\beta_{10}\beta_{20}}.$$

We consider the regularized $\mathrm{tr}\,\alpha^{B,F}$ for two important trajectories.

1. Quasihyperbolic trajectory, given by formula (22), is time-reversed to itself. Its representation in the $(u,\,v)$-variables is

$$\begin{gathered}u^{mir} = g(v) = v\,\mathrm{ch}\,\theta - \frac{\beta_1}{w_0}\mathrm{sh}\,\theta + \\ + \mathrm{sh}\,\theta\sqrt{\left(v-\frac{\beta_1^2}{w_0}\right)^2 + a^2}, \\ a = \frac{\beta_1\sqrt{1-\beta_1^2}}{w_0}, \quad \beta_1 = \mathrm{th}\,\frac{\theta}{2}.\end{gathered} \tag{108}$$

The initial β_1 and final $\beta_2 = -\beta_1$ velocities are subluminal.

Using this expression in representation (2) and introducing the variable

$$x = v - \frac{\beta_1^2}{w_0},$$

we obtain

$$\begin{gathered}\alpha^B_{\omega'\omega} = 2\sqrt{\frac{\omega'}{\omega}}\exp\left(i\frac{\omega+\omega'}{w_0}\beta_1^2\right)\times \\ \times\int\limits_0^\infty dx\,\cos[(\omega'-\omega\,\mathrm{ch}\,\theta)x]\times \\ \times\exp\left(-i\omega\,\mathrm{sh}\,\theta\sqrt{x^2+a^2}\right).\end{gathered} \tag{109}$$

According to formulas (9) and (15) in Sec. 2.5.25 in [27], this integral reduces to the modified Bessel and Hankel functions, and we finally have

$$\alpha^B_{\omega'\omega} = 2ia\,\mathrm{sh}\,\theta\sqrt{\frac{\omega\omega'}{Q}}\exp\left(i\frac{\omega+\omega'}{w_0}\beta_{10}^2\right)\times \times K_1\left(a\sqrt{Q}\right), \quad \text{if} \quad Q=\omega^2+\omega'^2-2\omega\omega'\,\mathrm{ch}\,\theta>0,$$
$$\alpha^B_{\omega'\omega} = -\pi a\,\mathrm{sh}\,\theta\sqrt{\frac{\omega\omega'}{-Q}}\exp\left(i\frac{\omega+\omega'}{w_0}\beta_{10}^2\right)\times \times H_1^{(2)}\left(a\sqrt{-Q}\right), \quad \text{if} \quad Q=\omega^2+\omega'^2-2\omega\omega'\,\mathrm{ch}\,\theta<0. \tag{110}$$

As usual,

$$\theta = \mathrm{Arth}\,\beta_{12}$$

is a Lorentz-invariant parameter defined by the relative velocity of the ends.

The corresponding Bogoliubov coefficient for the Fermi case is more complicated:

$$\alpha^F_{\omega'\omega} = a\exp\left(i\frac{\omega+\omega'}{w_0}\beta_{10}^2\right)\int\limits_{-\infty}^{\infty}dt\sqrt{\mathrm{sh}^2 t+\mathrm{ch}^2\frac{\theta}{2}}\times \times\exp\left[ia\left((\omega'-\omega)\mathrm{ch}\frac{\theta}{2}\,\mathrm{sh}\,t - (\omega'+\omega)\,\mathrm{sh}\,\frac{\theta}{2}\,\mathrm{ch}\,t\right)\right]. \tag{111}$$

Because the velocity of the mirror at the point $u=v=0$ (and hence the detector velocity) is equal to zero, $\beta_0=0$, the initial and final velocities β_1 and β_2 can be regarded as invariant relative velocities $\beta_1=\beta_{10}$ and $\beta_2=\beta_{20}$ of the mirror and detector at $t=\mp\infty$.

