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QUANTUM TELEPORTATION THROUGH AN ENTANGLED STATE COMPOSED OF DISPLACED VACUUM AND SINGLE-PHOTON STATES

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We study the teleportation protocol of an unknown macroscopic qubit by means of a quantum channel composed of the displaced vacuum and single-photon states. The scheme is based on linear optical devices such as a beam splitter and photon number resolving detectors. A method based on conditional measurement is used to generate both the macroscopic qubit and entangled state composed from displaced vacuum and single-photon states. We show that such a qubit has both macroscopic and microscopic properties. In particular, we investigate the quantum teleportation protocol from a macroscopic object to a microscopic state.

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1. INTRODUCTION

Quantum teleportation, first proposed by Bennett et al. [1], is a technique for moving an unknown state around in the presence of a quantum communication channel linking the sender to the recipient. Several experiments were implemented to demonstrate quantum teleportation [2, 3]. In the teleportation experiment in [2], entanglement of photons with different polarizations was used. In the teleportation experiment in [3], the quantum channel was a two-mode squeezed state. Presently, there is growing interest in the use of Schrödinger-cat states [4] for quantum information processing. Entangled coherent states were proposed to teleport a qubit encoded in a Schrödinger-cat state in [5]. It was shown in [6] how quantum information processing can be implemented using even and odd coherent superposition states. Quantum information processing based on entangled coherent states is described in [7]. There is a sole drawback of the application of entangled coherent states to quantum computation: it is extremely difficult to prepare them in practice [8].

In this paper, we study a quantum teleportation protocol with the help of an entangled state constructed from displaced vacuum and single-photon states [9–11]. The conditional mechanism of generation of both the macroscopic qubit and the entangled state composed of the displaced vacuum and single-photon states is used [12–14]. We show that such a macroscopic qubit reveals both microscopic and macroscopic properties. A scheme of quantum teleportation from a macroscopic object to a microscopic one is studied. The problem of the use of displaced states in quantum information processing is fresh and enables an additional degree of freedom. The photon state is determined by its number, while the phase is completely random. The displaced state is obtained from a number state by adding a nonzero value to the field amplitude. By displacing in phase space, a field amplitude is added to this state, and the photon number has a contribution from the coherent component of the field. The displaced state becomes phase dependent, which allows using new pairs of quasi-orthogonal entangled states (which may be used in other quantum protocols). The study of the quantum teleportation through the entangled states composed of the displaced state may be considered an extension of the previously proposed protocol based on entangled coherent states [5], which can be performed only in the presence of highly efficient and precise photocounting. Highly efficient photodetectors that precisely distinguish between m and $m+1$ must be used in [5]. Our proposal reduces the requirements on the parameters of registering devices.

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2. GENERATION OF AN ENTANGLED CHANNEL FROM DISPLACED VACUUM AND SINGLE-PHOTON STATES

We start with the problem of generation of an entangled state composed of the displaced vacuum and single-photon states. Parametric down conversion in a nonlinear crystal is used for the production of the desired state. A high-energy pump photon may split into two lower-energy photons that are normally emitted into symmetrically oriented directions. The modes into which the photons are emitted are called the signal and idler modes. Starting from the input vacuum field in the signal and idler modes, spontaneous parametric down conversion occurs. To generate a single-photon added state, a seed coherent state must be ejected, e.g., into the signal mode. We use the simple model where the pump mode is classical and interacts with two modes at frequencies ω_1 and ω_2. Two nonlinear $\chi^{(2)}$ crystals are placed in the modes of two powerful fields. Ancilla coherent states $|\alpha\rangle_1|\alpha\rangle_3$ are used as shown in Fig. 1. Dynamical description of the system of coupled converters involves four modes with the corresponding annihilation operators $\hat{a}_1$, $\hat{a}_2$, $\hat{a}_3$, and $\hat{a}_4$ described by the Hamiltonian

$$\hat{H} = i\hbar\chi\left(\hat{a}_1^\dagger\hat{a}_2^\dagger - \hat{a}_1\hat{a}_2 + \hat{a}_3^\dagger\hat{a}_4^\dagger - \hat{a}_3\hat{a}_4\right) \qquad (1)$$

in the interaction picture [15, 16]. The coupling coefficient χ in (1) is connected with the nonlinear second-order susceptibility tensor $\chi^{(2)}$ and also involves a classical pump. The coupling constants of two converters are assumed to be identical to each other, $\chi_1 = \chi_2 = \chi$. The input wave function to Hamiltonian (1) is $|\Psi_{IN}\rangle = |\alpha\rangle_1|0\rangle_2|\alpha\rangle_3|0\rangle_4$. If the parametric gain of the system of the coupled converters ($g \ll 1$, where $g = \chi t$) is sufficiently low, the final output state is described by

