ЖЭТФ, 2008, том 133, вып. 2, стр. 429–437

EFFECT OF PRESSURE ANISOTROPY ON MAGNETOROTATIONAL INSTABILITY

A. B. Mikhailovskii[a], *J. G. Lominadze*[b*], *A. P. Churikov*[c],
N. N. Erokhin[a], *N. S. Erokhin*[d], *V. S. Tsypin*[e]

[a] *Institute of Nuclear Fusion, Russian Research Centre Kurchatov Institute 123182, Moscow, Russia*

[b] *Georgian National Astrophysical Observatory 0160, Tbilisi, Georgia*

[c] *Syzran Branch of Samara Technical University 446001, Syzran, Samara Region, Russia*

[d] *Space Research Institute, Russian Academy of Sciences 117810, Moscow, Russia*

[e] *Brazilian Center for Physics Research BR-22290180, Rio de Janeiro, Brazil*

Received April 11, 2007

It is shown that two new instabilities of hybrid type can occur in a rotating magnetized plasma with anisotropic pressure, i. e., the rotational firehose instability and the rotational mirror instability. In the case of $\beta_\parallel > \beta_\perp$, where $\beta_\parallel$ and $\beta_\perp$ are the ratios of the parallel and perpendicular plasma pressure to the magnetic field pressure, the pressure anisotropy tends to suppress both new instabilities; in the case $\beta_\perp > \beta_\parallel$, it leads to their strengthening. In the last case, the perturbations considered can be unstable even if the Velikhov instability criterion is not satisfied.

PACS: 52.35.Bj, 94.30.cq

1. INTRODUCTION

The magnetorotational instability (MRI) [1] seems to be important for the physics of accretion disks [2]. It is necessary to consider instabilities there because, according to [3], the viscosity in the disks should be anomalous, i. e., caused by turbulence, which should in turn be a result of an instability.

In accordance with recent ideas, the accretion disks can contain collisionless plasma. The pressure of such a plasma can be anisotropic. According to [4–8], this can lead to a family of pressure-anisotropy-driven instabilities (a detailed bibliography of these instabilities can be found in review [9]). Therefore, it seems reasonable to develop a theory of the MRI in a plasma with anisotropic pressure, thereby generalizing the results in [4–8] to the case of rotating plasma. This is the goal of the present paper.

The pressure-anisotropy-driven instabilities discussed in [4–8] can be studied using both the kinetics and the Chew–Goldberger–Low (CGL) approach [10] (the so-called CGL approximation). The CGL approach has been reviewed in [11]. We expose results found in both these approaches.

In Sec. 2, based on the kinetic approach in [12], we derive a dispersion relation describing the MRI in a collisionless plasma with anisotropic pressure. We note that steps in the same direction have been made in [13, 14]. In Sec. 3, the dispersion relation for MRI is derived using the CGL approach. In Sec. 4, permittivity of a rotating plasma with anisotropic pressure is discussed. The goal of Sec. 5 is to present the theory of pressure-anisotropy-driven instabilities in a nonro-

*E-mail: j.lominadze@astro-ge.org

tating plasma. We arrive at the two known versions of the firehose instability, one of which is related to the Alfvén oscillation branch and the second to the magnetoacoustic branches, and to the known mirror instability, which is related to the magnetoacoustic oscillation branches. The main goal of the paper, i. e., the study of the MRI in the case of anisotropic pressure, is realized in Secs. 6 and 7. We predict two new instabilities of hybrid type: rotational firehose Sec. 6 and rotational mirror (Sec. 7). Discussions are given in Sec. 8.

2. KINETIC DISPERSION RELATION

For the description of plasma dynamics, we start from the equation of motion in the form [12, 15]

$$\rho\frac{d\mathbf{V}}{dt} = -\boldsymbol{\nabla}\cdot\mathbf{p} - \frac{1}{4\pi}\left[\frac{1}{2}\boldsymbol{\nabla}\mathbf{B}^2 - (\mathbf{B}\cdot\boldsymbol{\nabla})\mathbf{B}\right], \qquad (2.1)$$

where $d/dt = \partial/\partial t + \mathbf{V}\cdot\boldsymbol{\nabla}$, and ρ, $\mathbf{V}$, $\mathbf{p}$, and $\mathbf{B}$ are respectively the total mass density, velocity, pressure tensor, and the total magnetic field, given by

$$\rho = \rho_0 + \delta\rho, \quad \mathbf{V} = \mathbf{V}_0 + \delta\mathbf{V},$$

$$\mathbf{p} = \mathbf{I}p_0 + \delta\mathbf{p}, \quad \text{and} \quad \mathbf{B} = \mathbf{B_0} + \delta\mathbf{B},$$

$\mathbf{I}$ is the unit tensor and δ denotes the perturbed parts.

We consider cylindrical geometry characterized by the coordinates (R,ϕ,z). The equilibrium magnetic field $\mathbf{B}_0$ is assumed to be directed along the z axis, $\mathbf{B}_0 = (0,0,B_0)$. Plasma rotates in the azimuthal direction ϕ with an angular frequency Ω, and hence the equilibrium plasma velocity $\mathbf{V}_0$ is given by $\mathbf{V}_0 = (0, R\Omega, 0)$.

The perturbed velocity $\delta\mathbf{V}$ and the perturbed magnetic field $\delta\mathbf{B}$ are represented as

$$\delta\mathbf{V} = (\delta V_R, \delta V_\phi, \delta V_z)\,, \quad \delta\mathbf{B} = (\delta B_R, \delta B_\phi, \delta B_z)\,.$$

We assume that the perturbations are independent of the azimuthal coordinate ϕ. Then dependence of each perturbed value $F(\mathbf{r},t)$ can be taken in the form

$$F = F(R)\exp(-i\omega t + ik_R R + ik_z z)\,, \qquad (2.2)$$

where ω is the wave frequency, and k_R and k_z are the perpendicular and parallel projections of the wave vector.

