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NONLINEAR INTERACTION OF A TWO-LEVEL ATOM WITH COUNTER-PROPAGATING RADIATION BEAMS OF DIFFERENT FREQUENCIES

G. F. Mkrtchian *

Department of Quantum Electronics, Yerevan State University
0025, Yerevan, Armenia

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The theoretical and numerical results on the nonlinear dynamics of an atom in the fields of two counter-propagating radiation beams of different frequencies are presented. Both resonant and nonresonant interaction regimes are investigated. The atom center-of-mass energy dependence on the field amplitudes manifests the nonlinear threshold effect of an atom reflection in the interference field. This phenomenon leads to the atom acceleration or deceleration depending on its initial state. This acceleration/deceleration is of a shock character because of the impact with the moving potential barrier; it occurs at ultrashort distances of the order of radiation field wavelengths. Furthermore, the role of initial conditions is discussed and analyzed numerically.

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1. INTRODUCTION

Laser manipulation of atom center-of-mass motion has been extensively studied both theoretically and experimentally with the appearance of lasers [1, 2]. Since the first theoretical works [3–7], a continuous experimental progress in storing and controlling of ultracold atoms has led to a variety of spectacular results in the last decades (see, e.g., [8–13] for a review and the references therein). The growing interest in this subject can be largely attributed to the problems of quantum informatics, a variety of atomic and laser spectroscopic issues, especially at very low temperatures (it is worth noting the unique experiments with the trapping of separate atoms or Bose condensation of supercooled atomic gas in optical-dipole or magnetic traps).

Nevertheless, the spectrum of probable mechanisms for laser acceleration of atoms with respect to charged particles is very restricted, and the main reason is the neutrality of atom for direct electromagnetic interaction. It is clear that in this case, acceleration of atoms by laser fields is possible due to the interaction of the induced dipole moment of an atom with laser radiation. In the scope of the latter, there are two acceleration mechanisms, i.e., two types of radiative forces, dissipative and dispersive, acting on an atom interacting with laser fields [3–7]. The atom is then represented as a classical object — a complex particle with internal degrees of freedom.

The first-type force, also called the radiation pressure force, results from the transfer of momentum from the light beam to the atom at the resonant scattering and is proportional to the scattering rate Γ. The corresponding acceleration/deceleration of an atom with mass m is $\sim \hbar k\Gamma/m$, where $\hbar k$ is the momentum of the absorbed photon. With such a force, an atom at rest can be accelerated up to thermal velocities, or the thermal atomic beam can be stopped at the distance of the order of one meter during a few milliseconds.

The second-type, dispersive force, also called the dipole or gradient force, arises from the dispersive interaction of the induced atomic dipole moment with the intensity gradient of the laser beam:

$$F \sim \nabla I(\mathbf{r}),$$

where $I(\mathbf{r})$ is the intensity envelope of the incident laser beam. Because of its conservative character, this force can serve as an optical trap for neutral atoms [8–10]. As a great achievement, the optical dipole traps of atoms have been successfully realized [14–16].

Interesting effects can also be obtained in the field of two counter-propagating light beams. As a signifi-

*E-mail: mkrtchian@ysu.am

cant application of radiation pressure forces, Doppler cooling of neutral atoms [17] and trapped ions [18] have been realized. The latter results from a Doppler-induced imbalance between two opposite radiation pressure forces caused by the laser beams of the same frequency. This allows damping the atomic velocity in a few microseconds, achieving what is called an "optical molasses" [19].

We do not attempt to review the extensive literature on the laser manipulation of atoms by the counter-propagating light beams, apart from mentioning works [20, 21], which consider the acceleration of atoms in a moving periodic potential trap. This relies on the "conveyor belt" provided by a frequency-chirped optical lattice formed by two counter-propagating laser beams. Another regime of atom acceleration has been reported in [22] for the far-off-resonant waves. It has been found in [22] that in the field of two counter-propagating light beams of different frequencies, a critical intensity of radiation field exists, above which the atom "reflection" from the slowed interference wave occurs. The combined wave field becomes a moving potential barrier with respect to the atom, resulting in the atom acceleration or deceleration depending on its initial velocity. This is a shock acceleration/deceleration, which is independent of the interaction length.

In this paper, the results obtained in Ref. [22] are developed further. The theoretical and numerical results on the nonlinear dynamics of an atom in the fields of counter-propagating radiation beams of the different frequencies are presented. Both resonant and far-off-resonant regimes of interaction are investigated and the role of initial conditions is discussed and analyzed by numerical simulations.

The organization of the paper is as follows. In Sec. 2, we derive the basic equations of motion and briefly review distinct regimes of interaction. In Sec. 3, we present some numerical calculations and compare them with analytic results. Finally, conclusions are given in Sec. 4.

2. BASIC MODEL AND THEORY

We study the dynamics of interaction of a two-level atom with the two quasi-monochromatic counter-propagating plane waves of different frequencies in the given-field approximation (the magnitudes of the wave fields are assumed so strong that the radiation/absorption processes cannot change the given values). In the actual cases of strong wave pulses, this approximation is satisfied with great accuracy.

