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e^+e^- PAIR PRODUCTION IN ULTRARELATIVISTIC HEAVY-ION COLLISIONS AT INTERMEDIATE IMPACT PARAMETERS

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Using the semiclassical Green's function in the Coulomb field, we analyze the probabilities of single and multiple e^+e^- pair production at a fixed impact parameter b between colliding ultrarelativistic heavy nuclei. We perform calculations in the Born approximation with respect to the parameter $Z_B\alpha$ and exactly in $Z_A\alpha$, where Z_A and Z_B are the charge numbers of the corresponding nuclei. We also obtain the approximate formulas for the probabilities valid for $Z_A\alpha$, $Z_B\alpha \lesssim 1$.

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1. INTRODUCTION

The cross section of the e^+e^- pair production in ultrarelativistic heavy-ion collisions is very large, and this process can be a serious background for many experiments. Besides, it is also important in the problem of beam lifetime and luminosity of hadron colliders. This means that various corrections to the Born cross section for one-pair production, as well as the cross section for n-pair production ($n > 1$), are very important. Recently, the process was discussed in numerous papers, see reviews [1–3], but some important aspects of the problem are not yet entirely understood, and we elucidate them in the present paper.

For our purpose, it is convenient to consider a collision of nuclei A and B with the corresponding charge numbers Z_A and Z_B in the rest frame of nucleus A. Nucleus B is assumed to move in the positive direction of the z axis having the Lorentz factor γ. For $\gamma \gg 1$, it is possible to treat the nuclei as sources of the external field and calculate the probability $P_n(b)$ of n-pair production in the collision of two nuclei at a fixed impact parameter b. The corresponding cross section σ_n is obtained by integrating over the impact parameter,

$$\sigma_n = \int d^2b\, P_n(b). \tag{1}$$

The average number of the produced pairs at a given b is given by

$$W(b) = \sum_{n=1}^{\infty} nP_n(b). \tag{2}$$

The function $W(b)$ defines the number-weighted cross section

$$\sigma_T = \int d^2b\, W(b) = \sum_{n=1}^{\infty} n\sigma_n. \tag{3}$$

A closed expression for σ_T was obtained in Refs. [4–6], although the correct meaning of this expression was recognized later in Ref. [7].

The cross section σ_T can be represented as

$$\sigma_T = \sigma_T^0 + \sigma_T^C + \sigma_T^{CC}, \tag{4}$$

where σ_T^0 is the Born cross section, i.e., the cross section calculated in the lowest-order perturbation theory with respect to the parameters $Z_{A,B}\,\alpha$ ($\sigma_T^0 \propto (Z_B\alpha)^2(Z_A\alpha)^2$, $\alpha = e^2$ is the fine-structure constant, e is the electron charge, $\hbar = c = 1$), σ_T^C is the Coulomb correction with respect to one of the nuclei (containing the terms proportional to $(Z_B\alpha)^2(Z_A\alpha)^{2n}$ or $(Z_B\alpha)^{2n}(Z_A\alpha)^2$, $n \geqslant 2$), and σ_T^{CC} is the Coulomb correction with respect to both nuclei (containing the terms proportional to $(Z_B\alpha)^n(Z_A\alpha)^l$ with $n, l > 2$). The cross section σ_T^0 coincides with the Born cross section of one-pair production, which was calculated many years ago in Refs. [8, 9].

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The expression for $W(b)$ derived in Refs. [4–6] requires regularization. The correct regularization was made in Refs. [10, 11], where the expressions for σ_T^C and σ_T^{CC} were obtained in the leading logarithmic approximation:

$$\begin{aligned} \sigma_T^C &= -\frac{28}{9\pi}\frac{\zeta}{m^2}L^2\left[f(Z_B\alpha)+f(Z_A\alpha)\right], \\ \sigma_T^{CC} &= \frac{56}{9\pi}\frac{\zeta}{m^2}L\, f(Z_B\alpha)f(Z_A\alpha), \\ &\zeta=(Z_A\alpha)^2(Z_B\alpha)^2, \quad \psi(x)=\Gamma'(x)/\Gamma(x), \\ &L=\ln\gamma, \quad f(x)=\mathrm{Re}[\psi(1+iZ_A\alpha)+C], \end{aligned} \tag{5}$$

where m is the electron mass and $C = 0.577\ldots$ is the Euler constant. The expression for σ_T^C coincides with that obtained in Ref. [12] by means of the Weizsäcker – Williams approximation. The accuracy of expression (4) with σ_T^C and σ_T^{CC} given in (5) and σ_T^0 in Refs. [8, 9] is determined by the relative order of the omitted terms $\sim (Z_{A,B}\alpha)^2/L^2$. This accuracy is better than 0.4 % for the RHIC and LHC colliders. In recent papers [13, 14], the Coulomb corrections were calculated numerically for a few values of γ. We emphasize that the accuracy of the results in Refs. [13, 14] is the same as in (5). The uncertainty is related to the contribution of the region where the energies of the electron and the positron are of the order of the electron mass in the rest frame of one of the nuclei.

It was claimed in Refs. [15–18] that the factorization of the multiple pair production probability is valid with a good accuracy, resulting in the Poisson distribution for multiplicities:

$$P_n(b) = \frac{W^n(b)}{n!}\exp(-W(b)). \tag{6}$$

The factor $\exp(-W)$ is nothing but the vacuum–to–vacuum transition probability

$$P_0 = 1 - \sum_{n=1}^{\infty} P_n.$$

Strictly speaking, the factorization does not take place due to interference between the diagrams corresponding to the permutation of the electron (or positron) lines (see, e.g., [7]). Nevertheless, one can show that this interference makes the contribution that contains at least one power of L less than that of the amplitude squared. Therefore, in the leading logarithmic approximation, one can use expression (6). Thus, to obtain P_n, it suffices to know the function $W(b)$.