$$|\Psi_{OUT}\rangle = \exp\left(-i\frac{\hat{H}t}{\hbar}\right)|\Psi_{IN}\rangle \approx$$
$$\approx \left(1 + g\left(\hat{a}_1^\dagger\hat{a}_2^\dagger - \hat{a}_2\hat{a}_1 + \hat{a}_3^\dagger\hat{a}_4^\dagger - \hat{a}_3\hat{a}_4\right)\right) \times$$
$$\times|\alpha\rangle_1|0\rangle_2|\alpha\rangle_3|0\rangle_4 = |\alpha\rangle_1|0\rangle_2|\alpha\rangle_3|0\rangle_4 + g\sqrt{1+|\alpha|^2}\times$$
$$\times\left(|\alpha,1\rangle_1|\alpha\rangle_3|10\rangle_{24} + |\alpha\rangle_1|\alpha,1\rangle_3|01\rangle_{24}\right). \qquad (2)$$

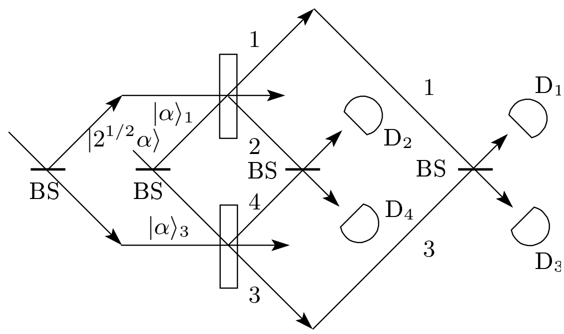

Fig. 1. Experimental arrangement of the system of coupled converters. Two parametric converters with type-I phase matching are inserted in the routes of powerful fields. An additional coherent state with the amplitude $|\sqrt{2}\,\alpha\rangle$ is injected into the system through the beam splitter (BS) with the Hadamard unitary operation

We now use a beam splitter to superimpose the following modes. Input–output relations at a lossless beam splitter can be characterized by the $SU(2)$ Lie algebra [25]. A beam splitter can be considered a four-port device with the input–output relations in the Heisenberg picture given by

$$\begin{bmatrix}\hat{a}_1^\dagger\\ \hat{a}_2^\dagger\end{bmatrix}_{out} = B\begin{bmatrix}\hat{a}_1^\dagger\\ \hat{a}_2^\dagger\end{bmatrix}_{in} = \begin{bmatrix}T & R\\ -R^* & T^*\end{bmatrix}\begin{bmatrix}\hat{a}_1^\dagger\\ \hat{a}_2^\dagger\end{bmatrix}_{in}, \qquad (3a)$$

where T and R are the transmittance and reflectivity of the beam splitter. A very simple way to describe the action of a beam splitter is to fix the phase relations by using two $\pi/2$ phase shifters inserted into both the input and output modes such that the beam splitter is described by the Hadamard transformation

$$\begin{bmatrix}\hat{a}_1^\dagger\\ \hat{a}_2^\dagger\end{bmatrix}_{out} = H\begin{bmatrix}\hat{a}_1^\dagger\\ \hat{a}_2^\dagger\end{bmatrix}_{in} = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}\hat{a}_1^\dagger\\ \hat{a}_2^\dagger\end{bmatrix}_{in}. \qquad (3b)$$

It is worth mentioning that the standard Hadamard gate in quantum computation is a single-qubit gate, while we use unitary matrix (3b) as a two-qubit operation. For a single incident particle, the action of the beam splitter with matrix (3b) is described by the standard Hadamard transformation ($H|0\rangle \to (|0\rangle+|1\rangle)/\sqrt{2}$, $H|1\rangle \to (|0\rangle - |1\rangle)/\sqrt{2}$). The general case of the beam splitter with matrix (3b) involves both incoming modes occupied by photons.

We combine modes 2 and 4 on the beam splitter with matrix (3b), which gives

$$\hat{H}_{24}|\Psi_{OUT}\rangle = |\alpha\rangle_1|0\rangle_2|\alpha\rangle_3|0\rangle_4 +$$
$$+ g\sqrt{1+2\alpha^2}\,|\Psi_+\rangle_{13}|10\rangle_{24} - g|\Psi_-\rangle_{13}|01\rangle_{24}, \qquad (4)$$

where we introduce the macroscopic entangled states

$$|\Psi_\pm\rangle_{13} = \frac{(|\alpha\rangle|\alpha,1\rangle \pm |\alpha,1\rangle|\alpha\rangle)_{13}}{\sqrt{2(1\pm|\alpha|^2/(1+|\alpha|^2))}}, \quad (5)$$