The Hamiltonian of the two-level atom in the field of two quasi-monochromatic counter-propagating plane electromagnetic waves can be represented in the form

$$\widehat{H} = \frac{\widehat{\mathbf{p}}^2}{2m} + \varepsilon_1 |1\rangle\langle 1| + \varepsilon_2 |2\rangle\langle 2| + \widehat{V}, \tag{1}$$

where

$$\widehat{V} = -d_{12} \left(E_1 \cos\varphi_1 (t, \mathbf{r}) + \right. \\ \left. + E_2 \cos\varphi_2 (t, \mathbf{r}) \right) |1\rangle\langle 2| + \text{H.c.} \tag{2}$$

is the interaction Hamiltonian.

The operator $|s\rangle\langle s|$ $(s = 1, 2)$ projects onto the state $|s\rangle$ with an energy ε_s. The operators $|1\rangle\langle 2|$ and $|2\rangle\langle 1|$ describe the transitions in the atomic system that are driven by the counter-propagating waves with the carrier frequencies ω_1 and ω_2 (let $\omega_1 > \omega_2$), wave numbers $\mathbf{k}_1$ and $\mathbf{k}_2$, and slowly varying amplitudes E_1 and E_2. The corresponding phases are

$$\varphi_{1,2} (t, \mathbf{r}) = \omega_{1,2} t - \mathbf{k}_{1,2} \cdot \mathbf{r}.$$

The fields of both pulses are assumed to be linearly polarized along the same direction; d_{12} is the projection of the atomic transition dipole moment on the polarization direction of the waves (we assume d_{12} to be real). Here, $\mathbf{r}$ and $\widehat{\mathbf{p}}$ are the operators of the position and momentum of the atom center-of-mass (m).

In the process of emitting and absorbing photons, atoms change not only their internal states but also their external translational states, due to photon recoil. If the atomic momentum change is large compared to the photon momenta $\hbar k_{1,2}$, the atom center-of-mass motion can be described classically. In this case, the position and momentum of the atom center-of-mass obey the Hamilton canonical equations of motion

$$\frac{d\mathbf{r}}{dt} = \frac{\mathbf{p}}{m}, \quad \frac{d\mathbf{p}}{dt} = -\nabla V_{eff} (\mathbf{r}, t), \tag{3}$$

with the effective potential

$$V_{eff} (\mathbf{r}, t) = \text{Sp} \left(\widehat{\rho} \widehat{V} \right). \tag{4}$$

Here, $\widehat{\rho}$ is the density matrix corresponding to the internal degrees of freedom of the atomic system. The density matrix $\widehat{\rho}$ can be written in the form

$$\widehat{\rho} = \rho_{11} |1\rangle\langle 1| + \rho_{22} |2\rangle\langle 2| + \left(\rho_{12} e^{i\omega_0 t} |1\rangle\langle 2| + \text{H.c.} \right), \tag{5}$$

where $\omega_0 = (\varepsilon_2 - \varepsilon_1)/\hbar$ is the frequency of the atomic transition. The dynamics of the density matrix $\widehat{\rho}$ in the interaction picture are determined by the von Neumann equation

$$i\hbar \frac{\partial \widehat{\rho}}{\partial t} = \left[\widehat{V}, \widehat{\rho} \right]. \tag{6}$$

The resulting equations for the density matrix elements are

$$\frac{d\rho_{11}}{dt} = -i\rho_{21}e^{-i\omega_0 t} \times \times \left(\frac{\Omega_1}{2}e^{i\varphi_1(t,\mathbf{r})} + \frac{\Omega_2}{2}e^{i\varphi_2(t,\mathbf{r})}\right) + \text{c.c.}, \quad (7a)$$

$$\frac{d\rho_{22}}{dt} = i\rho_{21}e^{-i\omega_0 t} \times \times \left(\frac{\Omega_1}{2}e^{i\varphi_1(t,\mathbf{r})} + \frac{\Omega_2}{2}e^{i\varphi_2(t,\mathbf{r})}\right) + \text{c.c.}, \quad (7b)$$

$$\frac{d\rho_{12}}{dt} = ie^{-i\omega_0 t} \times \times \left(\frac{\Omega_1}{2}e^{i\varphi_1(t,\mathbf{r})} + \frac{\Omega_2}{2}e^{i\varphi_2(t,\mathbf{r})}\right)(\rho_{11} - \rho_{22}), \quad (7c)$$

$$\frac{d\rho_{21}}{dt} = -ie^{i\omega_0 t} \times \times \left(\frac{\Omega_1}{2}e^{-i\varphi_1(t,\mathbf{r})} + \frac{\Omega_2}{2}e^{-i\varphi_2(t,\mathbf{r})}\right)(\rho_{11} - \rho_{22}). \quad (7d)$$

Using Eqs. (2), (4), and (5), we can obtain the following expression for the effective potential of interaction:

$$V_{eff}(\mathbf{r},t) = = \left(\frac{\hbar\Omega_1}{2}e^{-i\varphi_1(t,\mathbf{r})} + \frac{\hbar\Omega_2}{2}e^{-i\varphi_2(t,\mathbf{r})}\right)e^{i\omega_0 t}\rho_{12} + \text{c.c.} \quad (8)$$

Here, $\Omega_{1,2} = E_{1,2}d_{12}/\hbar$ are the Rabi frequencies.