In Refs. [19–23], the function $W_0(b)$ (the Born approximation for $W(b)$) was calculated numerically for $mb \lesssim 1$ and a few particular values of γ. The correct dependence of $W_0(b)$ on b at $mb \gg 1$ was obtained analytically in Ref. [24] by two different methods. Both methods give the result

$$W_0(b) = \frac{28}{9\pi^2}\frac{\zeta}{(mb)^2}\left[2\ln\gamma - 3\ln(mb)\right]\ln(mb) \tag{7}$$

in the region $1 \ll mb \le \sqrt{\gamma}$ and

$$W_0(b) = \frac{28}{9\pi^2}\frac{\zeta}{(mb)^2}\left(\ln\frac{\gamma}{mb}\right)^2 \tag{8}$$

in the region $\sqrt{\gamma} \le mb \ll \gamma$. We note that the function $W_0(b)$ given by Eqs. (7) and (8) is continuous at $mb = \sqrt{\gamma}$ together with its first derivative. Certainly, the integration of $W_0(b)$ over $\mathbf{b}$, $b = |\mathbf{b}|$, gives the leading term ($\propto L^3$) in σ_T^0. In the recent paper [23], an ansatz for $W_0(b)$ was suggested that has a quite different dependence of $W_0(b)$ on γ and b for $1 \ll mb \ll \sqrt{\gamma}$. In the present paper, we confirm the result (7) once more and unambiguously disprove the ansatz suggested in Ref. [23].

The one-pair production cross section σ_1 can be represented as

$$\begin{aligned} \sigma_1 = \sigma_T + \sigma_{unit} = \int d\mathbf{b} W(b) - \\ - \int d\mathbf{b} W(b)\left(1-\exp(-W(b))\right). \end{aligned} \tag{9}$$

Therefore, the difference between σ_1 and σ_T is due to the unitarity correction σ_{unit}. The leading contribution to the term σ_T comes from $b \gg 1/m$. It was shown in Ref. [24] that the leading contribution to the second term, σ_{unit}, as well as the leading contribution to the cross sections for the n-pair production ($n \ge 2$), comes from $b \sim 1/m$. As shown in Ref. [24], in this region, the function $W(b)$ has the form

$$W(b) = \zeta L \mathcal{F}(mb), \tag{10}$$

where the function $\mathcal{F}(mb)$ depends on the parameters $Z_B\alpha$ and $Z_A\alpha$ and is independent of γ. We represent $\mathcal{F}(x)$ as

$$\mathcal{F}(x) = \mathcal{F}_0(x) + \mathcal{F}_A(x) + \mathcal{F}_B(x) + \mathcal{F}_{AB}(x), \tag{11}$$

where $\mathcal{F}_0(x)$ is independent of Z_A and Z_B (the Born term), $\mathcal{F}_A(x)$ contains terms $\propto (Z_A\alpha)^{n>0}(Z_B\alpha)^0$ (Coulomb corrections with respect to nucleus A), $\mathcal{F}_B(x)$ contains terms $\propto (Z_A\alpha)^0(Z_B\alpha)^{n>0}$ (Coulomb corrections with respect to nucleus B), and $\mathcal{F}_{AB}(x)$ contains terms $\propto (Z_A\alpha)^{n>0}(Z_B\alpha)^{l>0}$ (Coulomb corrections with respect to both nuclei).

In the present paper, we calculate the function $\mathcal{F}(x)$ for $Z_B\alpha \ll 1$, $Z_A\alpha \lesssim 1$, and $x \lesssim 1$. In this limit, we can neglect the terms $\mathcal{F}_B(x)$ and $\mathcal{F}_{AB}(x)$ in Eq. (11). Although $Z_B\alpha \ll 1$, we cannot expand the exponential in (6) if $\zeta L \sim 1$. Our method is based on the use of the semiclassical Green's function of the Dirac equation in the Coulomb field.

2. GENERAL DISCUSSION

In the leading order in $Z_B\alpha$, the matrix element M of the e^+e^- pair production has the form

$$M = -e\int dt\, d\mathbf{r} \exp[-i(\varepsilon_p+\varepsilon_q)t] \times \\ \times \overline{\Psi}_{p_-}(\mathbf{r})\, \hat{\mathcal{A}}(t,\mathbf{r})\, \Psi_{-p_+}(\mathbf{r}), \quad (12)$$

where $\mathcal{A}^\mu(t,\mathbf{r})$ is the four-vector potential of the moving nucleus B, $\overline{\Psi}_{p_-}$ and Ψ_{-p_+} are the positive- and negative-energy solutions of the Dirac equation in the Coulomb field of nucleus A and $p_- = (\varepsilon_p, \mathbf{p})$ and $p_+ = (\varepsilon_q, \mathbf{q})$ are the four-momenta of the electron and positron, respectively.

We then use the Fourier transform $\mathcal{A}_k^\mu$ of the vector potential $\mathcal{A}^\mu(t,\mathbf{r})$,

$$\mathcal{A}_k^\mu = -\frac{4\pi e Z_B}{\mathbf{k}_\perp^2 + (k^0/\gamma\beta)^2} \times \\ \times \exp(-i\mathbf{k}_\perp\cdot\mathbf{b}) 2\pi\delta\left(\gamma k^0 - \gamma\beta k^z\right) u^\mu, \quad (13)$$

where $u^\mu = (\gamma, 0, 0, \gamma\beta)$ is the four-velocity of nucleus B and $\mathbf{b}$ is the impact parameter. Taking the integrals over t, k^0, and k^z, we obtain

$$M = -\frac{4\pi Z_B\alpha}{\gamma\beta}\int \frac{d\mathbf{k}_\perp}{(2\pi)^2}\frac{\exp(-i\mathbf{k}_\perp\cdot\boldsymbol{\beta})}{\mathbf{k}_\perp^2 + (E/\gamma\beta)^2} \times \\ \times \int d\mathbf{r}\, \exp\left[i\mathbf{k}_\perp\cdot\boldsymbol{\rho} + \frac{iEz}{\beta}\right] \overline{\Psi}_{p_-}(\mathbf{r})\hat{u}\, \Psi_{-p_+}(\mathbf{r}), \quad (14)$$

where

$$E = \varepsilon_p + \varepsilon_q, \quad \mathbf{r} = (\boldsymbol{\rho}, z).$$