where

$$|\alpha\rangle = \exp\left(-\frac{|\alpha|^2}{2}\right)\sum_{l=1}^{\infty}\frac{\alpha^l}{\sqrt{l!}}|l\rangle \quad (6a)$$

is the coherent state and

$$|\alpha,1\rangle = \frac{\hat{a}^\dagger|\alpha\rangle}{\sqrt{\langle\alpha|\hat{a}\hat{a}^\dagger|\alpha\rangle}} = \frac{\exp(-|\alpha|^2/2)}{\alpha\sqrt{1+|\alpha|^2}}\sum_{l=1}^{\infty}\frac{\alpha^l l}{\sqrt{l!}}|l\rangle \quad (6b)$$

is the one-photon added coherent state [17, 18]. Depending on the measurement outcome of two photodetectors placed behind the beam splitters in modes 2 and 4, the output can be divided into two groups if detector D_2 fires or if D_4 does. Each outcome of the measurement projects the state $\hat{H}_{24}|\Psi_{OUT}\rangle$ on either $|\Psi_+\rangle_{13}$ or $|\Psi_-\rangle_{13}$ depending on which detector clicked. The states $|\Psi_\pm\rangle$ are orthogonal to each other: $\langle\Psi_-|\Psi_+\rangle = 0$. It is worth noting that some methods to conditionally generate a one-photon added coherent state in the pumping modes were studied in [19]. We deal with conditional generation of the entangled state composed of the coherent and single-photon added coherent states in the signal and idler modes in the scheme in Fig. 1, not affecting transformation processes in the pumping modes as is the case in most experimental situations [20].

We note some properties of the $|\Psi_\pm\rangle$ states [15, 16]. The concurrence, as a measure of the amount of entanglement, of the state $|\Psi_-\rangle$ is equal to one independently of the value of the parameter α, while the concurrence of the state $|\Psi_+\rangle$ is $C(|\Psi_+\rangle) = 1/1+2|\alpha|^2$, which is less than one if $|\alpha| > 0$ and rapidly decreases as α increases. If the size α of the state $|\Psi_+\rangle$ approaches infinity ($\alpha \to \infty$), the concurrence tends to zero ($C(|\Psi_+\rangle) \to 0$). Because the state $|\Psi_-\rangle$ has the maximum possible amount of entanglement, it can be presented in terms of orthogonal states, namely, displaced vacuum and single-photon states [9–11]

$$|\Psi\rangle_{13} = \frac{(|0,\alpha\rangle|1,\alpha\rangle - |1,\alpha\rangle|0,\alpha\rangle)_{13}}{\sqrt{2}}, \quad (7)$$

where the components of the state are given by

$$|0,\alpha\rangle = \hat{D}(\alpha)|0\rangle \equiv |\alpha\rangle, \quad (8a)$$

$$|1,\alpha\rangle = \hat{D}(\alpha)|1\rangle = -\alpha^*\left(|\alpha\rangle - \frac{\sqrt{1+|\alpha|^2}}{\alpha^*}|\alpha,1\rangle\right), \quad (8b)$$

where $\hat{D}(\alpha) = \exp(\alpha\hat{a}^\dagger - \alpha^*\hat{a})$ is the unitary displacement operator [20].

A remark on the notation used is in order. The displaced single-photon $|1,\alpha\rangle$ state should not be confused with the single-photon added coherent state $|\alpha,1\rangle$. There is a difference between them because either annihilation or creation operators do not commute with the unitary displacement operator ($[\hat{a},\hat{D}(\alpha)] \neq 0$). In the limit case as $\alpha \to 0$, the states $|1,\alpha\rangle$ and $|\alpha,1\rangle$ approach the one-photon state $|1\rangle$. Coherent and one-photon added coherent states are not orthogonal to each other ($\langle\alpha|\alpha,1\rangle = \alpha^*/\sqrt{1+|\alpha|^2}$) and their scalar product approaches unity as $\alpha \to \infty$. On the contrary, the state $|1,\alpha\rangle$ is always orthogonal to the coherent state with the same amplitude α, irrespective of the value of α ($\langle\alpha|1,\alpha\rangle = 0$). Therefore, we can interpret the displaced vacuum and single-photon states as basis states (logical zero and logical one) for a logical qubit in the framework of quantum information. Below, we let $|0,\alpha\rangle \equiv |\alpha\rangle$ denote a coherent state by analogy with $|1,\alpha\rangle$. Then the well-known Schrödinger-cat state [7, 8] can be rewritten in our notation as $|\alpha\rangle + |-\alpha\rangle \equiv |0,\alpha\rangle + |0,-\alpha\rangle$ up to normalization factor. Such a representation of the Schrödinger-cat state reflects the difference of the mechanisms of generating the states. The Schrödinger-cat state is generated due to a nonlinear phase shift of one of the superposition components [8], while the single-photon state is created owing to single-photon addition and displacement. We sometimes call the states $|1,\alpha\rangle$ and $|0,\alpha\rangle$ a macroscopic single-photon state and a macroscopic vacuum, respectively.