According to Refs. [12, 15],

$$\boldsymbol{\nabla}\cdot\mathbf{p} = \boldsymbol{\nabla}p_\perp + \frac{p_\parallel - p_\perp}{B^2}\left\{\frac{1}{2}\boldsymbol{\nabla}_\perp B^2 + [\boldsymbol{\nabla}\times\mathbf{B}]\times\mathbf{B}\right\} + \frac{\mathbf{B}}{B}(\mathbf{B}\cdot\boldsymbol{\nabla})\left[\frac{1}{B}(p_\parallel - p_\perp)\right], \qquad (2.3)$$

where $p_\perp = p_{\perp 0} + \delta p_\perp$ and $p_\parallel = p_{\parallel 0} + \delta p_\parallel$ are the perpendicular and parallel pressures. The projections of Eqs. (2.1) are given by

$$-i\omega\delta V_R\left(\alpha_A^{AN} + \frac{k_z^2 v_A^2}{\omega^2}\right) - 2\Omega\delta V_\phi - \frac{iv_A^2}{B_0}\frac{k^2}{k_z}\delta B_R = 0, \qquad (2.4)$$

$$-i\omega\delta V_\phi\left(\alpha_A^{AN} + \frac{k^2 v_A^2}{\omega^2}\right) + ik_R\frac{\delta p_\perp}{\rho_0} + \delta V_R\frac{\kappa^2}{2\Omega} + \frac{iv_A^2}{B_0}k_z\delta B_\phi = 0, \qquad (2.5)$$

where $k^2 = k_R^2 + k_z^2$ and v_A is the Alfvén velocity,

$$v_A^2 = \frac{B_0^2}{4\pi\rho_0}. \qquad (2.6)$$

The parameter κ is introduced by

$$\kappa^2 = \frac{2\Omega}{R}\frac{d(R^2\Omega)}{dR}. \qquad (2.7)$$

We set

$$\alpha_A^{AN} = 1 - \frac{k_z^2 v_A^2}{\omega^2}\left(1 + \frac{\beta_\perp - \beta_\parallel}{2}\right), \qquad (2.8)$$

where

$$(\beta_\perp,\ \beta_\parallel) = 8\pi(p_{0\perp},\ p_{0\parallel})/B_0^2. \qquad (2.9)$$

2.1. Kinetic approach

The perturbed perpendicular pressure $\delta p_\perp$ is expressed in terms of the perturbed distribution function δf as [9, 12]

$$\delta p_\perp = M\int\frac{v_\perp^2}{2}\delta f\,d\mathbf{v}, \qquad (2.10)$$

where $v_\perp$ are the perpendicular particle velocities, $\mathbf{v}$ is the velocity space volume, and M is the ion mass. According to [9], the function δf is given by

$$\delta f = \frac{Mv_\perp^2}{2T_\perp}\left(1 - \frac{T_\perp}{T_\parallel} + \frac{\omega}{\omega - k_z v_\parallel}\frac{T_\perp}{T_\parallel}\right)f_0\frac{\delta B_z}{B_0}, \qquad (2.11)$$

where $T_\perp$ and $T_\parallel$ are the equilibrium perpendicular and parallel temperatures. For δf given by (2.11), Eq. (2.10) for $\delta p_\perp$ takes the form

$$\delta p_\perp = 2p_{\perp 0}\left\{1 - \frac{T_\perp}{T_\parallel}\left[1 + \frac{i\sqrt{\pi}\omega}{|k_z|\,v_{T\parallel}}W\left(\frac{\omega}{|k_z|\,v_{T\parallel}}\right)\right]\right\}\times\frac{\delta B_z}{B_0}, \qquad (2.12)$$

where $v_{T\|} = \sqrt{2T_\|/M}$ is the ion parallel thermal velocity and

$$W(x) = \exp(-x^2)\left(1 + \frac{i}{\sqrt{\pi}}\int_0^x \exp(t^2)dt\right) \quad (2.13)$$

is the plasma dispersion function [16]. According to [16], the function $W(x)$ has the following asymptotic forms:

$$W(x) = \frac{i}{\sqrt{\pi}x}, \quad x \gg 1, \quad (2.14)$$

$$W(x) = 1, \quad x \ll 1. \quad (2.15)$$

It then follows from Eq. (2.12) that

$$\delta p_\perp = 2p_{\perp 0} \times \\ \times \begin{cases} 1, & \omega \gg |k_z|\,v_{T\|}, \\ 1 - \dfrac{T_\perp}{T_\|}\left(1 + \dfrac{i\sqrt{\pi}\omega}{|k_z|\,v_{T\|}}\right), & \omega \ll |k_z|\,v_{T\|}. \end{cases} \quad (2.16)$$

Using (2.12), we rewrite Eq. (2.5) as

$$- i\omega\delta V_\phi\left(\alpha_M^{AN} + \frac{k^2v_A^2}{\omega^2}\right) + \\ + \delta V_R\frac{\kappa^2}{2\Omega} + \frac{iv_A^2}{B_0}\frac{k^2}{k_z}\delta B_\phi = 0, \quad (2.17)$$

where

$$\alpha_M^{AN} = \alpha_M^{AN(kin)} \equiv 1 - \frac{k^2v_A^2}{\omega^2}\left(1 + c_M^{AN}\right), \quad (2.18)$$

$$c_M^{AN} = 1 + \frac{k_z^2}{2k^2}(\beta_\perp - \beta_{||}) + \frac{k_R^2}{k^2}\beta_\perp \times \\ \times\left\{1 - \frac{T_\perp}{T_{||}}\left[1 + \frac{i\sqrt{\pi}\omega}{|k_z|v_{T||}}W\left(\frac{\omega}{|k_z|v_{T||}}\right)\right]\right\}. \quad (2.19)$$