According to definition (105), we obtain

$$\mathrm{tr}\,\alpha^B|_{reg} = \frac{\mathrm{cth}\,\theta/2}{2\pi}\left[-\frac{\pi}{2}-i(\ln\frac{2}{\gamma\varepsilon}-1)\right], \quad \varepsilon=\frac{\nu}{w_0}, \tag{112}$$

$$\mathrm{tr}\,\alpha^F|_{reg} = \frac{1}{2\pi}\left\{\frac{1}{\mathrm{sh}\,\theta/2}\left[-\frac{\pi}{2}-i(\ln\frac{2}{\gamma\varepsilon}-1)\right] + i\left[\mathrm{th}\,\frac{\theta}{2}\,\mathbf{B}(k)+\frac{\ln\,\mathrm{ch}\,\theta/2}{\mathrm{sh}\,\theta/2}\right]\right\}, \tag{113}$$

where

$$\mathbf{B}(k) = \int\limits_0^{\pi/2}\frac{\cos^2\varphi\,d\varphi}{\sqrt{1-k^2\sin^2\varphi}}, \quad k=\mathrm{th}\,\frac{\theta}{2},$$

is one of the elliptic integrals [14].

In both $\mathrm{tr}\,\alpha^{B,F}$, the infrared singularities were removed by introducing the small parameter ε (the least momentum transfer in w_0 units), while the ultraviolet singularities were eliminated as was written above.

The function

$$R(\theta) = \mathrm{th}\,\frac{\theta}{2}\,\mathbf{B}(k)+\frac{\ln\,\mathrm{ch}\,\theta/2}{\mathrm{sh}\,\theta/2}$$

is equal to zero at $k=0$, grows almost linearly with k, reaches the maximum value $R\approx 1.28$ at $k\approx 0.97$ and then decays rapidly to 1 as $k\to 1$.

2. The Airy semiparabola with the in-tangent line to the inflection point is given by

$$\varkappa u^{mir}(v) = \begin{cases} \left(1-\dfrac{b^2}{c}\right)\varkappa' v-\dfrac{b^3}{3c^2}, & -\infty<v\leqslant v_0,\\ \varkappa' v+b\,\varkappa'^2v^2+\dfrac{1}{3}c\varkappa'^3v^3, & v_0\leqslant v<\infty, \end{cases} \tag{114}$$

where the inflection point is $v_0=-b/\varkappa' c$, $b>0$, and $c>b^2$ because the trajectory is time-like. The initial velocity is subluminal, but the final one is luminal.

Using this trajectory in integral representations for the Bogoliubov coefficients, we find that

$$\alpha^B_{\omega'\omega} = \sqrt{\frac{s'/s}{\varkappa\varkappa'}}(cs)^{-1/3}\exp\left(i\frac{b}{c}(s-s')-i\frac{2}{3}w^{3/2}\right)\times \times\left[\pi\,\mathrm{Ai}(z)-i\left(\pi\,\mathrm{Gi}(z)-\frac{1}{z}\right)\right], \tag{115}$$

$$\alpha^F_{\omega'\omega} = \frac{(cs)^{-1/3}}{\sqrt{\varkappa\varkappa'(\alpha+1)}}\exp\left(i\frac{b}{c}(s-s')-i\frac{2}{3}w^{3/2}\right)\times \times\left[\frac{i\sqrt{\alpha}}{z}+\frac{1}{\sqrt{w}}\int\limits_0^{\infty}dt\sqrt{t^2+\alpha w}\times \times\exp\left(-izt-\frac{it^3}{3}\right)\right], \tag{116}$$

where $\mathrm{Ai}(z)$ and $\mathrm{Gi}(z)$ are the well-known Airy and Scorer functions defined as in [28], and

$$z=(cs)^{-1/3}(s-s')-w, \quad w=(b/c)^2(cs)^{2/3},$$
$$s=\omega/\varkappa, \quad s'=\omega'/\varkappa', \quad \alpha=c/b^2-1.$$

The parameter

$$\alpha=\frac{1-\beta_{10}}{2\beta_{10}}$$

is defined by the initial relative velocity β_{10} of the mirror and the detector, $\beta_{20} = -1$.