To be more precise, we should add the terms describing spontaneous transitions and other relaxation processes in the set of equations (7). Since we have not taken the relaxation processes into account, our consideration is correct only for the times $T < \tau_{min}$, where τ_{min} is the minimum of all relaxation times. Therefore, full dynamics in the absence of any losses are governed by Eqs. (3), (7), and (8). These equations are a nonlinear set of equations with the atomic internal ($\widehat{\rho}$) and translational ($\mathbf{r}$, $\mathbf{p}$) variables defined self-consistently. However, in some cases, it is possible to decouple the translational variables and to identify the nonlinear dynamics of an atom center-of-mass motion.

The case of large resonance detunings was considered in Ref. [22]. We briefly repeat the simple results for the sake of self-consistency. For large resonance detunings (or not very strong wave fields), when $|\Delta_{1,2}| \gg |\Omega_{1,2}|$ ($\Delta_{1,2} = \omega_{1,2} - \omega_0$ are the resonance detunings for atomic internal transitions), and if the atom is initially in the ground state, the excited state population remains small and can be neglected. Then, setting $\rho_{11} \approx 1$ and $\rho_{22} \approx 0$ in Eq. (7c), we obtain

$$\rho_{12} \approx e^{-i\omega_0 t}\left(\frac{\Omega_1}{2\Delta_1}e^{i\varphi_1(t,\mathbf{r})} + \frac{\Omega_2}{2\Delta_2}e^{i\varphi_2(t,\mathbf{r})}\right), \quad (9)$$

and, correspondingly, effective potential (8) is reduced to

$$V_{eff}(\mathbf{r},t) = \frac{\hbar\Omega_1\Omega_2}{2}\left[\frac{1}{\Delta_1} + \frac{1}{\Delta_2}\right] \times \times \cos\left[\delta\omega\left(t - \frac{z}{v_{ph}}\right)\right]. \quad (10)$$

In Eq. (10), only the time dependent terms are dropped, $\delta\omega = \omega_1 - \omega_2 > 0$, and it is assumed that the waves propagate along the z axis. As we see, the atomic translational motion is governed by the slowed interference wave. This wave propagates with the phase velocity $v_{ph} = c/n < c$ (c is the speed of light in vacuum). The quantity

$$n = \frac{\omega_1 + \omega_2}{\omega_1 - \omega_2} > 1 \quad (11)$$

is the "effective refractive index" for a slowed interference wave. Hence, the resonant interaction of an atom with two traveling vacuum waves affects the atom center-of-mass translational motion in the slowed wave field, which is of a nonlinear-threshold nature over the interference wave intensity, as we show in that follows.

Next, we consider the nonlinear dynamics of translational motion of the atom center-of-mass in the field of the slowed traveling wave (10), at the near-resonant transitions between the atomic internal quantum states: $|\Delta_{1,2}| \ll |\Omega_{1,2}|$. In this case, the internal and translational variables are also separated, which allows integrating the reduced equations of motion. This is clear if the resonance condition for two waves

$$\omega_0 = \frac{\omega_2 + \omega_1}{2} \quad (12)$$

holds, which requires inverse symmetric detunings $\Delta_1 = -\Delta_2$. For simplicity, we also assume $\Omega_1 = \Omega_2 \equiv \Omega$. The set of equations (7) can then be rewritten as

$$\frac{d\rho_{12}}{dt} = i\Omega\cos\left[\frac{\delta\omega}{2}\left(t - \frac{z}{v_{ph}}\right)\right] \times \times \exp\left(-i\frac{\delta\omega z}{2c}\right)(\rho_{11} - \rho_{22}), \quad (13a)$$

$$\frac{d\rho_{11}}{dt} = i\Omega\cos\left[\frac{\delta\omega}{2}\left(t - \frac{z}{v_{ph}}\right)\right] \times \times \left[\exp\left(i\frac{\delta\omega z}{2c}\right)\rho_{12} - \text{c.c.}\right], \quad (13b)$$

$$\rho_{22} = 1 - \rho_{11}, \quad \rho_{21} = \rho_{12}^*, \tag{13c}$$

and effective potential (8) is reduced to

$$V_{eff}(\mathbf{r},t) = \hbar\Omega \cos\left[\frac{\delta\omega}{2}\left(t - \frac{z}{v_{ph}}\right)\right] \times \times \left[\exp\left(-i\frac{\delta\omega z}{2c}\right)\rho_{21} + \text{c.c.}\right]. \tag{14}$$

If $v_{ph} \ll c$, which is satisfied with great accuracy for the considered setup, the slow oscillations of the exponential function $\exp[\pm i\delta\omega z/2c]$ can be ignored in Eqs. (13) and (14). This is justified if the condition

$$|z| \ll \frac{2c}{\delta\omega} \approx n\frac{c}{\omega_0} \tag{15}$$

is satisfied, which practically does not limit the interaction length for actual pulses because of very large values of the effective refractive index $n \gg 1$ (this is equivalent to the condition $v_{ph} \ll c$).