3. CREATION OF THE MACROSCOPIC QUBIT AND ITS PROPERTIES

We consider generation of an unknown macroscopic qubit composed of the displaced vacuum and single-photon states. The superposition state

$$|\tau\rangle = A|0,\alpha\rangle + B|1,\alpha\rangle, \quad (9)$$

where A and B are arbitrary amplitudes (with $|A|^2 + |B|^2 = 1$), can be generated through a photon-number conditional measurement. We take a beam splitter with unitary matrix (3a) and combine the single-photon added coherent state with an amplitude α_1 (mode 1) with the ancilla coherent state with an amplitude α_2 (mode 2) through a beam splitter. The mixing result is given by

$$\hat{U}(|\alpha_1, 1\rangle_1 |\alpha_2\rangle_2) = \frac{1}{\sqrt{1+|\alpha_1|^2}} \left(\begin{array}{c} T\sqrt{1+|\alpha_1 T - \alpha_2 R^*|^2}\, |\alpha_1 T - \alpha_2 R^*, 1\rangle_1 |\alpha_1 R + \alpha_2 T^*\rangle_2 + \\ + R\sqrt{1+|\alpha_1 R + \alpha_2 T^*|^2}\, |\alpha_1 T - \alpha_2 R^*\rangle_1 |\alpha_1 R + \alpha_2 T^*, 1\rangle_2 \end{array} \right). \quad (10)$$

In the case where $\alpha_1 R + \alpha_2 T^* \ll 1$, we can neglect higher-order photon-number states in the $|\alpha_1 R + \alpha_2 T^*\rangle$ and $|\alpha_1 R + \alpha_2 T^*, 1\rangle$ states and replace them with finite superpositions up to $c_0^{(i)}|0\rangle + c_1^{(i)}|1\rangle + c_2^{(i)}|2\rangle$, where $c_j^{(j)}$ are the respective amplitudes ($i = 0, 1$ and $j = 0, 1, 2$) of the coherent and single-photon added coherent states. When the state $|0\rangle_2$ is registered in the second output mode of the beam splitter, this detection projects the first mode into a single-photon added coherent state with the amplitude $\alpha_1 T - \alpha_2 R^*$ [13]. State (8) is conditionally generated if either the state $|1\rangle$ or the state $|2\rangle$ is registered in the second mode, which can be performed by using a photon-number resolving detectors [21].

We note some properties of the superposition of displaced vacuum and single-photon states (9). We consider readout of qubit (9) in the framework of quantum information processing and assume that somebody gave us either the $|0, \alpha\rangle$ or the $|1, \alpha\rangle$ state, which we do not know exactly. Our task is to determine which state we were given. We prepare a second input channel in the beam splitter in a coherent state with the amplitude α_1, $|0, \alpha_1\rangle_2$, and impose the restriction $\alpha R + \alpha_1 T^* = 0$ on the input conditions. Then, combining the input states $|0, \alpha\rangle_1$ and $|0, \alpha_1\rangle_2$, we have the output $|0, \alpha/T^*\rangle_1 |0\rangle_2$, while combining the input states $|1, \alpha\rangle_1$ and $|0, \alpha_1\rangle_2$, we have

$$\hat{U}_{\alpha R + \alpha_1 T^* = 0}\left(|1, \alpha\rangle_1 |0, \alpha_1\rangle_2\right) = = T \left|1, \frac{\alpha}{T^*}\right\rangle_1 |0\rangle_2 + R \left|0, \frac{\alpha}{T^*}\right\rangle_1 |1\rangle_2. \quad (11)$$

If an ideal detector following the beam splitter in the second output mode registers a single-photon impact, this definitely means that we obtained the $|1, \alpha\rangle_1$ state. If the detector does not click, we can tell nothing definite about the state that was sent. But we can decrease the influence of the $T|1, \alpha/T^*\rangle_1 |0\rangle_2$ term in (11) by choosing a beam splitter with the transmittance T as low as possible ($|T| \ll 1$) such that $\lim\limits_{T^* \to \infty} \alpha_1 T^* = -\alpha$. If we take the transmittance T very small, then it is possible to claim that the state $|1, \alpha\rangle_1$ was given in the case we registered nothing. The probability to mistake the $|0, \alpha\rangle_1$ state for the $|1, \alpha\rangle_1$ state is $|T|^2 \ll 1$.