In finding $\operatorname{tr}\alpha^B$, the integral

$$\operatorname{tr}\alpha^B = \frac{1}{2\pi}\frac{3}{2}(\alpha+1)\int_0^\infty dw\,\exp\left(-i\frac{2}{3}w^{3/2}\right)\times \left[\pi\mathrm{Ai}(-w) - i\left(\pi\mathrm{Gi}(-w) + \frac{1}{w}\right)\right] \quad (117)$$

appears, which diverges at both the lower and the upper limits. The infrared divergence is removed by introducing the nonzero lower limit

$$w_1 = \left(\frac{\varepsilon}{2(\alpha+1)^2}\right)^{2/3},$$

where $\varepsilon = \nu/w_0 \ll 1$. To eliminate the ultraviolet divergence, we subtract from the integrand the first term $\sqrt{\pi}e^{-i\pi/4}w^{-1/4}$ of its asymptotic expansion as $w \to \infty$. It is then possible to turn the integration contour by the angle $-\pi/3$ and, introducing the integration variable

$$t = \exp(i\pi/3)w,$$

to bring the regularized integral to the form

$$\operatorname{tr}\alpha^B|_{reg} = \frac{1}{2\pi}\frac{3}{2}(\alpha+1)\times \left\{-\frac{\pi}{3} - i\int_{t_1}^\infty \frac{dt}{t}\exp\left(-\frac{2}{3}t^{3/2}\right) + i\int_0^\infty dt\,\pi\mathrm{Gi}(t)\exp\left(-\frac{2}{3}t^{3/2}\right)\right\}. \quad (118)$$

In these transformations, we used the formulas

$$\begin{aligned} &\mathrm{Ai}\left(\exp\left(\frac{2\pi i}{3}\right)t\right) = \\ &= \frac{1}{2}\exp\left(\frac{i\pi}{3}\right)[\mathrm{Ai}(t) - i\mathrm{Bi}(t)], \\ &\mathrm{Gi}\left(\exp\left(\frac{2\pi i}{3}\right)t\right) = -\exp\left(\frac{i\pi}{3}\right)\mathrm{Gi}(t) + \\ &+ \frac{1}{2}\exp\left(-\frac{i\pi}{6}\right)[\mathrm{Ai}(t) + i\mathrm{Bi}(t)], \\ &\int_0^\infty dt\left(\pi\mathrm{Bi}(t)\exp\left(-\frac{2}{3}t^{3/2}\right) - \frac{\sqrt{\pi}}{t^{1/4}}\right) = 0. \end{aligned} \quad (119)$$

The last integral in (118) is equal to

$$\frac{2}{3} + \frac{2}{9}\ln 2.$$

As a result, we finally obtain

$$\operatorname{tr}\alpha^B|_{reg} = \frac{1}{2\pi}(\alpha+1)\left\{-\frac{\pi}{2} - i\left[\ln\frac{3(\alpha+1)^2}{\gamma\varepsilon} - 1 - \frac{1}{3}\ln 2\right]\right\}, \quad \varepsilon = \frac{\nu}{w_0}. \quad (120)$$

The evaluation of $\operatorname{tr}\alpha^F$ follows a similar way. The integral

$$\operatorname{tr}\alpha^F = \frac{1}{2\pi}\frac{3}{2}\sqrt{\alpha+1}\int_0^\infty dw\exp\left(-i\frac{2}{3}w^{3/2}\right)\times \left[\frac{1}{\sqrt{w}}\int_0^\infty dt\,\sqrt{t^2+\alpha w}\,\times \exp\left(iwt - \frac{it^3}{3}\right) - \frac{i}{w}\sqrt{\alpha}\right] \quad (121)$$