In calculating the probabilities integrated over the angles of the final particles, it is convenient to use the Green's functions of the Dirac equation in an external field. Using the relations (see, e.g., [25])

$$\sum_\sigma \int d\Omega_\mathbf{q}\, \Psi_{-p_+}(\mathbf{r}_2)\overline{\Psi}_{-p_+}(\mathbf{r}_1) = \\ = -i\frac{(2\pi)^2}{q\,\varepsilon_q}\delta G\left(\mathbf{r}_2, \mathbf{r}_1 | -\varepsilon_q\right), \\ \sum_\sigma \int d\Omega_\mathbf{p} \Psi_{p_-}(\mathbf{r}_1)\overline{\Psi}_{p_-}(\mathbf{r}_2) = \\ = i\frac{(2\pi)^2}{p\,\varepsilon_p}\delta G\left(\mathbf{r}_1, \mathbf{r}_2 | \varepsilon_p\right), \quad (15)$$

where $\delta G(\mathbf{r}, \mathbf{r}'|\varepsilon)$ is the discontinuity of the Green's function on the cut and the summation is performed over spin states, we obtain the total probability

$$W(b) = \sum_{\sigma_\pm} |M|^2 \frac{d\mathbf{p}\, d\mathbf{q}}{(2\pi)^6} = \left(\frac{2Z_B\alpha}{\gamma\beta}\right)^2 \times \\ \times \int \frac{d\varepsilon_q d\varepsilon_p d\mathbf{k}_{1\perp} d\mathbf{k}_{2\perp}}{(2\pi)^4} \times \\ \times \frac{\exp\left[i(\mathbf{k}_{1\perp} - \mathbf{k}_{2\perp})\cdot\mathbf{b}\right]}{\left[\mathbf{k}_{1\perp}^2 + (E/\gamma\beta)^2\right]\left[\mathbf{k}_{2\perp}^2 + (E/\gamma\beta)^2\right]} \int d\mathbf{r}_1 d\mathbf{r}_2 \times \\ \times \exp\left[i\mathbf{k}_{2\perp}\cdot\boldsymbol{\rho}_2 - i\mathbf{k}_{1\perp}\cdot\boldsymbol{\rho}_1 + i\frac{E}{\beta}(z_2 - z_1)\right] \times \\ \times \mathrm{Sp}\left[\hat{u}\,\delta G(\mathbf{r}_2, \mathbf{r}_1| -\varepsilon_q)\hat{u}\delta G(\mathbf{r}_1, \mathbf{r}_2|\varepsilon_p)\right]. \quad (16)$$

Using gauge invariance and the condition $\gamma \gg 1$, it is possible to make the replacement

$$\mathrm{Sp}\left[\hat{u}\,\delta G(\mathbf{r}_2, \mathbf{r}_1| -\varepsilon_q)\hat{u}\delta G(\mathbf{r}_1, \mathbf{r}_2|\varepsilon_p)\right] \to \frac{\gamma^2}{E^2} \times \\ \times\ \mathrm{Sp}\left[\hat{k}_{2\perp}\,\delta G(\mathbf{r}_2, \mathbf{r}_1| -\varepsilon_q)\,\hat{k}_{1\perp}\delta G(\mathbf{r}_1, \mathbf{r}_2|\varepsilon_p)\right] \quad (17)$$

in Eq. (16).

In the leading logarithmic approximation, the leading contribution to the probability $W(\mathbf{b})$ comes from the region $\varepsilon_\pm \gg m$, where the semiclassical approximation is applicable. Besides, it is convenient to perform the calculations in terms of the Green's function $D(\mathbf{r}, \mathbf{r}'|\varepsilon)$ of the squared Dirac equation [25, 26]. Using the transformations similar to those in Ref. [26], we obtain

$$W(b) = 4\left(Z_B\alpha\right)^2 \int \frac{d\varepsilon_q d\varepsilon_p d\mathbf{k}_{1\perp} d\mathbf{k}_{2\perp}}{E^2\,(2\pi)^4} \times \\ \times \frac{\exp\left[i(\mathbf{k}_{1\perp} - \mathbf{k}_{2\perp})\cdot\mathbf{b}\right]}{\left[\mathbf{k}_{1\perp}^2 + (E/\gamma\beta)^2\right]\left[\mathbf{k}_{2\perp}^2 + (E/\gamma\beta)^2\right]} \int d\mathbf{r}_1 d\mathbf{r}_2 \times \\ \times \exp\left[i\mathbf{k}_{2\perp}\cdot\boldsymbol{\rho}_2 - i\mathbf{k}_{1\perp}\cdot\boldsymbol{\rho}_1 + i\frac{E}{\beta}(z_2 - z_1)\right] \times \\ \times \mathrm{Sp}\left\{\left[\left[-2i\mathbf{k}_{2\perp}\cdot\nabla_2 + \hat{k}_2\hat{k}_{2\perp}\right] D(\mathbf{r}_2, \mathbf{r}_1| -\varepsilon_q)\right] \times \right.\\ \left. \times \left[\left[-2i\mathbf{k}_{1\perp}\cdot\nabla_1 - \hat{k}_1\hat{k}_{1\perp}\right] D(\mathbf{r}_1, \mathbf{r}_2|\varepsilon_p)\right]\right\}, \quad (18)$$

where

$$k_1 = (E, \mathbf{k}_{1\perp}, E), \quad k_2 = (E, \mathbf{k}_{2\perp}, E).$$

In the semiclassical approximation, the function D is given by [25]

$$D(\mathbf{r}_2,\mathbf{r}_1|\varepsilon) = \frac{i\kappa\exp(i\kappa r)}{8\pi^2 r_1 r_2}\times \times\int d\mathbf{q}\exp\left[i\frac{\kappa r(\mathbf{q}+\mathbf{f})^2}{2r_1r_2}\right]\left(\frac{4r_1r_2}{q^2}\right)^{iZ_A\alpha\lambda}\times \times\left[1+\frac{\lambda r}{2r_1r_2}\boldsymbol{\alpha}\cdot(\mathbf{q}+\mathbf{f})\right], \quad (19)$$

$$\kappa=\sqrt{\varepsilon^2-m^2},\quad \lambda=\frac{\varepsilon}{\kappa},\quad \boldsymbol{\alpha}=\gamma^0\boldsymbol{\gamma},$$

$$\mathbf{f}=\frac{[\mathbf{r}_1\times\mathbf{r}_2]\times\mathbf{r}}{r^2},\quad \mathbf{r}=\mathbf{r}_1-\mathbf{r}_2,$$

where $\mathbf{q}$ is a two-dimensional vector in the plane perpendicular to $\mathbf{r}$. The explicit form (19) of the semiclassical Green's function is very convenient for analytical investigation of high-energy processes in the Coulomb field.