The same argument is applicable to state (9). If a single photon hits a photodetector in the second mode, we definitely know that the $|1, \alpha\rangle_1$ state was measured; if we do not register anything, we know it was the $|0, \alpha\rangle_1$ state with fidelity

$$F = |A|^2 / (|A|^2 + |B|^2 |T|^2) \approx 1 - |B|^2 |T|^2 / |A|^2.$$

The fidelity approaches 1 as $|T| \to 0$, which confirms the possibility to almost definitely distinguish the states $|0, \alpha\rangle_1$ and $|1, \alpha\rangle_1$ from each other.

We consider the interference properties of qubit (9). We calculate the Wigner function of superposition (8) with $A = 1/\sqrt{2}$ and $B = -\exp(i\varphi_a)/\sqrt{2}$. The Wigner function of such a state is evaluated as

$$W_{|\Psi_B\rangle}(x, y) = \frac{4}{\pi}\left((x - x_\alpha)(x - x_\alpha - \cos\varphi_\alpha) + \right. + (y - y_\alpha)(y - y_\alpha - \sin\varphi_\alpha)) \times \times \exp\left(-2\left((x - x_\alpha)^2 + (y - y_\alpha)^2\right)\right), \quad (12a)$$

where $\cos\varphi_\alpha = x_\alpha / \sqrt{|x_\alpha|^2 + |y_\alpha|^2}$ and $\sin\varphi_\alpha = y_\alpha / \sqrt{|x_\alpha|^2 + |y_\alpha|^2}$. The Wigner function of the statistical mixture described by the density matrix $\rho = (|0, \alpha\rangle\langle 0, \alpha| + |1, \alpha\rangle\langle 1, \alpha|)/2$ is given by

$$W_\rho(x, y) = \frac{4}{\pi}\left((x - x_\alpha)^2 + (y - y_\alpha)^2\right) \times \times \exp\left(-2\left((x - x_\alpha)^2 + (y - y_\alpha)^2\right)\right). \quad (12b)$$

The Wigner functions of the balanced superposition and statistical mixture are plotted in Fig. 2. The Wigner function of the statistical mixture has two peaks, while one of the peaks is destroyed in the case of the balanced superposition due to the interference effect inherent to superposition (9).

For example, the marginal distribution, as a function of x, can be obtained by integrating the Wigner function over y. We consider the case of a balanced superposition. The corresponding quadrature distribution $|\Psi(x)|^2$ of the pure state is given by

$$|\Psi(x)|^2 = 2\sqrt{\frac{2}{\pi}} \times \times ((x - x_\alpha)(x - x_\alpha - \cos\varphi_\alpha) + 0.25) \times \times \exp\left(-2(x - x_\alpha)^2\right). \quad (13)$$

The corresponding dependences of the quadrature distributions $|\Psi(x)|^2$ are plotted in Fig. 3 for both superposition state (13) and the statistical mixture. The plots clearly show the difference in behavior of pure macroscopic qubit (9) and the statistical mixture. The quadrature components of the pure state and of the statistical mixture can be measured in experiments to observe the difference.

4. GENERATION OF AN ENTANGLED CHANNEL WITH DIFFERENT AMPLITUDES

In Sec. 2, we discussed a method of conditional generation of the entangled state composed of displaced vacuum and single photon states (Eq. (7)) with equal amplitudes. We now consider conditional generation of entangled state (7) with different amplitudes. The method used in Sec. 2 is not applicable to the generation of entangled state (7) with distinct amplitudes. We assume that the modes of the beam splitter are prepared in a coherent state with the amplitude $\sqrt{2}\,\alpha$ ($|\sqrt{2}\,\alpha\rangle_1$) (mode 1) and in a state of a single-mode qubit consisting of the vacuum and single photon

$$|\tau\rangle_2 = \frac{|\alpha|}{\sqrt{2+|\alpha|^2}} \left(|0\rangle - \frac{\sqrt{2}}{\alpha^*}\,|1\rangle \right)_2 . \qquad (14)$$

Experimental generation of an arbitrary superposition of the vacuum and single-photon states has been accomplished using parametric down conversion with the input signal mode prepared in a coherent state [22], employing the quantum scissor scheme [23], or conditioning on homodyne measurements on one part of a nonlocal single photon in two spatial modes [24].