To describe the behavior of the perturbed magnetic field, we use the freezing condition

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times [\mathbf{V}\times\mathbf{B}] = 0. \quad (2.20)$$

Then we find

$$-i\omega\delta B_R - ik_zB_0\delta V_R = 0, \quad (2.21)$$

$$-i\omega\delta B_\phi - \frac{d\Omega}{d\ln R}\delta B_R - ik_zB_0\delta V_\phi = 0. \quad (2.22)$$

In addition, using the Maxwell equation $\nabla\cdot\mathbf{B} = 0$, we arrive at

$$\delta B_z = -k_R\delta B_R/k_z. \quad (2.23)$$

By means of Eqs. (2.20)–(2.22), we express $(\delta V_R, \delta V_\phi)$ in terms of $(\delta B_R, \delta B_\phi)$. Equations (2.4) and (2.17) then become

$$\alpha_A^{AN}\delta B_\phi - \frac{2i\Omega}{\omega}\delta B_R = 0, \quad (2.24)$$

$$\frac{2i\Omega}{\omega}\delta B_\phi + \left(\alpha_M^{AN} - \frac{1}{\omega^2}\frac{d\Omega^2}{d\ln R}\right)\delta B_R = 0. \quad (2.25)$$

Using Eqs. (2.24) and (2.25), we obtain the dispersion relation

$$\begin{vmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{vmatrix} = 0, \quad (2.26)$$

where

$$\alpha_{11} = \alpha_A^{AN}, \quad (2.27)$$

$$\alpha_{12} = -\alpha_{21} = -2i\Omega/\omega, \quad (2.28)$$

$$\alpha_{22} = \alpha_M^{AN} - \frac{1}{\omega^2}\frac{d\Omega^2}{d\ln R}. \quad (2.29)$$

Dispersion relation (2.26) can be represented in the form

$$\alpha_A D^{kin} - \kappa^2\omega^2 + \\ + k_z^2v_A^2\left(1 + \frac{\beta_\perp - \beta_\|}{2}\right)\frac{d\Omega^2}{d\ln R} = 0, \quad (2.30)$$

where

$$D^{kin} = \omega^4\Lambda^{kin}, \quad (2.31)$$

$$\Lambda^{kin} = \left[1 - \frac{k_z^2v_A^2}{\omega^2}\left(1 + \frac{\beta_\perp - \beta_\|}{2}\right)\right] \times \\ \times\left[1 + \frac{k^2v_A^2}{\omega^2}\left(\Delta - c_M^{AN}\right)\right] - 4\frac{\Omega^2}{\omega^2}. \quad (2.32)$$

The quantity Δ describing the Velikhov effect [1] is given by

$$\Delta = -\left(1 + \frac{d\Omega^2/d\ln R}{k^2v_A^2}\right). \quad (2.33)$$

3. DISPERSION RELATION DERIVED BY MEANS OF THE CGL APPROACH

3.1. The essence of the CGL approach

There is a problem to express the perturbed plasma pressure in terms of the perturbed magnetic field. In the scope of the one-fluid magnetohydrodynamic approach, this problem is solved using the adiabaticity condition (cf. [12])

$$\frac{d}{dt}\left(\frac{p}{\rho^\Gamma}\right) = 0, \tag{3.1}$$

where Γ is the adiabatic exponent.

But there is no mechanism to equalize the perpendicular and parallel pressures, $p_\perp$ and $p_\parallel$, in a collisionless plasma. The authors of Ref. [10] therefore suggested using two conditions (see also [11])

$$\frac{d}{dt}\left(\frac{p_\perp}{nB}\right) = 0, \tag{3.2}$$

$$\frac{d}{dt}\left(\frac{p_\parallel B^2}{n^3}\right) = 0, \tag{3.3}$$

where n is a plasma number density, instead of a single adiabatic condition (3.1). These conditions are the essence of the CGL approach.

According to [10, 11], Eqs. (3.2) and (3.3) are similar to the one-fluid magnetohydrodynamic equation (3.1) describing the behavior of a plasma in a strong magnetic field. At the same time, Eqs. (3.2) and (3.3) are heuristic, and therefore the validity of the results obtained by means of the CGL approach should be verified for any particular problem using the kinetic approach.

According to [10, 11], Eqs. (3.2) and (3.3) can be interpreted as follows. The quantities $p_\perp$ and B remain unchanged when plasma is compressed in the direction of the magnetic field. The quantities $p_\parallel$ and n are found to be related by an adiabatic law with $\Gamma = 3$ in accordance with an energy increase for the longitudinal degree of freedom. When the plasma is compressed in the direction perpendicular to the magnetic field, $p_\parallel$ remains unchanged. It follows from the freezing condition that $B \propto n$. Consequently, Eq. (3.2) can be interpreted as an adiabaticity relation with $\Gamma = 2$ indicating that the energy of two perpendicular degrees of freedom is increased.