Then, these equations can be solved exactly subject to certain initial conditions. The general solution for the density matrix elements is

$$\rho_{11} = \frac{1}{2} + \frac{\operatorname{Im}[\rho_{12}(0)]}{\sin\vartheta_0}\cos\vartheta(t), \tag{16}$$

$$\operatorname{Im}[\rho_{12}(t)] = \frac{\operatorname{Im}[\rho_{12}(0)]}{\sin\vartheta_0}\sin\vartheta(t), \tag{17}$$

$$\operatorname{Re}[\rho_{12}(t)] = \text{const}, \tag{18}$$

where

$$\vartheta(t) = 2\int_0^t \Omega(t')\cos\left[\frac{\delta\omega}{2}\left(t' - \frac{z(t')}{v_{ph}}\right)\right]dt' + \vartheta_0 \tag{19}$$

and

$$\operatorname{tg}\vartheta_0 = \frac{\operatorname{Im}[\rho_{12}(0)]}{\rho_{11}(0) - 1/2}. \tag{20}$$

This solution represents Rabi oscillations with a modulated Rabi frequency. For the effective potential, we obtain

$$V_{eff}(\mathbf{r},t) = 2\hbar\Omega \operatorname{Re}[\rho_{12}(0)] \times \times \cos\left[\frac{\delta\omega}{2}\left(t - \frac{z}{v_{ph}}\right)\right]. \tag{21}$$

As can be seen from Eqs. (10) and (21) in these two distinct cases, translational motion of an atom is governed by the slowed interference wave. For the near-resonant interaction, in contrast to the far-off-resonant case, the amplitude of the effective interaction potential depends on the initial internal atomic state. For a nonvanishing interaction, the atom must be prepared in a superposition state, and to maximize the interaction potential, the equal superposition of the states $|1\rangle$ and $|2\rangle$ must be achieved. For the same wave intensities, the amplitude of effective interaction potential (21) is then at least one order of magnitude larger than the amplitude expected in the nonresonant interaction regime.

We now turn to the solution of the equation of motion for the center-of-mass motion of an atom. Equations (3) imply the conservation of transversal momentum of the atom: $p_{x,y} = \text{const}$. Then, with the dependence of the effective potential on time and coordinate taken into account in both resonant and nonresonant cases for the monochromatic waves in Eqs. (3), we can find the integral of motion

$$\mathcal{E} - v_{ph}p_z = \text{const} = \mathcal{E}_0 - v_{ph}p_{0z}, \tag{22}$$

where $\mathcal{E}_0$ and p_{0z} are the initial energy and the longitudinal momentum of the atom. For the quasi-monochromatic waves with slowly varying envelopes, Eq. (22) represents an adiabatic integral, when the waves are turned on and off adiabatically.

Using Eq. (22), we can obtain the velocity of the atom in the field

$$v_z = v_{ph}\left[1 \mp \sqrt{\left(1 - \frac{v_{0z}}{v_{ph}}\right)^2 - \frac{V_{eff}(z,t)}{\mathcal{E}_{ph}}}\right], \tag{23}$$

$$v_x = v_{0x}, \quad v_y = v_{0y}, \tag{24}$$

where $\mathbf{v}_0 = (v_{0x}, v_{0y}, v_{0z})$ is the initial velocity of the atom and $\mathcal{E}_{ph} = mv_{ph}^2/2$ is the kinetic energy of a particle corresponding to the velocity v_{ph}.

As can be seen from Eq. (23), when the maximal value of the interaction potential $V_{eff}(z,t)_{max} = |V_0|$ is larger than the value (which is called critical in what follows)

$$V_{cr} = \mathcal{E}_{ph}\left(1 - \frac{v_{0z}}{v_{ph}}\right)^2, \tag{25}$$

expression (23) for the atom velocity may become complex. This complexity is bypassed in the complex plane by continuously passing from one Riemann sheet to another, at which the root changes its sign. Hence, the atom velocity remains real everywhere and the multivaluedness of expression (23) also disappears. Indeed,

if $|V_0| < V_{cr}$, we should take the root with the sign "$-$" in the Eq. (23) if $v_{0z} \le v_{ph}$ and with the sign "$+$" if $v_{0z} \ge v_{ph}$, to satisfy the initial condition $v_z = v_{0z}$ at $V_{eff}(z, t = -\infty) = 0$. Then, after the interaction ($V_{eff}(z, t = +\infty) = 0$), the energy of the atom remains unchanged. However, when $|V_0| > V_{cr}$, the value $V_{eff}(z(t_0), t_0) = V_{cr}$ (where $z(t_0)$ is the atom coordinate at the instant $t = t_0$) becomes a turning point, and we should change the sign of the root for $t > t_0$ compared with the instants $t \le t_0$.