The result of the unitary transformation with matrix (3b) is

$$\hat{U}_H \left(|\sqrt{2}\,\alpha\rangle_1 \frac{|\alpha|}{\sqrt{2+|\alpha|^2}} \left(|0\rangle - \frac{\sqrt{2}}{\alpha^*}\,|1\rangle \right)_2 \right) = \\ = \frac{1}{\sqrt{2+|\alpha|^2}} \left(|1,\alpha\rangle_1 |\alpha\rangle_2 + \right. \\ \left. + \frac{|\alpha|}{\alpha^*} \sqrt{1+|\alpha|^2}\,|\alpha\rangle_1 |\alpha,1\rangle_2 \right), \qquad (15)$$

where $\hat{U}_H$ is the evolution operator corresponding to the Hadamard unitary transformation. We restrict our attention to the events in which no photons are recorded in the second output channel (Eq. (15)). Then the first output channel of the beam splitter is conditionally prepared in the state $|1,\alpha\rangle$ because one-photon added coherent state (6(b)), unlike the coherent one, does not contain the vacuum state. The conditionally prepared displaced single-photon state $|1,\alpha\rangle$ can serve as a basis for generation of the entangled state with different amplitudes. To show this, we combine $|1,\alpha\rangle_1$ with the ancilla coherent state $|0,\alpha_1\rangle_2$ on a beam splitter with arbitrary T and R parameters,

$$\hat{U}\,(|1,\alpha\rangle_1 |0,\alpha_1\rangle_2) = \\ = T|1,\alpha T - \alpha_1 R^*\rangle_1 |0,\alpha R + \alpha_1 T^*\rangle_2 + \\ + R|0,\alpha T - \alpha_1 R^*\rangle_1 |1,\alpha R + \alpha_1 T^*\rangle_2. \qquad (16)$$

Choosing $|T| = |R| = 1/\sqrt{2}$, we obtain a maximally entangled state. Imposing the condition $\alpha R + \alpha_1 T^* = 0$, we prepare a maximally entangled state consisting of macroscopic and microscopic objects

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left(|1,\sqrt{2}\,\alpha\rangle|0\rangle + |0,\sqrt{2}\,\alpha\rangle|1\rangle \right) \qquad (17)$$

if $T = R = -R^* = 1/\sqrt{2}$ and $T^* = -1/\sqrt{2}$.

5. TELEPORTATION

We consider a teleportation scheme as an example of the use of the entangled state like (17). A typical setup of the teleportation problem is as follows. Alice wishes to teleport unknown state (9) in mode 1 to remote colleague Bob by prior sharing state (16). Modes 1 and 2 are at Alice's hands, while mode 3 is at Bob's side. The initial state of the joint system is given by

$$|\Omega\rangle_{123} = |\tau\rangle_1 |\Psi\rangle_{23}. \qquad (18)$$

We consider the case where all three amplitudes of the state $|\Omega\rangle_{123}$ are not equal to each other. We apply the $\hat{U}_H$ operation to state (18) and finally obtain

$$\hat{U}_H|\Omega\rangle_{123} = \frac{1}{2}|1,(\alpha_1+\alpha_2)/\sqrt{2}\,\rangle_1 \times \\ \times |0,(\alpha_1-\alpha_2)/\sqrt{2}\,\rangle_2 \,(A|0,\alpha_3\rangle_3 + B|1,\alpha_3\rangle_3) - \\ - \frac{1}{2}|0,(\alpha_1+\alpha_2)/\sqrt{2}\,\rangle_1 |1,(\alpha_1-\alpha_2)/\sqrt{2}\,\rangle_2 \times \\ \times (A|0,\alpha_3\rangle_3 - B|1,\alpha_3\rangle_3) + \frac{A}{\sqrt{2}} \times \\ \times |0,(\alpha_1+\alpha_2)/\sqrt{2}\,\rangle_1 |0,(\alpha_1-\alpha_2)/\sqrt{2}\,\rangle_2 |1,\alpha_3\rangle_3 + \\ + \frac{B}{\sqrt{2}} \left(|2,(\alpha_1+\alpha_2)/\sqrt{2}\,\rangle_1 |0,(\alpha_1-\alpha_2)/\sqrt{2}\,\rangle_2 - \right. \\ \left. - |0,(\alpha_1+\alpha_2)/\sqrt{2}\,\rangle_1 |2,(\alpha_1-\alpha_2)/\sqrt{2}\,\rangle_2 \right) |0,\alpha_3\rangle_3. \qquad (19)$$

Alice must now perform a photon-number measurement by placing detectors behind the beam splitter and then send her results to Bob, to do the corresponding unitary operations. To simplify Alice and Bob's aim, we take $\alpha_1 = \alpha_2$ and $\alpha_3 = 0$ ($|n,\alpha=0\rangle \equiv |n\rangle$). Then formula (19) becomes