3. ANALYTIC RESULTS

For $mb \lesssim 1$, the leading contribution to the integrals in Eq. (18) is given by the region of small angles between the vectors $\mathbf{r}_1$ and $-\mathbf{r}_2$ and the z axis. Using these conditions and the semiclassical Green's function (19), we obtain the following representation for

$$\mathcal{F}(mb)=\mathcal{F}_0(mb)+\mathcal{F}_A(mb)$$

(details of the calculation are presented in the Appendix):

$$\mathcal{F}(mb)=\frac{1}{\pi^4(Z_A\alpha)^2}\int_0^1 dx\int d^2Q\times \times\int\frac{d^2\beta}{\beta^2}\left[1-\left(\frac{|\mathbf{R}+x\mathbf{Q}|}{|\mathbf{R}-\bar{x}\mathbf{Q}|}\right)^{2iZ_A\alpha}\right]\times \times\left\langle 4\sqrt{x\bar{x}}\,(x-\bar{x})\,\boldsymbol{\beta}\cdot\mathbf{Q}\left(\frac{K_1^2(\tilde{Q})}{\tilde{Q}^2}-\frac{K_1(Q)K_1(\tilde{Q})}{Q\tilde{Q}}\right)+ +\left[K_0(\tilde{Q})-K_0(Q)\right]^2+4x\bar{x}\beta^2\times \times\frac{K_1^2(\tilde{Q})}{\tilde{Q}^2}+\left(Q^2-4x\bar{x}\frac{(\boldsymbol{\beta}\cdot\mathbf{Q})^2}{\beta^2}\right)\times \times\left[\frac{K_1(\tilde{Q})}{\tilde{Q}}-\frac{K_1(Q)}{Q}\right]^2\right\rangle, \quad (20)$$

$$\tilde{Q}^2=Q^2+\beta^2,\quad \mathbf{R}=\sqrt{x\bar{x}}\boldsymbol{\beta}+m\mathbf{b},\quad \bar{x}=1-x,$$

where $K_n(x)$ is a modified Bessel function of the third kind. The form (20) is suitable for investigation of the asymptotic behavior of $\mathcal{F}(mb)$. For numerical evaluation, it is convenient to pass from the integration over the angle ϕ of the vector $\mathbf{Q}$ to the integration over the parameter v using the identities

$$\int\frac{d\phi}{2\pi}\left[1-\left(\frac{1+a\cos\phi}{1-b\cos\phi}\right)^{i\nu}\right]\begin{Bmatrix}1\\ \cos\phi\\ \cos 2\phi\end{Bmatrix}= =\frac{\nu\,\mathrm{sh}\,\pi\nu}{\pi}\lim_{\delta\to 0}\int_0^1\frac{dv}{v^{1-\delta}\,\bar{v}^{1-\delta}}\left(\frac{v}{\bar{v}}\right)^{-i\nu}\times \times\begin{Bmatrix}\ln\dfrac{1+\sqrt{1-s^2}}{2}\\ \dfrac{s}{1+\sqrt{1-s^2}}\\ -\dfrac{1}{2}\left(\dfrac{s}{1+\sqrt{1-s^2}}\right)^2\end{Bmatrix},$$

$$\bar{v}=1-v,\quad s=av-b\bar{v}. \quad (21)$$

Making the substitution

$$v=\frac{u}{u+\xi\bar{u}},$$

where

$$\bar{u}=1-u,\quad \xi=\frac{R^2+\bar{x}^2Q^2}{R^2+x^2Q^2},$$

and taking the symmetry of the integrand under the substitution $u\to\bar{u}$, $x\to\bar{x}$ into account, we obtain

$$\mathcal{F}(mb)=4\frac{\mathrm{sh}(\pi Z_A\alpha)}{\pi^4 Z_A\alpha}\int_0^1 dx\int_0^\infty dQ\,Q\int\frac{d^2\beta}{\beta^2}\times \times\int_0^{1/2}\frac{du}{u\,\bar{u}}\cos\left(Z_A\alpha\ln\frac{u}{\bar{u}}\right)\left\langle\ln\frac{s+\sqrt{s^2-t^2}}{g}\times \times\left\{(1-2x\bar{x})\left[\frac{QK_1(\tilde{Q})}{\tilde{Q}}-K_1(Q)\right]^2+ +\left[K_0(\tilde{Q})-K_0(Q)\right]^2+4x\bar{x}\frac{K_1^2(\tilde{Q})\beta^2}{\tilde{Q}^2}\right\}+ +x\bar{x}\left[2\left(\frac{\boldsymbol{\beta}\cdot\mathbf{R}}{\beta R}\right)^2-1\right]\left[\left(\frac{t}{s+\sqrt{s^2-t^2}}\right)^2- -\left(\frac{RQ\bar{x}}{g}\right)^2\right]\left[\frac{QK_1(\tilde{Q})}{\tilde{Q}}-K_1(Q)\right]^2-$$

$$-4\sqrt{x\bar{x}}\,(\bar{x}-x)\,\frac{\boldsymbol{\beta}\cdot\mathbf{R}}{R}\left[\frac{t}{s+\sqrt{s^2-t^2}}+\frac{RQ\bar{x}}{g}\right]\times$$
$$\times\left[\frac{QK_1^2(\tilde{Q})}{\tilde{Q}^2}-\frac{K_1(Q)K_1(\tilde{Q})}{\tilde{Q}}\right]\Bigg\rangle, \tag{22}$$
$$\tilde{Q}^2 = Q^2+\beta^2\,,\quad g=\max(R^2,Q^2\bar{x}^2)\,,$$
$$t=2QR\,(xu-\bar{x}\bar{u})\,,\quad s=R^2+Q^2\left(x^2u+\bar{x}^2\bar{u}\right).$$