now appears instead of integral (117). The leading terms of the asymptotic expansions of the integrand as $w \to 0$ and $w \to \infty$ are identical to those of the integrand in (117) and differ from them only by extra factors $\sqrt{\alpha}$ and $\sqrt{\alpha+1}$ correspondingly. After elimination of the infrared and ultraviolet divergences and turning the integration contour by the angle $-\pi/3$, we obtain

$$\operatorname{tr}\alpha^F|_{reg} = \frac{1}{2\pi}\left\{\sqrt{\alpha(\alpha+1)}\times \left(-\frac{\pi}{2} - i\ln\frac{3(\alpha+1)^2}{\gamma\varepsilon}\right) + i\sqrt{\alpha+1}\,J(\alpha)\right\}, \quad (122)$$

where

$$J(\alpha) = -3\int_0^\infty dx\left[\exp\left(-\frac{2}{3}x^3\right)\times \int_0^\infty d\tau\,\sqrt{\tau^2+\alpha x^2}\exp\left(x^2\tau - \frac{\tau^3}{3}\right) - \sqrt{\pi(1+\alpha)x}\right]. \quad (123)$$

S. L. Lebedev called the author's attention to the fact that the integral $J(\alpha)$ can be reduced to elementary functions. Indeed, it can be shown that

$$J(\alpha) = 1 + \sqrt{\alpha} + \frac{\alpha-2}{3\sqrt{\alpha+1}}\ln\frac{\alpha+\sqrt{\alpha(\alpha+1)}}{1+\sqrt{\alpha+1}} + \frac{\sqrt{4+\alpha}}{3}\ln\frac{\sqrt{\alpha(4+\alpha)}-\alpha}{4+2\sqrt{4+\alpha}}. \quad (124)$$

The function $J(\alpha)$ is equal to

$$J(\alpha) = 1 - \frac{2}{3}\ln 2 = 0.5379\ldots \quad \text{at} \quad \alpha = 0,$$

attains the minimum value

$$J(\alpha) \approx 0.39 \quad \text{at} \quad \alpha \approx 0.3,$$

and then grows and behaves as

$$J(\alpha) = \left(1 + \frac{1}{3}\ln 2\right)\sqrt{\alpha} \quad \text{as} \quad \alpha \to \infty.$$

We note that $\alpha^{B,F}_{\omega'\omega}$ depend on two dimensionless parameters b and c, but the traces $\operatorname{tr}\alpha^{B,F}$ depend only on their combination α, i.e., only on the subluminal relative velocity β_{10}.

The Airy semiparabola with an out-tangent line is time-reversed to the considered trajectory and can be obtained from it by the changes

$$v \rightleftarrows -u, \quad \varkappa \rightleftarrows \varkappa'.$$

This leads to the change $s \rightleftarrows s'$ in the expressions for $\alpha^{B,F}_{\omega'\omega}$. The quantities $\operatorname{tr}\alpha^{B,F}$ do not change at all, but it must be understood that the parameter α is now defined by the final (and negative) relative velocity β_{20} of the mirror and detector:

$$\alpha = -\frac{1+\beta_{20}}{2\beta_{20}} > 0, \quad \text{while} \quad \beta_{10} = 1.$$

The infrared logarithmic singularities of $\operatorname{tr}\alpha^{B,F}$ were regularized by a nonzero momentum transfer $\nu \ll w_0$. Their coefficients are in accordance with the general consideration in Sec. 6. These singularities disappear from $\operatorname{tr}\alpha^F|_{reg}$ at luminal velocities of the ends, and $\operatorname{tr}\alpha^F|_{reg}$ becomes purely imaginary positive. The positive sign of $\operatorname{Im}\operatorname{tr}\alpha^F|_{reg}$ in this case can be explained by the large momentum transfer to the mirror during its contact with the detector, while the negative signs of $\operatorname{Im}\Delta m_0$ and $\operatorname{Im}\Delta m_1$ are related to energy–momentum losses by the charge due to the change of self-interaction at acceleration.