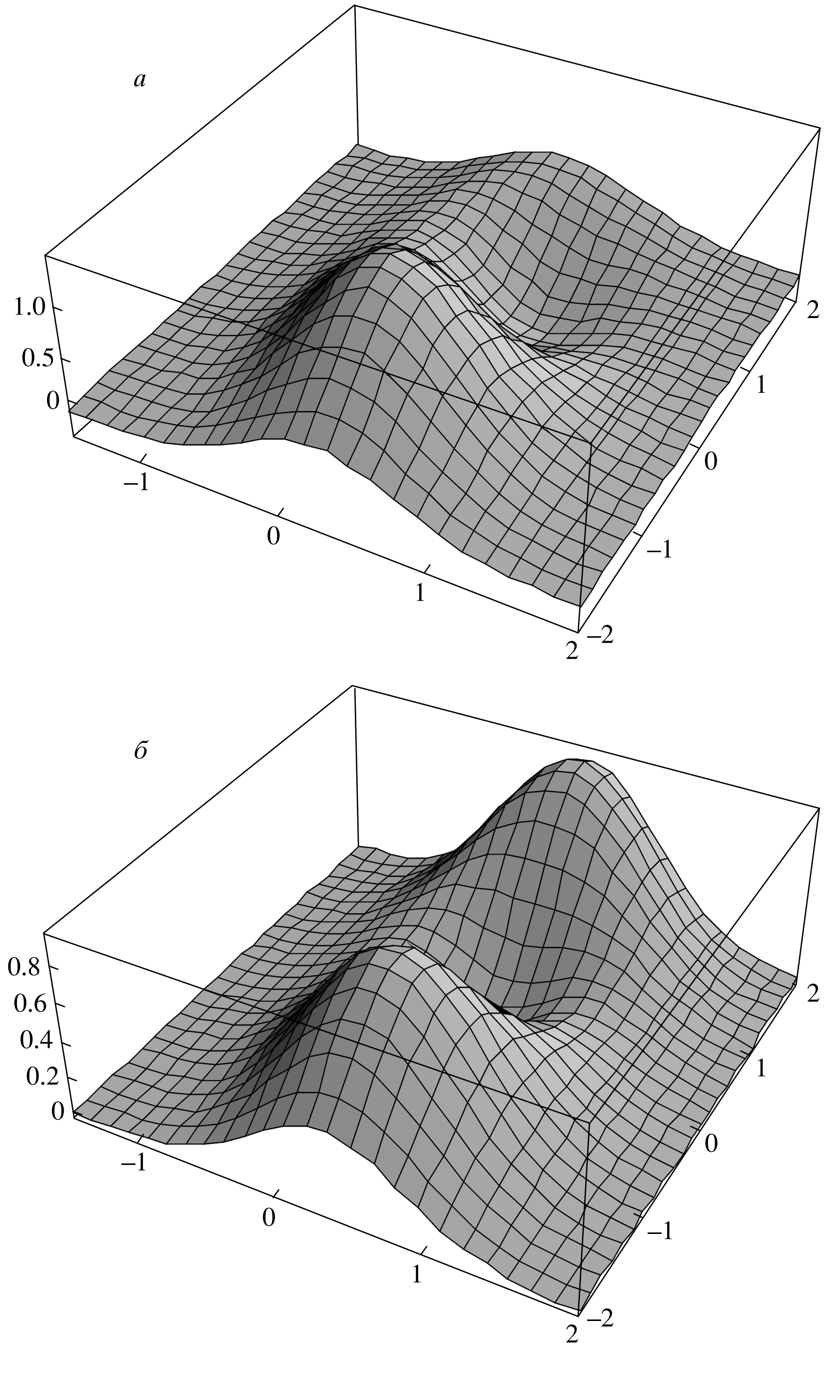

Fig. 2. Wigner function of the superposition state $|\Psi\rangle = (|0,\alpha\rangle - \exp(i\varphi_\alpha)|1,\alpha\rangle)/\sqrt{2}$ (a) and statistical mixture described by the density operator $\rho = (|0,\alpha\rangle\langle 0,\alpha| + |1,\alpha\rangle\langle 1,\alpha|)/2$ (b) for $\alpha = 0.5$ and $Q_\alpha = \pi/4$

$$\hat{U}_H|\Omega\rangle_{123} = \frac{1}{2}|1,\sqrt{2}\,\alpha\rangle_1|0\rangle_2\,(A|0\rangle_3 + B|1\rangle_3) - \\ - \frac{1}{2}|0,\sqrt{2}\,\alpha\rangle_1|1\rangle_2\,(A|0\rangle_3 - B|1\rangle_3) + \\ + \frac{A}{\sqrt{2}}|0,\sqrt{2}\,\alpha\rangle_1|0\rangle_2|1\rangle_3 + \\ + \frac{B}{\sqrt{2}}\left(|2,\sqrt{2}\,\alpha\rangle_1|0\rangle_2 - |0,\sqrt{2}\,\alpha\rangle_1|2\rangle_2\right)|0\rangle_3. \quad (20)$$