We now consider the behavior of the atom in the field in this situation. As we see, the atom cannot penetrate the region of the field $V_{eff}(z, t) > V_{cr}$, where expression (23) becomes complex. The slowed interference wave then becomes a potential barrier for the atom and the reflection of the atom from such a moving barrier occurs. To explain the physics of this phenomenon, it is necessary to clarify the meaning of the critical field.

This is an essentially nonlinear phenomenon of threshold nature, and the critical intensity of the interference wave is the threshold value for this process. Namely, Eq. (23) shows that the critical value V_{cr} is the value of the potential at which the longitudinal velocity of the atom in the field $v_z(t)$ becomes equal to the phase velocity of the slowed interference wave: $v_z(t) = v_{ph}$, irrespective of the atom initial velocity v_{0z}. The last formula is the condition of resonance with the Doppler-shifted waves frequencies, at which the coherent scattering, that is, the induced scattering of counter-propagating waves on an atom occurs:

$$\omega_1 \left(1 - \frac{v_z(t)}{c}\right) = \omega_2 \left(1 + \frac{v_z(t)}{c}\right). \qquad (26)$$

Under this condition, the nonlinear resonance occurs because the resonant velocity of the atom $v_z(t) = v_{ph}$ is acquired in the field at the value $V_{eff} = V_{cr}$ (due to the wave intensity effect).

We note that the existence of a critical intensity in coherent wave fields is the feature of induced coherent processes, such as Cherenkov and Compton processes (as well as in an undulator), where nonlinear resonant phenomena have been revealed [23]. Then, at the critical point, the resonant absorption of photons from one wave and re-emission into the other wave occurs, resulting in a break of the synchronism $v_z(t) = v_{ph}$ between the atom and the slowed interference wave (either $v_z(t) > v_{ph}$ or $v_z(t) < v_{ph}$), and the atom abandons it: the reflection of the atom from the moving barrier occurs.

We note that this is actually a reflection in the frame of reference moving with the velocity $V = v_{ph}$, which is the rest frame of the slowed interference wave. In this frame, the atom with the velocity v'_{0z} swoops on the motionless barrier and, as is seen from Eq. (23), an elastic reflection of the atom occurs: $v'_z = -v'_{0z}$. Thus, if the maximal value of the interaction potential $|V_0| > V_{cr}$, then the atom velocity after the interaction is given by

$$v_{zf} = 2v_{ph} - v_{0z}. \qquad (27)$$

As we see from Eq. (27), if the slowed interference wave pulse initially overtakes the atom ($v_{0z} < v_{ph}$), then $v_{zf} > v_{0z}$ and the atom is accelerated. But if the atom initially overtakes the wave ($v_{0z} > v_{ph}$), then $v_{zf} < v_{0z}$ and the deceleration of the atom occurs. For the resonant atoms ($v_{0z} = v_{ph}$), $V_{cr} = 0$ and consequently the atom velocity does not change ($v_{zf} = v_{0z}$).

For the kinetic energy change of the atom center of mass, we have

$$\Delta\mathcal{E} = 4\mathcal{E}_{ph}\left(1 - \frac{v_{0z}}{v_{ph}}\right). \qquad (28)$$

It follows from this formula that the acceleration of the atom depends neither on the field magnitude (once it is above the threshold field) nor on the interaction length. Formulas (27) and (28) show that acceleration or deceleration of the atom is defined by the key parameters of this process — the atom initial velocity and the phase velocity of the slowed interference wave v_{ph}.

3. NUMERICAL TREATMENT

In this section, we present some numerical simulations that illustrate the nonlinear picture of interaction of the atom with the two counter-propagating waves. The time evolution of the system of equations (3), (7) is found with a Runge–Kutta method. The calculations were made for a quasi-monochromatic wave fields providing the adiabatic turn on/off of the interaction. This is achieved by describing the envelopes with the Gaussian functions

$$\Omega_{1,2}(t) = \Omega_0 \exp\left[-(t - 3\tau)^2/2\tau^2\right],$$

where τ and Ω_0 characterize the pulse duration and amplitudes. We consider the resonant interaction regime assuming the atom initially to be in an equal superposition of the states $|1\rangle$ and $|2\rangle$ ($\rho_{12}(0) = 1/2$). For all calculations, we took $\Omega_0/\delta\omega = 10^3$. We note that the qualitative picture of an atom center-of-mass motion practically is independent of this ratio. The pulse duration has been chosen as $\delta\omega\tau = 20$ (the pulse duration should be larger than the period of the interference