We consider the asymptotic form of Eq. (20). For $mb \gg 1$, there are two regions of integration over $\boldsymbol{\beta}$ giving the leading logarithmic contribution to $\mathcal{F}(mb)$:

$$1 \ll |\sqrt{x\bar{x}}\boldsymbol{\beta}+m\mathbf{b}| \ll mb \quad \text{and} \quad 1 \ll \beta \ll mb.$$

These regions give equal contributions, and the final result is

$$\mathcal{F}(mb) = \frac{56}{9\pi^2(mb)^2}\ln(mb). \tag{23}$$

Thus, the leading logarithmic contribution is given by the Born term $\mathcal{F}_0(mb)$. This asymptotic expression agrees with Eq. (7) under the condition

$$\ln(mb) \ll L.$$

The leading contribution to $\mathcal{F}_A(mb)$ comes from the region

$$|\sqrt{x\bar{x}}\boldsymbol{\beta}+m\mathbf{b}| \sim 1$$

and has the form

$$\mathcal{F}_A(mb) = -\frac{28}{9\pi^2(mb)^2}f(Z_A\alpha), \tag{24}$$

where the function $f(x)$ is defined in Eq. (5). Again, this asymptotic expression is valid under the condition $\ln(mb) \ll L$. For the Coulomb corrections to $W(b)$ with respect to nucleus A, $W_A(b)$, similarly to the derivation of Eq. (7) based on the equivalent photon approximation (see Ref. [24]), it is possible to obtain the expression valid in the wider region $\ln(mb) \lesssim L$ (but still $1 \ll mb \ll \gamma$). We have

$$W_A(b) = -\frac{28}{9\pi^2}\frac{\zeta}{(mb)^2}f(Z_A\alpha)\ln\frac{\gamma}{mb}. \tag{25}$$

Equation (24) evidently agrees with Eq. (25).

We consider the asymptotic regime of small impact parameters. For $mb \ll 1$, the leading logarithmic contribution comes from the region $mb \ll \beta \sim Q \ll 1$. Taking the integrals over this region, we obtain

$$\mathcal{F}(mb) = \frac{8}{3\pi^2(Z_A\alpha)^2}\ln\frac{1}{mb}\times$$
$$\times\,\mathrm{Re}\left[\psi(1+iZ_A\alpha)+C-(Z_A\alpha)^2+\right.$$
$$\left.+iZ_A\alpha(1+(Z_A\alpha)^2)\psi'(1+iZ_A\alpha)\right]. \tag{26}$$

This asymptotic expression is obtained for a zero nuclear radius R_n. To obtain $W(b)$ for extended nuclei, it is sufficient, within the logarithmic accuracy, to make the substitution

$$\ln(mb) \to \ln(mb+mR_n)$$

in (26). For $b \gg R_n$, the finite-nuclear-size correction to $W(b)$ is negligible.

4. NUMERICAL RESULTS

Using Eq. (18), we tabulated the function $\mathcal{F}(mb)$ for a few values of Z_A. The corresponding results are presented in the left plot in Fig. 1 and in the Table. We recall that these results are obtained in the Born approximation with respect to nucleus B. For most experiments, $Z_A = Z_B$, and it is necessary to know the function $\mathcal{F}(mb)$ beyond the Born approximation with respect to nucleus B. If we assume that the term $\mathcal{F}_{AB}$ in Eq. (11) is numerically small, then we can approximate the function $\mathcal{F}$ as $\mathcal{F}_0 + 2\mathcal{F}_A$ in this case. This function is shown in the right plot in Fig. 1. It is seen that the Coulomb corrections in the region $mb \lesssim 1$ are very important for the experimentally interesting case $Z_A = Z_B = 79$. The assumption of smallness of the contribution $\mathcal{F}_{AB}$ is supported by the comparison of our results for $W(b)$ with those obtained in Refs. [27, 14] for $Z_A = Z_B = 79$ and $\gamma = 2\cdot 10^4$ ($\gamma_{c.m.} = 100$).

As we have already pointed out, Eq. (10) has a logarithmic accuracy, which can be sufficient for very large γ. To go beyond the logarithmic accuracy, we represent $W(b)$ in the form

$$W(b) = \zeta\,[L-G(mb)]\,\mathcal{F}(mb)\,, \tag{27}$$

where $G(mb)$ is some function of mb and, generally speaking, of the parameters $Z_A\alpha$ and $Z_B\alpha$. The asymptotic form of $G(mb)$ for $1 \ll mb \ll \sqrt{\gamma}$ is known, see Eqs. (7) and (25). However, the calculation of the function $G(mb)$ at $mb \lesssim 1$ is a rather complicated problem. Instead, we use the results of numerical calculations performed for a few values of γ in Refs. [19, 27] in the Born approximation. We have found that the form

$$G(mb) = \frac{3}{2}\ln(mb+1)+1.9 \tag{28}$$

provides good agreement of Eq. (27) with the numerical results in Refs. [19, 28, 27] in a wide region of mb, see Fig. 2. The form (28) of $G(mb)$ is obtained by fitting the Born results and is therefore independent of $Z_{A,B}$. It provides the correct asymptotic expression for $W_0(b)$,