Equations (3.2) and (3.3) can be represented as [10, 11]

$$\frac{dp_\perp}{dt} + 2p_\perp \boldsymbol{\nabla}\cdot\mathbf{V} - \frac{p_\perp}{B^2}\mathbf{B}\cdot(\mathbf{B}\cdot\boldsymbol{\nabla})\,\mathbf{V} = 0, \tag{3.4}$$

$$\frac{dp_\parallel}{dt} + p_\parallel \boldsymbol{\nabla}\cdot\mathbf{V} + \frac{2p_\parallel}{B^2}\mathbf{B}\cdot(\mathbf{B}\cdot\boldsymbol{\nabla})\,\mathbf{V} = 0. \tag{3.5}$$

3.2. Calculation of the perturbed pressure

Using Eq. (3.4) and the continuity equation, we find [5]

$$\delta p_\perp = p_{\perp 0}\left(\frac{\delta n}{n_0} + \frac{\delta B_z}{B_0}\right), \tag{3.6}$$

where δn is the perturbed plasma number density. To find δn, we use the perturbed continuity and parallel motion equations of the form [5]

$$\frac{\partial}{\partial t}\delta n - \frac{n_0}{B_0}\frac{\partial}{\partial t}\delta B_z + n_0\frac{\partial}{\partial z}\delta V_z = 0, \tag{3.7}$$

$$\frac{\partial}{\partial t}\delta V_z + \frac{1}{Mn_0B_0}\frac{\partial}{\partial z}\left(\delta p_\parallel - \frac{p_{\parallel 0} - p_{\perp 0}}{B_0}\delta B_z\right) = 0. \tag{3.8}$$

To calculate this value we use the parallel adiabatic condition in (3.5), which yields [5]

$$\delta p_\parallel = p_{\parallel 0}\left(\frac{3\delta n}{n_0} - \frac{2\delta B_z}{B_0}\right). \tag{3.9}$$

We then find

$$\omega\left(\delta n - \frac{n_0}{B_0}\delta B_z\right) - n_0 k_z \delta V_z = 0, \tag{3.10}$$

$$\omega\delta V_z - \frac{k_R}{Mn_0}\left(\delta p_\parallel - \frac{p_{\parallel 0} - p_{\perp 0}}{B_0}\delta B_z\right) = 0. \tag{3.11}$$

It follows from Eqs. (3.10), (3.11), and (3.6) that

$$\delta n = \frac{n_0}{\alpha_S^{CGL}}\left[1 - \frac{k_z^2 T_\parallel}{M\omega^2}\left(3 - \frac{T_\perp}{T_\parallel}\right)\right]\frac{\delta B_z}{B_0}, \tag{3.12}$$

$$\delta V_z = \frac{k_z T_\perp}{M\omega\alpha_S^{CGL}}\frac{\delta B_z}{B_0}, \tag{3.13}$$

$$\delta p_\parallel = \frac{p_{\parallel 0}}{\alpha_S^{CGL}}\left(1 - \frac{6k_z^2 T_\perp}{M\omega^2}\right)\frac{\delta B_z}{B_0}, \tag{3.14}$$

$$\delta p_\perp = p_{\perp 0}\left(6 - \frac{T_\perp}{T_\parallel}\right)\frac{\delta B_z}{B_0}, \tag{3.15}$$

where

$$\alpha_S^{CGL} = 1 - \frac{3k_z^2 T_\parallel}{M\omega^2}. \tag{3.16}$$

3.3. Dispersion relation

As a result, we have dispersion relation (2.26) with α_{11}, α_{12}, and α_{21} given by (2.27) and (2.28), and the following modification of Eq. (2.29) for α_{22}:

$$\alpha_{22} = \alpha_M^{CGL} - \frac{1}{\omega^2}\frac{d\Omega^2}{d\ln R}. \quad (3.17)$$

Here (cf. (2.32))

$$\alpha_M^{CGL} = \alpha_S^{CGL}\left(\hat{\alpha}_M^{CGL} - \frac{k_R^2 v_A^2}{\omega^2}\beta_\perp\right) - \left(\frac{\beta_\perp}{2}\right)^2 \frac{k_z^2 k_R^2 v_A^4}{\omega^4}, \quad (3.18)$$

where

$$\hat{\alpha}_M^{CGL} = 1 - \frac{k^2 v_A^2}{\omega^2}\left(1 + \frac{\beta_\perp - \beta_\parallel}{2}\right). \quad (3.19)$$

The CGL dispersion relation considered can be represented in the form

$$\alpha_A D^{CGL} + \alpha_S^{CGL} \times \left[-\kappa^2\omega^2 + k_z^2 k^2 v_A^4 (\Delta+1)\left(1+\frac{\beta_\perp-\beta_\parallel}{2}\right)\right] = 0, \quad (3.20)$$

where D^{CGL} is given by [5]

$$D^{CGL} = \omega^4 \Lambda^{CGL} \quad (3.21)$$

with

$$\Lambda^{CGL} = \alpha_S^{CGL}\left[1 - \frac{k^2 v_A^2}{\omega^2}\left(1 + \frac{\beta_\perp - \beta_\parallel}{2}\right) - \frac{k_z^2 v_A^2}{\omega^2}\beta_\perp\right] - \left(\frac{\beta_\perp}{2}\right)^2 \frac{k_z^2 k_R^2 v_A^4}{\omega^4}. \quad (3.22)$$

We also note that as $\omega^2 \to 0$, the function D^{CGL} given by (3.21) reduces to

$$D^{CGL}_{\omega^2\to 0} = k_z^2 v_A^2 \left[3k^2 \bar{v}_\parallel^2 \left(1 + \frac{\beta_\perp - \beta_\parallel}{2}\right) + \frac{3}{2} k_R^2 \bar{v}_\parallel^2 (\beta_\perp + \beta_\parallel) - \frac{\beta_\perp}{2} k_R^2 \bar{v}_\perp^2\right], \quad (3.23)$$

where $\bar{v}_\parallel^2 = T_\parallel / M$.