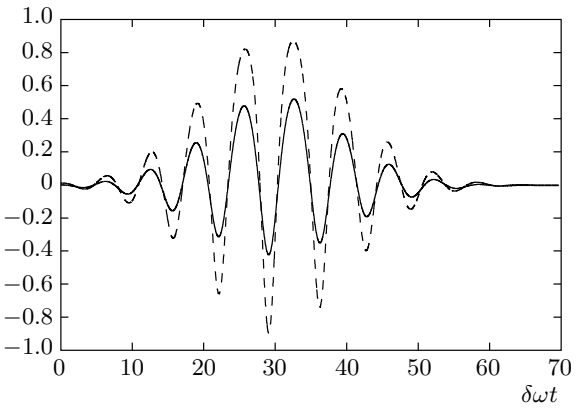

Fig. 1. The solid curve displays the temporal evolution of the atom scaled velocity v_z/v_{ph}. The dashed curve shows the variation of the scaled interaction potential V_{eff}/V_{cr}, sensed by the atom along the trajectory. The initial conditions are $v_0 = 0$ and $z_0 = 0$, the intensity below the critical point is $V_0 = 0.9V_{cr}$

wave). At $t = 0$, the wave intensities are reduced by the factor $1/e^9$ relative to their maximal values, providing the adiabatic switch on of the interaction. Then, to accentuate this acceleration mechanism caused by the nonlinear resonance in the fields, we present the atom dynamics in the case where the initial velocity of the atom is very far from the induced resonance in Eq. (26).

Figure 1 illustrates the temporal evolution of the atom center-of-mass velocity (solid curve) in the case where $v_0 = 0$ and the intensity is below the critical point: $V_0 = 0.9V_{cr}$. The dashed curve shows the variation of the scaled potential V_{eff}/V_{cr} along the atom trajectory. We see that the acceleration is negligibly small.

In Fig. 2, the atom dynamics is displayed in the case where the intensity is above the critical point: $V_0 = 1.3V_{cr}$. Figure 2*a* illustrates the acceleration of an atom at rest ($v_0 = 0$). The solid curve shows the temporal evolution of the atom velocity. The dashed curve is the variation of the scaled potential V_{eff}/V_{cr} along the atom trajectory. Figure 2*b* illustrates the deceleration in the case where $v_0 = 2v_{ph}$. It is clearly seen from these figures that at the critical point $V_{eff} \approx V_{cr}$, the longitudinal velocity of the atom becomes equal to the phase velocity of the interference wave: $v_z(t) = v_{ph} = c/n$, and it is a turning point for the solid curves. This corresponds to formulas (23), where the root changes its sign and the further evolution of the velocity proceeds along the second brunch of the root with the reversed sign. In the resonance range, the velocity of the atom strictly increases if $v_0 < v_{ph}$

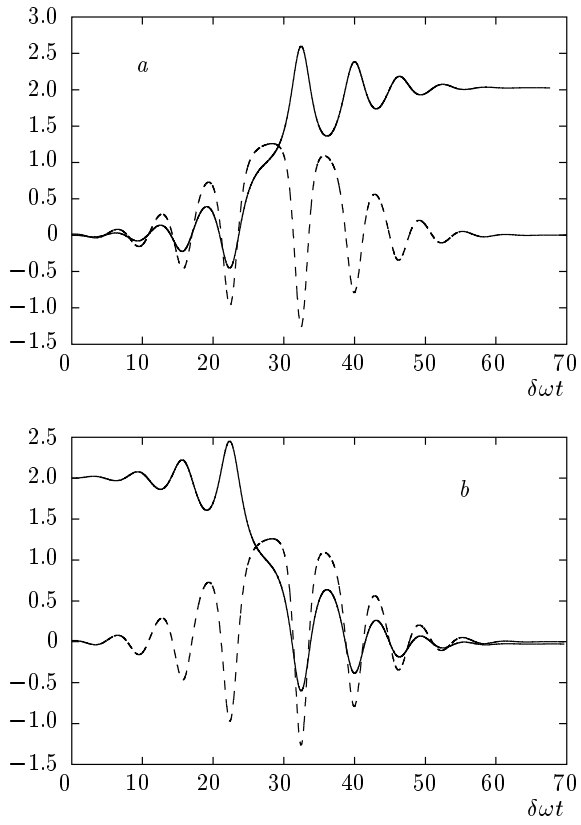

Fig. 2. Atom acceleration/deceleration. The intensity is above the critical point: $V_0 = 1.3V_{cr}$. *a*) The solid curve displays the temporal evolution of the atom scaled velocity v_z/v_{ph} with $v_0 = 0$ and $z_0 = 0$. The dashed curve is the scaled interaction potential V_{eff}/V_{cr}, sensed by the atom along the trajectory. *b*) Atom deceleration for $v_0 = 2v_{ph}$ and $z_0 = 0$

(Fig. 2*a*) or decreases if $v_0 > v_{ph}$ (Fig. 2*b*) due to the genuinely nonlinear character of the resonance in the field. Then, after leaving the resonance range, the final velocity of the atom becomes $v_{zf} = 2v_{ph}$ (acceleration) and $v_{zf} = 0$ (deceleration), in accordance with the analytic results (see Eqs. (23) and (27)).