Off-shelf photon counters have efficiencies around 65 % and can only differentiate between zero and more photons. However, Takeuchi et al. [21] developed an avalanched photodetector that can discern 0, 1, and 2 photons with 90% efficiency. We use such detectors assuming that they are ideal. Then, in the case of registration of a single photon in Alice's second mode, state

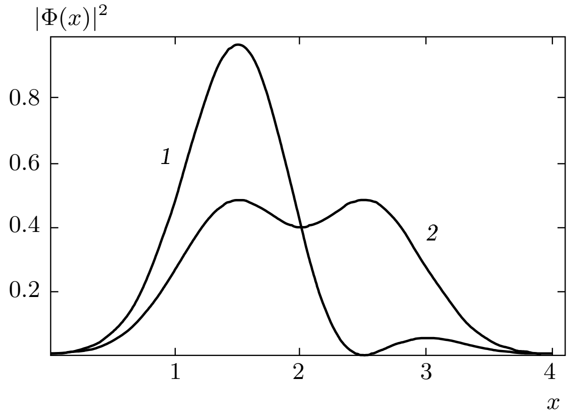

Fig. 3. Quadrature distribution $|\Phi(x)|^2$ of the superposition state and statistical mixture for $\alpha = 2$ and $Q_\alpha = 0$. Curves *1* and *2* respectively correspond to the superposition state and the statistical mixture. Two separated peaks of the statistical mixture transform to one peak of pure superposition

(20) collapses into an unknown state $A|0\rangle_3 - B|1\rangle_3$, but on Bob's side. Alice sends one bit of classical information informing him about the result of her measurement and, finally, Bob needs to shift a phase of a single photon by π. If Alice registers nothing, she must consider outcomes of her first mode, in other words, she must be able to distinguish the states $|0,\alpha\rangle$, $|1,\alpha\rangle$, and $|2,\alpha\rangle$ from each other. This can be done if she combines the states with the ancilla coherent state $|0,\alpha_1\rangle$ with the amplitude α_1 on a beam splitter with arbitrary transmittance and reflectivity (3a). The results of mixing of the states $|0,\alpha\rangle$, $|0,\alpha_1\rangle$ and $|1,\alpha\rangle$, $|0,\alpha_1\rangle$ (Eq. (11)) are presented above. Superimposing $|2,\alpha\rangle_1$ and $|0,\alpha_1\rangle_{1'}$, on a beam splitter, we obtain

$$\hat{U}_{\alpha R+\alpha_1 T^*=0}\left(|2,\alpha\rangle_1|0,\alpha_1\rangle_{1'}\right) = \\ = T^2\left|2,\frac{\alpha}{T^*}\right\rangle_1|0\rangle_{1'} + \sqrt{2}\,TR\left|1,\frac{\alpha}{T^*}\right\rangle_1|1\rangle_{1'} + \\ + R^2\left|0,\frac{\alpha}{T^*}\right\rangle_1|2\rangle_{1'} \quad (21)$$

if $\alpha R + \alpha_1 T^* = 0$. As $T \to 0$ ($R \to 1$), the photon-number resolving detector registering a single photon projects output state (20) on $(A|0\rangle_3 + B|1\rangle_3)$ on Bob's side. Alice has only to inform Bob about this and Bob has nothing to do. Thus, we described a type of quantum information delivered from a macroscopic state to a microscopic one. The success probability of the teleportation protocol is 0.5 and is independent of the amplitudes of participating states. Therefore, quantum teleportation through a coherent entangled channel requires photon detection being able to distinguish odd-photon-number states from even ones [7], which is impossible in practice with the current level of technology.

In conclusion, we have investigated the possibility to apply displaced states to quantum information. We showed different methods of conditional generation of both macroscopic qubits and entangled states constructed from the displaced states. All the methods are based on the use of linear optical elements including phonon-number-resolving photodetectors. We showed the possibility to teleport an unknown macroscopic qubit to a microscopic state if participants share a quantum channel of special form. It is plausible that the proposed scheme of quantum teleportation based on displaced states can be realized in practice. We address the displaced states as macroscopic objects because they are characterized by some parameter α that can take large values. But the measurements performed to distinguish the coherent state from the displaced single photon are nevertheless intrinsically microscopic, in that we must be able to distinguish between vacuum and one-photon impact, which are microscopically distinct. In this sense, the claim that the states are macroscopic may be disputed. To claim about macroscopic states, one should show that the states are distinguishable for photon-number measurements with a limited resolving power [26]. This may become the subject of the future investigation.

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