4. PERMITTIVITY OF ROTATING PLASMA WITH ANISOTROPIC PRESSURE

According to Ref. [12], dispersion relation (2.26) can be represented in the form

$$\begin{vmatrix} \varepsilon_{11} - c^2 k_z^2/\omega^2 & \varepsilon_{12} \\ \varepsilon_{21} & \varepsilon_{22} - c^2 k^2/\omega^2 \end{vmatrix} = 0, \quad (4.1)$$

where ε_{ik} $(i, k = 1, 2)$ are the components of the permittivity tensor related to the coefficients α_{ik} by

$$\varepsilon_{ik} = \frac{c^2}{v_A^2}\begin{pmatrix} \alpha_{11} + k_z^2 v_A^2/\omega^2 & \alpha_{12} \\ \alpha_{21} & \alpha_{22} + k^2 v_A^2/\omega^2 \end{pmatrix}. \quad (4.2)$$

It was explained in Ref. [12] that the permittivity tensor has the structure

$$\varepsilon_{ik} = \varepsilon_{ik}^{(0)} + \varepsilon_{ik}^{(r)}, \quad (4.3)$$

where $\varepsilon_{ik}^{(0)}$ and $\varepsilon_{ik}^{(r)}$ are the "nonrotational" and "rotational" parts of ε_{ik}. The rotational part is an invariant independent of detailed plasma properties. It is given by

$$\varepsilon_{ik}^{(r)} = \frac{c^2}{v_A^2}\begin{pmatrix} 0 & -2i\Omega/\omega \\ 2i\Omega/\omega & -(1/\omega^2)\, d\Omega^2/d\ln R \end{pmatrix}. \quad (4.4)$$

In contrast to this, the tensor $\varepsilon_{ik}^{(0)}$ essentially depends on plasma properties. In the kinetic description, it is given by

$$\varepsilon_{ik}^{(0)} = \varepsilon_{ik}^{(0)kin} = \begin{pmatrix} \varepsilon_{11}^{AN} & 0 \\ 0 & \varepsilon_{22}^{(0)kin} \end{pmatrix}, \quad (4.5)$$

where, in accordance with (2.8), ε_{11}^{AN} is equal to

$$\varepsilon_{11}^{AN} = \frac{c^2}{v_A^2}\left(\alpha^{AN} + \frac{k_z^2 v_A^2}{\omega^2}\right) = \frac{c^2}{v_A^2}\left[1 - \frac{k_z^2 v_A^2}{2\omega^2}(\beta_\perp - \beta_\parallel)\right], \quad (4.6)$$

and, in accordance with (2.18),

$$\varepsilon_{22}^{(0)kin} = \frac{c^2}{v_A^2}\left(1 + \frac{c_M^{AN} k^2 v_A^2}{\omega^2}\right). \quad (4.7)$$

In accordance with the above explanations, the CGL approach leads to the same expression for $\varepsilon_{11}^{(0)}$ as the kinetic approach, i. e.,

$$\varepsilon_{11}^{(0)} = \varepsilon_{11}^{(0)kin}. \quad (4.8)$$

In contrast to this, we have

$$\varepsilon_{22}^{(0)CGL} \neq \varepsilon_{22}^{(0)kin} \quad (4.9)$$

in the scope of this approach. Turning to Eqs. (3.18) and (3.19), we find

$$\varepsilon_{22}^{(0)CGL} = \frac{c^2}{v_A^2} \times \left\{\alpha_S^{CGL}\left[1 - \frac{k^2 v_A^2}{\omega^2}\left(1 + \frac{\beta_\perp - \beta_\parallel}{2}\right) - \frac{k_R^2 v_A^2}{\omega^2}\beta_\perp\right] - \left(\frac{\beta_\perp}{2}\right)^2 \frac{k_z^2 k_R^2 v_A^4}{\omega^4}\right\}. \quad (4.10)$$

5. PRESSURE-ANISOTROPY-DRIVEN INSTABILITIES IN NONROTATING PLASMA

5.1. Basic equations

In the absence of plasma rotation, i.e., at $\Omega = 0$, Eq. (4.1) splits into two dispersion relations:

$$\alpha_A^{AN} \equiv \frac{v_A^2}{c^2}\left(\varepsilon_{11} - \frac{c^2 k_z^2}{\omega^2}\right) = 0, \qquad (5.1)$$

$$\alpha_M^{AN} \equiv \frac{v_A^2}{c^2}\left(\varepsilon_{22} - \frac{c^2 k^2}{\omega^2}\right) = 0 \qquad (5.2)$$

respectively describing the Alfvén oscillation branches and the magnetoacoustic branches. As we have noted, the value of α_A^{AN} is the same in the kinetic and the CGL approaches. It is given by Eq. (2.8). In contrast to this, the value of α_M^{AN} in the kinetic approach differs from that in the CGL approach:

$$\alpha_M^{AN(kin)} = 1 - \frac{k^2 v_A^2}{\omega^2}\left(1 + c_M^{AN}\right), \qquad (5.3)$$

$$\alpha_M^{AN(CGL)} = \\ = 1 - \frac{k^2 v_A^2}{\omega^2}\left[1 + \frac{k_z^2}{2k^2}\left(\beta_\perp - \beta_\parallel\right) + \frac{k_R^2}{k^2}\frac{\beta_\perp}{\alpha_S^{CGL}}\right]. \qquad (5.4)$$

Physically, the identity of α_A^{AN} in the kinetic and CGL approaches means that the properties of perturbations predicted by both these approaches are the same. In contrast to this, the difference between $\alpha_M^{AN(kin)}$ and $\alpha_M^{AN(CGL)}$ results in different regularities for magnetoacoustic perturbations.