To illustrate the physical picture of the atom reflection from the interference wave and the shock character of acceleration/deceleration, we present the atom phase trajectory, velocity versus the coordinate $z'(t)$ (in units of the reduced wavelength $\lambda = 2\lambda_1\lambda_2/(\lambda_1 + \lambda_2)$), in the rest frame of the slowed interference wave. In this frame, the actual reflection occurs. The corresponding picture is given in Fig. 3 at $v_0 = 0$ (the symmetric picture occurs at $v_0 = 2v_{ph}$). In this frame, the atom swoops on the motionless barrier with the velocity $v'_{0z} = -v_{ph}$ and, as is seen from Fig. 3, the reflection of the atom occurs. Figure 3 also shows that the reflec-

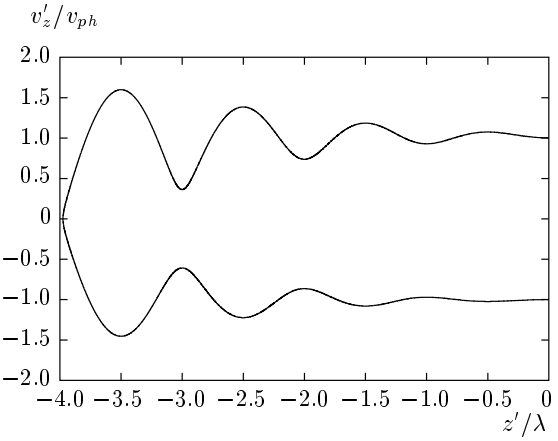

Fig. 3. Atom phase trajectory (velocity versus the coordinate $z'(t)$, in units of the reduced wavelength) in the rest frame of the slowed interference wave for the setup in Fig. 2*a*

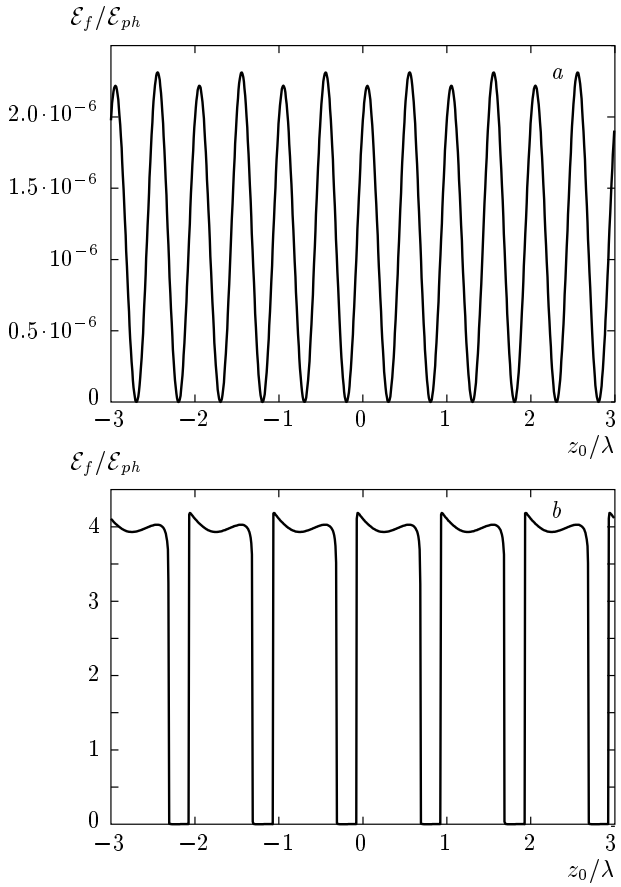

Fig. 4. The final scaled energy versus the initial position of the atom z_0 (in units of the reduced wavelength), when $v_0 = 0$. *a*) The intensity is below the critical point: $V_0 = 0.9V_{cr}$. *b*) The intensity is above the critical point: $V_0 = 1.3V_{cr}$

tion occurs at the distances smaller than the radiation wavelength, confirming the shock character of acceleration. This is also confirmed in the laboratory frame, which is well seen from Fig. 4, which displays the role of initial conditions, showing the final energy versus the initial position z_0 of the atom. We see that the acceleration is negligibly small below the nonlinear resonance threshold (Fig. 4*a*). The net gain is defined by the initial phase, which is in accordance with the perturbation theory. When the amplitude of the slowed interference wave is above the critical point (Fig. 4*b*), the final energy for reflected particles is almost constant ($\mathcal{E}_f = 4\mathcal{E}_{ph}$).