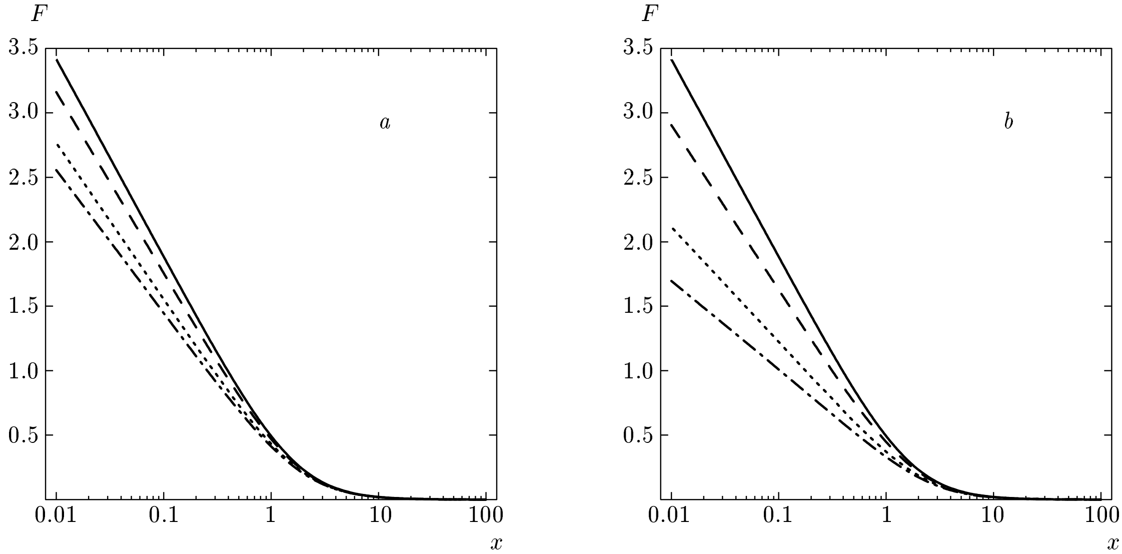

Fig. 1. The function $\mathcal{F}(x)$ in Eq. (22) for $Z_A = 92$ (dash-dotted line), 79 (dotted line), 47 (dashed line), and the Born approximation (solid line). a — Born approximation in $Z_B\alpha$, Eq. (22). b — Results obtained from Eqs. (11) and (22) for $Z_B = Z_A$ with the term $\mathcal{F}_{AB}(x)$ omitted

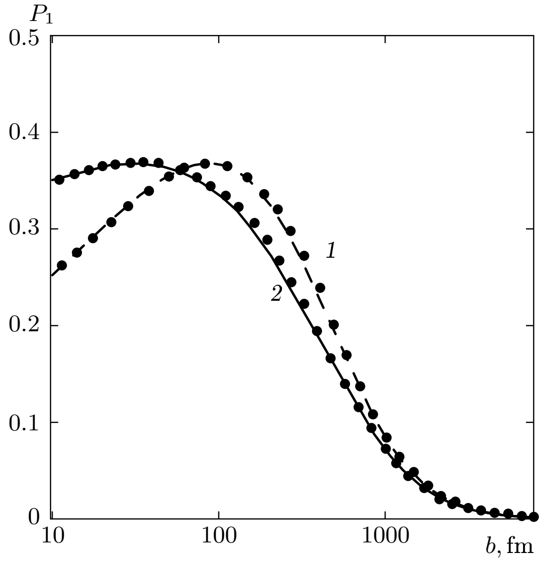

Fig. 2. The one-pair production probability $P_1(b)$ corresponding to the function $W(b)$ in Eq. (27), $\gamma = 2\cdot 10^4$, and $Z_A = Z_B = 79$. *1* — the function $\mathcal{F}$ is taken in the Born approximation, $\mathcal{F} = \mathcal{F}_0$; *2* — Coulomb corrections are taken into account, $\mathcal{F} = \mathcal{F}_0 + 2\mathcal{F}_A$. Dots show the corresponding results of numerical calculations in Ref. [27]

Eq. (7). It turns out that formula (27) with $G(mb)$ in Eq. (28) also has a high accuracy for $Z_A\alpha$, $Z_B\alpha \lesssim 1$ in the region $mb \lesssim 1$, where the Coulomb corrections are large. We have checked this fact by comparing our results with those in Ref. [27] obtained numerically for $Z_A = Z_B = 79$, see Fig. 2. We note that the tabulation of $W(b)$ and $P_N(b)$ performed in Refs. [13, 14, 19, 27, 28] for a few values of γ required the evaluation of a ninefold integral and was therefore very laborious. The calculation of $\mathcal{F}$ in Eq. (22) is essentially simpler. Besides, because this function is independent of γ, one can easily obtain predictions for $W(b)$ at any $\gamma \gg 1$ using Eqs. (27) and (28).

5. CONCLUSION

In the present paper, we have found a simple representation for the function $W(b)$ for $mb \lesssim 1$, $Z_B\alpha \ll 1$, and arbitrary $Z_A\alpha$ in the leading logarithmic approximation. Using the results of numerical calculation of $W(b)$ performed for a few values of γ and $Z_{A,B}$, we have obtained the approximate formula for $W(b)$ valid in a wide region of parameters:

$$mb \lesssim \sqrt{\gamma}, \quad Z_A\alpha \lesssim 1, \quad Z_B\alpha \lesssim 1, \quad \gamma \gg 1.$$

We estimate the accuracy of this formula to be a few percent. The results obtained clearly demonstrate the dependence of $W(b)$, as well as of $P_n(b)$, on the relativistic factor γ and the parameters $Z_{A,B}\alpha$.