5.2. Alfvén firehose instability

In explicit form, Eq. (5.1) becomes

$$\omega^2 = k_z^2 v_A^2\left(1 + \frac{\beta_\perp - \beta_\parallel}{2}\right). \qquad (5.5)$$

It follows that this equation describes an instability for

$$\beta_\parallel > \beta_\perp + 2. \qquad (5.6)$$

Then

$$\omega^2 = -\gamma^2, \qquad (5.7)$$

where γ is the growth rate of perturbations satisfying the condition

$$\gamma^2 = \frac{k_z^2 v_A^2}{2}\left(\beta_\parallel - \beta_\perp - 2\right). \qquad (5.8)$$

Relations (5.6) and (5.8) describe the Alfvén firehose instability.

5.3. Magnetoacoustic oscillation branches

5.3.1. Dispersion relations

The kinetic approach yields the following dispersion relation for magnetoacoustic oscillation branches [4]:

$$1 - \frac{k^2 v_A^2}{\omega^2}\left\{1 + \frac{k_z^2}{2k^2}\left(\beta_\perp - \beta_\parallel\right) + \right. \\ \left. + \frac{k_R^2}{k^2}\beta_\perp\left[1 - \frac{T_\perp}{T_\parallel}\left(1 + \frac{i\sqrt{\pi}\omega}{|k_z|\,v_{T\parallel}}W\right)\right]\right\} = 0. \qquad (5.9)$$

In the CGL approach, in contrast to (5.9), we find

$$D^{CGL} = 0, \qquad (5.10)$$

where the function D^{CGL} is given by Eqs. (3.21) and (3.22). In explicit form, Eq. (5.10) becomes

$$\left(1 - \frac{3k_z^2 T_\parallel}{M\omega^2}\right)\left\{1 - \frac{k^2 v_A^2}{\omega^2}\left[1 + \frac{k_z^2}{2k^2}\left(\beta_\perp - \beta_\parallel\right) + \right.\right. \\ \left.\left. + \frac{k_R^2}{k^2}\beta_\perp\right]\right\} - \left(\frac{\beta_\perp}{2}\right)^2\frac{k_z^2 k_R^2 v_A^4}{\omega^4} = 0. \qquad (5.11)$$

5.3.2. Magnetoacoustic firehose instability predicted by the kinetic approach

Using Eq. (5.9) and the asymptotic form (2.14) for $\omega \gg |k_z|\,v_{T\parallel}$, we arrive at the dispersion relation

$$1 - \frac{k^2 v_A^2}{\omega^2}\left[1 + \frac{k_z^2}{2k^2}\left(\beta_\perp - \beta_\parallel\right) + \frac{k_R^2}{k^2}\beta_\perp\right] = 0. \qquad (5.12)$$

Perturbations with $k_z \gg k_R$ described by this dispersion relation are unstable for condition (5.6). Their growth rate is given by Eq. (5.7). This is the magnetoacoustic firehose instability predicted by the kinetic approach.

5.3.3. Magnetoacoustic firehose instability predicted by the CGL approach

Dispersion relation (5.11) can be represented in the form [5]

$$\omega^4 - \omega^2 k_z^2 v_A^2\left(1 + \frac{k_R^2}{k^2}\beta_\perp + \frac{3}{2}\frac{k_z^2}{k^2}\beta_\parallel\right) + \\ + \frac{3}{2}k_z^2 k_R^2 v_A^4\left[1 + \frac{\beta_\perp - \beta_\parallel}{2} - \frac{\beta_\perp^2}{6\beta_\parallel}\frac{k_R^2}{k^2} + \right. \\ \left. + \frac{3}{2}\frac{k_R^2}{k^2}\left(\beta_\perp + \beta_\parallel\right)\right] = 0. \qquad (5.13)$$

It hence follows that for $k_z \gg k_R$, the perturbations are unstable for condition (5.6) and their growth rate is given by (5.8). These perturbations correspond to the magnetoacoustic firehose instability.

5.3.4. Kinetic firehose and mirror instabilities

With the asymptotic form (2.15), it follows from Eq. (5.9) that

$$1 - \frac{k^2 v_A^2}{\omega^2}\left\{1 + \frac{k_z^2}{2k^2}\left(\beta_\perp - \beta_\parallel\right) + \right.$$
$$\left. + \frac{k_R^2}{k^2}\beta_\perp\left[1 - \frac{T_\perp}{T_\parallel}\left(1 + \frac{i\sqrt{\pi}\omega}{|k_z|\, v_{T\parallel}}W\right)\right]\right\} = 0, \quad (5.14)$$

and hence [4]

$$\omega = -\frac{i}{\sqrt{\pi}\beta_\perp}\frac{|k_z|^3}{k_R^2}\frac{T_\parallel}{T_\perp}v_{T\parallel}\Delta^{VS}, \quad (5.15)$$

where

$$\Delta^{VS} = \frac{k^2}{k_z^2} +$$
$$+ \frac{\beta_\perp}{2}\left(1 - \frac{T_\parallel}{T_\perp}\right) + \frac{k_R^2}{k^2}\beta_\perp\left(1 - \frac{T_\perp}{T_\parallel}\right) \quad (5.16)$$

and the superscript "VS" refers to the authors of Ref. [4]. We see that perturbations considered are unstable for the condition

$$\Delta^{VS} < 0. \quad (5.17)$$

According to Eq. (5.5), perturbations with $k_z \gg k_R$ are unstable for condition (5.6) with the growth rate $\gamma \equiv \operatorname{Im}\omega$ [4] given by

$$\gamma = \frac{v_{T\parallel}}{\sqrt{\pi}}\frac{|k_z|^3}{k_R^2\beta_\perp}\left(\beta_\parallel - \beta_\perp - 2\right). \quad (5.18)$$

Relations (5.6) and (5.18) characterize the kinetic firehose instability.