We make some estimations. Best suited systems for the near-resonant interaction regime are the Rydberg atoms, i.e., the highly excited states of hydrogen or alkali metal atoms [24]. Here, we are mainly interested in circular Rydberg states. These are the states with the highest allowed angular momentum $l = n_0 - 1$ for a given principal quantum number n_0 (with $|m_0| = l$, where m_0 is the magnetic quantum number). For these states, only one resonant dipole transition is allowed:

$$n_0 \leftrightarrow n_0 + 1,$$

and therefore such states closely approximate a two-level system with an extremely long lifetime and are widely used in the microwave cavity quantum electrodynamics experiments [25]. Hence, with our notation, we set

$$|1\rangle \equiv |n_0, l = n_0 - 1, m_0 = n_0 - 1\rangle$$

and

$$|2\rangle \equiv |n_0 + 1, l = n_0, m_0 = n_0 - 1\rangle.$$

For a Rydberg atom state with a large n_0 and $\Delta n_0 = 1$, the transition frequency is

$$\omega_0 \approx \frac{\varepsilon_0}{\hbar n_0^3} = \frac{1}{n_0^3} \text{ at.u.},$$

where ε_0 is the atomic unit of energy (27.2 eV). Here, we have taken into account that the quantum defect that corrects for the deviation of the binding potential from a purely hydrogenic situation is small for high orbital moments l. The transition dipole moment between neighboring Rydberg states [26] is estimated as

$$d_{12} = \sqrt{2} e a_0 \frac{2^{2n_0+2} n_0^{n_0+3} (n_0+1)^{n_0+2}}{(2n_0+1)^{2n_0+3}} \approx \frac{n_0^2}{\sqrt{2}} \text{ at.u.},$$

where a_0 is the Bohr radius (e is the elementary charge). The rate of spontaneous emission is given by

$$\Gamma = \frac{\Gamma_0}{n_0^5},$$

where

$$\Gamma_0 = \frac{2\alpha^4 c}{3a_0} \approx 10^{10}\ \mathrm{s}^{-1}$$

is the characteristic rate and α is the fine-structure constant. Then, the pulse duration of the waves is assumed to be $\tau \approx 1/10\Gamma$, which gives $\delta\omega \approx 2 \cdot 10^2\Gamma$ in accordance with the condition $\delta\omega\tau = 20$. For the effective refractive index, we then obtain

$$n \approx \frac{2\omega_0}{\delta\omega} = 10^{-2}\frac{\varepsilon_0}{\hbar\Gamma_0}n_0^2 \approx 4 \cdot 10^4 n_0^2.$$

For an atom initially at rest (with an atomic weight A), the critical field and, consequently, Rabi frequency is given by

$$V_{cr} = \hbar\Omega_{min} = m\frac{c^2}{2n^2} \approx 1.2 \cdot 10^{-2}\frac{A}{n_0^4}\ \text{at.u.}$$

This value should be much smaller than the frequency difference between the main resonant and nonresonant transitions ($n_0 \leftrightarrow n_0 - 1$, $n_0 + 1 \leftrightarrow n_0 + 2$), which is of the order of $3/n_0^4$ at.u. This condition is satisfied for the hydrogen atom as well as for the light alkali atoms (lithium, sodium), and the model of supposed two-level atom is sufficiently well justified. We note that the required fields for this effect should be

$$E \gtrsim 2 \cdot 10^{-2}\frac{A}{n_0^6}\ \text{at.u.},$$

which are much smaller than the atomic fields

$$E_0 = \frac{1}{16n_0^4}\ \text{at.u.}$$

for the Rydberg atoms in the state with a large n_0.

In particular, for the principal quantum number $n_0 = 40$ and $\omega_0/2\pi \approx 103$ GHz (with the corresponding effective refractive index given by $n \approx 6.4 \cdot 10^7$), an atom initially at rest can be accelerated up to the velocities 10^3 cm/s. The required fields for this effect are $E \gtrsim 2.5 \cdot 10^{-2}A$ V/cm, which corresponds to wave intensities $I \approx 4 \cdot 10^{-5}$ W/cm^2, for lithium atoms with $A = 7$. In the inverse regime of deceleration, such an atomic beam can be stopped with the same fields.

4. CONCLUSION

We have presented a theoretical treatment of nonlinear dynamics of the two-level atom interacting with counter-propagating radiation fields of different frequencies. We have shown that in this induced coherent process, a critical intensity of the wave fields exists above which a nonlinear phenomenon of a neutral atom reflection from the slowed interference wave occurs. This results in the atom acceleration or deceleration depending on its initial velocity. We have examined various limits of atomic dynamics depending on the resonance detuning. For the near-resonant interaction, in contrast to the far-off-resonant case, the amplitude of the slowed interference wave depends on the initial internal atomic state. The role of initial conditions has been discussed and analyzed by numerical calculations. The numerical simulations are in good agreement with the analytic results.

In the considered scheme, acceleration/deceleration depends neither on the field magnitude (once it is above the threshold field) nor on the interaction length, and it may serve as a promising way for efficient manipulation of ultracold atoms. The threshold character of such an acceleration may be used for separation of atoms by velocities.

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