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The function $\mathcal{F}(x)$ in Eq. (22) calculated in the Born approximation ($Z_A\alpha \to 0$) and exactly in the parameter $Z_A\alpha$ for Au, Pb, and U

x	Born	Au	Pb	U	x	Born	Au	Pb	U
0.0100	3.420	2.760	2.71	2.560	1.26	0.391	0.347	0.343	0.332
0.0126	3.260	2.650	2.59	2.450	1.58	0.304	0.273	0.27	0.262
0.0158	3.110	2.520	2.47	2.340	2.00	0.231	0.209	0.207	0.202
0.0200	2.960	2.400	2.35	2.220	2.51	0.171	0.156	0.155	0.152
0.0251	2.800	2.280	2.24	2.110	3.16	0.124	0.114	0.114	0.111
0.0316	2.650	2.160	2.12	2.000	3.98	$8.78\cdot10^{-2}$	$8.20\cdot10^{-2}$	$8.15\cdot10^{-2}$	$8.01\cdot10^{-2}$
0.0398	2.500	2.040	2.0	1.890	5.01	$6.14\cdot10^{-2}$	$5.78\cdot10^{-2}$	$5.75\cdot10^{-2}$	$5.66\cdot10^{-2}$
0.0501	2.340	1.920	1.88	1.780	6.31	$4.25\cdot10^{-2}$	$4.02\cdot10^{-2}$	$4.00\cdot10^{-2}$	$3.95\cdot10^{-2}$
0.0631	2.190	1.800	1.76	1.670	7.94	$2.91\cdot10^{-2}$	$2.77\cdot10^{-2}$	$2.76\cdot10^{-2}$	$2.73\cdot10^{-2}$
0.0794	2.040	1.680	1.64	1.560	10.00	$1.99\cdot10^{-2}$	$1.90\cdot10^{-2}$	$1.89\cdot10^{-2}$	$1.87\cdot10^{-2}$
0.1000	1.880	1.550	1.52	1.450	12.60	$1.35\cdot10^{-2}$	$1.29\cdot10^{-2}$	$1.29\cdot10^{-2}$	$1.28\cdot10^{-2}$
0.1260	1.730	1.430	1.41	1.340	15.80	$9.07\cdot10^{-3}$	$8.75\cdot10^{-3}$	$8.72\cdot10^{-3}$	$8.64\cdot10^{-3}$
0.1580	1.580	1.310	1.29	1.230	20.00	$6.09\cdot10^{-3}$	$5.89\cdot10^{-3}$	$5.87\cdot10^{-3}$	$5.83\cdot10^{-3}$
0.2000	1.430	1.190	1.17	1.120	25.10	$4.07\cdot10^{-3}$	$3.95\cdot10^{-3}$	$3.94\cdot10^{-3}$	$3.91\cdot10^{-3}$
0.2510	1.280	1.070	1.06	1.010	31.60	$2.71\cdot10^{-3}$	$2.64\cdot10^{-3}$	$2.63\cdot10^{-3}$	$2.61\cdot10^{-3}$
0.3160	1.140	0.961	0.941	0.898	39.80	$1.80\cdot10^{-3}$	$1.75\cdot10^{-3}$	$1.75\cdot10^{-3}$	$1.74\cdot10^{-3}$
0.3980	0.993	0.842	0.829	0.793	50.10	$1.19\cdot10^{-3}$	$1.16\cdot10^{-3}$	$1.16\cdot10^{-3}$	$1.15\cdot10^{-3}$
0.5010	0.856	0.731	0.72	0.690	63.10	$7.90\cdot10^{-4}$	$7.71\cdot10^{-4}$	$7.69\cdot10^{-4}$	$7.65\cdot10^{-4}$
0.6310	0.725	0.625	0.616	0.591	79.40	$5.21\cdot10^{-4}$	$5.09\cdot10^{-4}$	$5.08\cdot10^{-4}$	$5.05\cdot10^{-4}$
0.7940	0.603	0.524	0.517	0.498	100.00	$3.43\cdot10^{-4}$	$3.35\cdot10^{-4}$	$3.34\cdot10^{-4}$	$3.33\cdot10^{-4}$
1.0000	0.491	0.431	0.426	0.411					

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APPENDIX

Calculation of the integrals

In this Appendix, we present some details of the derivation of Eq. (20) from Eq. (18). The leading contribution to the integrals comes from the region of small angles between the vectors $\mathbf{r}_1$ and $-\mathbf{r}_2$ and the z axis. Using this fact, we take the integrals over the angles of $\mathbf{r}_1$ and $\mathbf{r}_2$, make the substitution $r_{1,2} \to E\,r_{1,2}$, and change the variables as $\varepsilon_p = Ex$, $\varepsilon_q = E\bar{x} = E(1-x)$. Taking the integral over E in the logarithmic approximation with $\gamma \gg 1$ and $mb \lesssim 1$, we obtain

$$\begin{aligned} dW(b) = {} & \frac{(Z_B\alpha)^2}{(2\pi)^6}\ln\gamma \int \frac{d\mathbf{k}_{1\perp}}{k_{1\perp}^2}\frac{d\mathbf{k}_{2\perp}}{k_{2\perp}^2} \times \\ & \times \int dx\, x\bar{x}\frac{dr_1}{r_1}\frac{dr_2}{r_2}\int d\mathbf{Q}\, d\mathbf{q}\left(\frac{|\mathbf{q}+\mathbf{Q}|}{|\mathbf{q}-\mathbf{Q}|}\right)^{2iZ_A\alpha} \times \\ & \times \exp\left[-\frac{i}{2}m^2(r_1+r_2) - i\,\boldsymbol{\Delta}\cdot\boldsymbol{\beta} - \right. \\ & \left. -\frac{i}{2}x\bar{x}(r_1k_{1\perp}^2 + r_2k_{2\perp}^2) + \frac{i(r_1+r_2)\mathbf{Q}^2}{2r_1r_2}\right] \times \\ & \times \left\langle 2\,(\bar{x}-x)\left(\frac{k_{1\perp}^2\mathbf{k}_{2\perp}\cdot\mathbf{Q}}{r_2} - \frac{k_{2\perp}^2\mathbf{k}_{1\perp}\cdot\mathbf{Q}}{r_1}\right) - \right. \\ & -4x\bar{x}k_{1\perp}^2k_{2\perp}^2 - \frac{4\,(\mathbf{k}_{1\perp}\cdot\mathbf{Q})\,(\mathbf{k}_{2\perp}\cdot\mathbf{Q})}{r_1r_2} - \\ & -(\mathbf{k}_{1\perp}\cdot\mathbf{k}_{2\perp})\left(\frac{m^2(r_1+r_2)^2}{2x\bar{x}r_1r_2} + \frac{r_1k_{1\perp}^2}{2r_2} + \right. \\ & \left.\left. + \frac{r_2k_{2\perp}^2}{2r_1}\right)\right\rangle, \end{aligned} \tag{A.1}$$

$$\boldsymbol{\Delta} = \mathbf{k}_{1\perp} - \mathbf{k}_{2\perp}, \quad \beta = \mathbf{q}/2 + (\bar{x}-x)\,\mathbf{Q}/2 - \mathbf{b}\,.$$