In the opposite limit case, $k_R \gg k_z$, instability condition (5.17) reduces to [4]

$$\beta_\perp > \beta_\parallel. \quad (5.19)$$

The growth rate is then given by [4]

$$\gamma = \frac{v_{T\parallel}}{\sqrt{\pi}}\frac{k_R^2}{|k_z|\,\beta_\perp}\left(\beta_\perp - \beta_\parallel\right). \quad (5.20)$$

Equations (5.19) and (5.20) characterize the kinetic mirror instability.

5.3.5. Mirror instability predicted by the CGL approach

According to Eq. (5.14), near the stability boundary, the perturbations with $k_R \gg k_z$ are described by the dispersion relation [5]

$$\omega^2 = \frac{3}{2}k_R^2 v_A^2 \beta_\parallel \frac{1 + \beta_\perp - \beta_\perp^2/6\beta_\parallel}{1 + \beta_\perp}. \quad (5.21)$$

Then we find that the perturbations are unstable for [5]

$$\beta_\perp > \frac{6\beta_\parallel}{\beta_\perp}\left(1 + \beta_\perp\right). \quad (5.22)$$

Their growth rate is given by

$$\gamma^2 = \frac{3}{2}k_R^2 v_A^2 \frac{\beta_\perp^2/6 - (1 + \beta_\perp)\,\beta_\parallel}{1 + \beta_\perp}. \quad (5.23)$$

Relations (5.21) and (5.23) characterize the mirror instability predicted by the CGL approach. Comparing (5.21) and (5.22) with (5.18) and (5.19), we see that the predictions of the kinetic and CGL approaches regarding the mirror instability are essentially different.

6. ROTATIONAL FIREHOSE INSTABILITY

6.1. General expressions for growth rate near the stability boundary

Starting with Eq. (2.26) as $k_R \to 0$, both the kinetic and the CGL approaches result in the dispersion relation

$$\left[1 - \frac{k_z^2 v_A^2}{\omega^2}\left(1 + \frac{\beta_\perp - \beta_\parallel}{2}\right)\right] \times$$
$$\times \left[1 + \frac{k_z^2 v_A^2}{\omega^2}\left(\Delta + \frac{\beta_\parallel - \beta_\perp}{2}\right)\right] - \frac{4\Omega^2}{\omega^2} = 0. \quad (6.1)$$

According to (6.1), the plasma rotation leads to the two following effects. First, it mixes the Alfvén firehose mode with the magnetoacoustic one (see the term with $4\Omega^2$ in (6.1)). Second, it modifies both these modes by the Velikhov effect (see the term with Δ).

To elucidate the results of these effects, we consider Eq. (6.1) near the stability boundary. We then have

$$\omega^2\left(2 + \frac{\kappa^2}{k_z^2 v_A^2} + \beta_\perp - \beta_\parallel\right) =$$
$$= -k_z^2 v_A^2\left(1 + \frac{\beta_\perp - \beta_\parallel}{2}\right)\left(\Delta + \frac{\beta_\parallel - \beta_\perp}{2}\right). \quad (6.2)$$

The growth rate is given by

$$\gamma^2 = k_z^2 v_A^2 \times$$
$$\times \frac{\left[1 + \left(\beta_\perp - \beta_\parallel\right)/2\right]\left[\Delta + \left(\beta_\parallel - \beta_\perp\right)/2\right]}{2 + \kappa^2/k_z^2 v_A^2 + \beta_\perp - \beta_\parallel}. \quad (6.3)$$

In the case of isotropic pressure, $\beta_\perp = \beta_\parallel$, this relation becomes [13]

$$\gamma^2 = k_z^2 v_A^2 \Delta/\Delta_1, \quad (6.4)$$

where

$$\Delta_1 = 2 + \kappa^2/k^2 v_A^2. \quad (6.5)$$

Then the modes considered are unstable only due to the Velikhov effect.

We introduce the rotational Mach number M_R as

$$M_R^2 = \Omega^2/k^2 v_A^2. \quad (6.6)$$

For large $M_R \gg 1$ and $\beta_\perp, \beta_\parallel \gg 1$ with $\beta_\perp \neq \beta_\parallel$, Eq. (6.3) takes the form

$$\gamma^2 = \frac{k_z^2 v_A^2}{2} \times \times \frac{\left[-\left(d\Omega^2/d\ln R\right)/k^2 v_A^2 + \left(\beta_\parallel - \beta_\perp\right)/2\right]}{\kappa^2/k_z^2 v_A^2 + \beta_\perp - \beta_\parallel} \times \times \left(\beta_\perp - \beta_\parallel\right). \quad (6.7)$$

In the limit case of very strong rotation,

$$M_R^2 \gg \left|\beta_\parallel - \beta_\perp\right|, \quad (6.8)$$

it follows from Eq. (6.7) that

$$\gamma^2 = \frac{k_z^4 v_A^2}{k^2} \frac{\left(-d\Omega^2/d\ln R\right)}{\kappa^2} \left(\beta_\perp - \beta_\parallel\right). \quad (6.9)$$

6.2. The case where the Velikhov instability criterion is satisfied

We analyze the above formulas in the case when the Velikhov instability criterion

$$\Delta > 0 \quad (6.10)$$

is satisfied. Then the MRI is suppressed for

$$\beta_\parallel > \beta_\perp. \quad (6.11)$$

In the opposite case

$$\beta_\perp > \beta_\parallel, \quad (6.12)$$

it is enhanced, with the growth rate given by

$$\gamma = \frac{k_z^2 v_A}{|k|} \frac{\sqrt{-d\Omega^2/d\ln R}}{\kappa} \sqrt{\beta_\perp - \beta_\parallel}. \quad (6.13)$$

6.3. The case where the Velikhov instability criterion is not satisfied

According to Eq. (6.7), the perturbations considered can be unstable even if the Velikhov instability criterion is not satisfied,

$$\Delta < 0. \quad (6.14)$$

This case includes the situation where

$$d\Omega^2/d\ln R > 0. \quad (6.15)$$

In this case, for example, it follows from (6.13) that the perturbations are unstable for condition (6.11).