We do not consider the coefficients $\beta^{B,F*}_{\omega'\omega}$ here. They can be obtained from $\alpha^{B,F}_{\omega'\omega}$ by the changes

$$\omega \to -\omega, \quad \sqrt{\omega} \to -i\sqrt{\omega},$$

and division by i in the Bose case, see Eq. (2).

8. CONCLUSION

The symmetry being discussed reveals itself in the coincidence of the quantities bilinear in $\beta_{\omega'\omega}$, such as

$$|\beta_{\omega'\omega}|^2, \quad (\beta^+\beta)_{\omega\omega} = \int_0^\infty \frac{d\omega'}{2\pi}\,\beta^*_{\omega'\omega}\,\beta_{\omega'\omega},$$

$$\bar N = \operatorname{tr}\beta^+\beta = \int_0^\infty \frac{d\omega}{2\pi}\,(\beta^+\beta)_{\omega\omega},$$

with the corresponding quantities describing the emission of vector (scalar) quanta by an electric (scalar) charge in 3 + 1-dimensional space, see the Introduction. Only similarly transforming frequencies are involved in each summation entering these quantities and the equality $\omega = \omega''$ for the diagonal elements of the matrix

$$(\beta^+\beta)_{\omega\omega''} = \int_0^\infty \frac{d\omega'}{2\pi}\,\beta^*_{\omega'\omega}\beta_{\omega'\omega''}.$$

On the other hand, the definition of the trace of the matrix $\alpha_{\omega'\omega}$ with differently transforming indices ω and ω' required the Lorentz-invariant frequencies Ω and Ω' coinciding with ω and ω' in the proper system of the detector, moving along the tangent line to the mirror trajectory at the characteristic point. As a result, $\operatorname{tr}\alpha$ becomes a functional of not only the mirror trajectory but also the detector one. This allows considering $\operatorname{tr}\alpha$ as an experimentally measurable quantity.

The symmetry under discussion has been embodied in several exact mathematical relations between important physical quantities. The most important of them are, of course, the fundamental relations (11) and (12) between the Bogoliubov coefficients for the processes induced by a mirror in 1 + 1-dimensional space and the current and charge densities for the processes induced by a charge in 3 + 1-dimensional space. Another is the integral relation in Eq. (16) between the propagator of a pair of massless particles scattered in 1 + 1-dimensional space in opposite directions with frequencies ω and ω' (such that the pair has a mass $m = 2\sqrt{\omega\omega'}$), and the propagator of a single particle in 3 + 1-dimensional space. This relation provides the connection

$$\Delta W_{1,0} = e^2 \Delta W^{B,F}$$

between the self-action changes of a charge in 3 + 1-dimensional space and of a mirror in 1 + 1-dimensional space if $\operatorname{tr}\beta^+\beta \ll 1$.

The other relations in which the symmetry manifests itself are the spectral representations for the real parts of self-action changes (32) and of mass shifts (34) and (38) of electric and scalar charges in quasihyperbolic motion. The mass shifts of charges, the sources of Bose fields with spins 1 and 0 in 3+1-dimensional space, are represented by the spectral distributions of the heat capacity or the energy of Bose and Fermi gases of massless particles in $1+1$-dimensional space. The spectral representations allow considering the mass shift formation as the balance between the radiation and excitation of the proper energy at acceleration.

The symmetry between processes induced by the mirror in 2-dimensional and by the charge in 4-dimensional space–times predicts not only the value $e_0^2 = 1$ for the bare charge squared, which corresponds to the bare fine structure constant $\alpha_0 = 1/4\pi$, but also the appearance of scalar particles in ultra high-energy collisions in $3+1$-dimensional space and a decrease in their interaction with a scalar source with increasing the energy.

It is very interesting that the bare fine structure constant has a purely geometric origin, and, also, that its value is small:

$$\alpha_0 = 1/4\pi \ll 1.$$

The smallness of α_0 has the essential meaning for the quantum electrodynamics, where it explains the smallness of α and a priori justifies the applicability of the perturbation theory.

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