The integration over the two-dimensional vectors $\mathbf{k}_{1\perp}$ and $\mathbf{k}_{2\perp}$ can be easily performed. The result is given by

$$
\begin{aligned}
dW(b) = -\frac{(Z_B\alpha)^2}{(2\pi)^4}\ln\gamma\int dx\,\frac{dr_1}{r_1^2}\frac{dr_2}{r_2^2}\times \\
\times\int d\mathbf{Q}\,d\mathbf{q}\exp\left[-\frac{i}{2}m^2(r_1+r_2)+\frac{i(r_1+r_2)\mathbf{Q}^2}{2r_1r_2}\right]\times \\
\times\left(\frac{|\mathbf{q}+\mathbf{Q}|}{|\mathbf{q}-\mathbf{Q}|}\right)^{2iZ_A\alpha}\left\langle\left[2\,(\bar{x}-x)\,\frac{\boldsymbol{\beta}\cdot\mathbf{Q}}{\beta^2}+\frac{1}{2x\bar{x}}\right]\times\right. \\
\times\,(2\mathcal{E}_1\mathcal{E}_2-\mathcal{E}_1-\mathcal{E}_2)-4\mathcal{E}_1\mathcal{E}_2+ \\
+\left[\frac{4\,(\boldsymbol{\beta}\cdot\mathbf{Q})^2}{\beta^4}x\bar{x}+\frac{m^2(r_1+r_2)^2}{2\beta^2}\right]\times \\
\left.\times\,(\mathcal{E}_1-1)\,(\mathcal{E}_2-1)\right\rangle, \qquad \text{(A.2)}
\end{aligned}
$$

$$
\mathcal{E}_i=\exp\left[\frac{i\beta^2}{2x\bar{x}r_i}\right].
$$

Taking the integrals over $r_{1,2}$ and passing from the variable $\mathbf{q}$ to

$$
\boldsymbol{\beta}=\frac{\mathbf{q}}{2}+\frac{(\bar{x}-x)\,\mathbf{Q}}{2}-\mathbf{b},
$$

we obtain Eq. (20).

REFERENCES

1. C. Bertulani and G. Baur, Phys. Rep. **163**, 299 (1988).
2. G. Baur, K. Hencken, D. Trautmann, S. Sadovsky, and Yu. Kharlov, Phys. Rep. **364**, 359 (2002).
3. C. A. Bertulani, S. R. Klein, and J. Nystrand, Annu. Rev. Nucl. Part. Sci. **55**, 271 (2005).
4. B. Segev and J. C. Wells, Phys. Rev. A **57**, 1849 (1998); E-print archives, physics/9805013.
5. A. J. Baltz and L. McLerran, Phys. Rev. C **58**, 1679 (1998).
6. U. Eichmann, J. Reinhardt, S. Schramm, and W. Greiner, Phys. Rev. A **59**, 1223 (1999).
7. A. J. Baltz, F. Gelis, L. McLerran, and A. Peshier, Nucl. Phys. A **695**, 395 (2001).
8. L. D. Landau and E. M. Lifshitz, Phys. Z. Sowjet. **6**, 244 (1934).
9. G. Racah, Nuovo Cim. **14**, 93 (1937).
10. R. N. Lee and A. I. Milstein, Phys. Rev. A **61**, 032103 (2000).
11. R. N. Lee and A. I. Milstein, Phys. Rev. A **64**, 032106 (2001).
12. D. Yu. Ivanov, A. Schiller, and V. G. Serbo, Phys. Lett. B **454**, 155 (1999).
13. A. J. Baltz, Phys. Rev. C **71**, 024901 (2005), Erratum-ibid. C **71**, 039901 (2005).
14. A. J. Baltz, E-print archives, nucl-th/0608006.
15. G. Baur, Phys. Rev. D **41**, 3535 (1990).
16. M. J. Rhoades-Brown and J. Weneser, Phys. Rev. A **44**, 330 (1991).
17. C. Best, W. Greiner, and G. Soff, Phys. Rev. A **46**, 261 (1992).
18. K. Hencken, D. Trautmann, and G. Baur. Phys. Rev. A **51**, 998 (1995).
19. K. Hencken, D. Trautmann, and G. Baur. Phys. Rev. A **51**, 1874 (1995).
20. M. C. Güçlü, J. C. Wells, A. S. Umar, M. R. Strayer, and D. J. Ernst, Phys. Rev. A **51**, 1836 (1995).
21. M. C. Güçlü, Nucl. Phys. A **668**, 149 (2000).
22. K. Hencken, G. Baur, and D. Trautmann, Phys. Rev. C **69**, 054902 (2004).
23. M. C. Güçlü, G. Kovankaya, and M. Yilmaz, Phys. Rev. A **72**, 022724 (2005).
24. R. N. Lee, A. I. Milstein, and V. G. Serbo, Phys. Rev. A **65**, 022102 (2002).
25. R. N. Lee, A. I. Milstein, and V. M. Strakhovenko, Phys. Rev. A **69**, 022708 (2004).
26. R. N. Lee, A. I. Milstein, V. M. Strakhovenko, and O. Ya. Schwartz, Zh. Eksp. Teor. Fiz. **127**, 5 (2005).
27. K. Hencken, D. Trautmann, and G. Baur, Phys. Rev. C **59**, 841 (1999).
28. A. Alscher, K. Hencken, D. Trautmann, and G. Baur, Phys. Rev. A **55**, 396 (1997).