7. ROTATIONAL MIRROR INSTABILITY

We now consider the perturbations with $k_R \gg k_z$ near their stability boundary. Then, in contrast to the rotational firehose instability, the mixing between the Alfvén and magnetoacoustic oscillation branches is unimportant, and therefore we have the dispersion relation

$$\varepsilon_{22} - k_R^2 v_A^2/\omega^2 = 0. \quad (7.1)$$

Using the kinetic expression for ε_{22} and taking $\omega \ll |k_z| v_{T\parallel}$, we arrive at the following generalization of dispersion relation (5.18):

$$\omega = \frac{i\,|k_z|\, v_{T\parallel}}{\sqrt{\pi}} \left(1 - \frac{T_\parallel}{T_\perp} + \frac{\Delta}{\beta_\perp}\frac{T_\parallel}{T_\perp}\right). \quad (7.2)$$

In the case of isotropic pressure ($T_\parallel = T_\perp$), this oscillation branch is unstable only due to the Velikhov effect [12]:

$$\omega = \frac{i\,|k_z|\, v_{T\parallel}}{\sqrt{\pi}} \frac{\Delta}{\beta_\perp}. \quad (7.3)$$

This corresponds to the kinetic MRI. We see that the MRI is suppressed for

$$\Delta < \left(\beta_\parallel - \beta_\perp\right) T_\perp/T_\parallel. \quad (7.4)$$

Such a suppression can occur for condition (6.11). In the opposite case, i. e., for condition (6.12), the MRI instability is enhanced and the perturbations can be unstable even if the Velikhov instability criterion is not satisfied, i. e., for condition (6.14). In the opposite case, i. e., for condition (6.12), the MRI is enhanced.

8. DISCUSSIONS

The main result in the present paper consists in pointing out two new instabilities in rotating plasma with anisotropic pressure of the hybrid type: rotational firehose instability and rotational mirror instability. They are described by the respective dispersion relations (6.3) and (7.2). In both these cases, the pressure anisotropy of the type $\beta_\parallel > \beta_\perp$ (see Eq. (6.11)) is stabilizing, while that of type $\beta_\perp > \beta_\parallel$ (see Eq. (6.12)) is destabilizing. In other words, the anisotropy leading to the mirror instability in a nonrotating plasma is destabilizing for both types of the hybrid instabilities, while that leading to the firehose instability is stabilizing.

We are grateful to G. D. Chagelishvili, S. V. Konovalov, E. A. Pashitskii, A. I. Smolyakov, A. V. Stepanov, K. N. Stepanov, S. V. Vladimirov, and L. M. Zelenyi for useful comments. This work was supported by the Russian Foundation for Basic Research (grant No. 06-02-16767), the International Science and Technology Center (ISTC), grant No. G-1217, the Science and Technology Center in Ukraine (grant No. 3473), the Research Foundation of the State of São Paulo (FAPESP), the National Council of Scientific and Technological Development (CNPq), and the Ministry of Science and Technology, Brazil.

REFERENCES

1. E. P. Velikhov, Zh. Eksp. Teor. Fiz. **36**, 1398 (1959).
2. S. A. Balbus and J. F. Hawley, Astrophys. J. **376**, 214 (1991).
3. N. I. Shakura and R. A. Sunyaev, Astron. Astrophys. **24**, 337 (1973).
4. A. A. Vedenov and R. Z. Sagdeev, in *Plasma Physics and the Problem of Controlled Thermonuclear Reactions*, Vol. 3, Pergamon Press, New York (1958), p. 332.
5. L. I. Rudakov and R. Z. Sagdeev, in *Plasma Physics and the Problem of Controlled Thermonuclear Reactions*, Vol. 3, Pergamon Press, New York (1958), p. 321.
6. S. Chandrasekhar, A. N. Kaufman, and K. M. Watson, Proc. Roy. Soc. London A **245**, 435 (1958).
7. E. N. Parker, Phys. Rev. **109**, 1874 (1958).
8. A. B. Kitsenko and K. N. Stepanov, Zh. Eksp. Teor. Fiz. **38**, 1840 (1960).
9. A. B. Mikhailovskii, in *Reviews of Plasma Physics*, Vol. 6, ed. by M. A. Leontovich, Consultants Bureau, New York (1975), p. 77.
10. G. F. Chew, M. L. Goldberger, and F. E. Low, Proc. Roy. Soc. London A **236**, 112 (1956).
11. T. F. Volkov, in *Reviews of Plasma Physics*, Vol. 4, ed. by M. A. Leontovich, Consultants Bureau, New York (1966), p. 1.
12. A. B. Mikhailovskii, J. G. Lominadze, A. P. Churikov et al., Zh. Eksp. Teor. Fiz. **133**, 183 (2007).
13. Y. M. Huang and A. B. Hassam, Phys. Plasmas **10**, 204 (2003).
14. K. Ferriere, Space Sci. Rev. **122**, 247 (2006).
15. A. B. Mikhailovskii, *Instabilities in a Confined Plasma*, Institute of Physics, Bristol (1998).
16. A. B. Mikhailovskii, *Theory of Plasma Instabilities*, Vol. 1, Consultants Bureau, New York (1974).