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A STABILIZED WARPED BRANE WORLD WITH EQUAL GRAVITY STRENGTHS ON THE BRANES

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We discuss a stabilized brane world model with two branes, allowing a solution to the hierarchy problem due to the warped extra dimension and having a remarkable feature: the strength of gravitational interaction is of the same order on both branes, contrary to the case of the Randall–Sundrum model with a hierarchical difference of the gravitational strength on the branes. The solution also admits the existence of two branes with equal gravity strengths.

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1. INTRODUCTION

Since the appearance of papers [1, 2], where theories with warped extra dimensions were shown to allow a solution to the hierarchy problem, such theories are widely discussed in the literature (see, e. g, reviews [3, 4]). It turned out that the RS1 model [2] involves a massless radion, which contradicts the experimental data even at the classical level, and therefore this model requires a stabilization. The first stabilization mechanism was proposed in paper [5], where the size of the extra dimension is determined by the minimum of the effective potential of a five-dimensional scalar field. But the backreaction of the scalar field on the background metric is not taken into account by this mechanism. This problem was solved in the model proposed in [6]. We note that in the method proposed in [6], the size of the extra dimension is defined not by the minimum of the effective four-dimensional scalar field potential but by the boundary conditions on the branes. With an appropriate choice of the model parameters, it is possible to obtain the background solution for the metric that is close to the original Randall–Sundrum solution [7]. This model can be considered the stabilized Randall–Sundrum model.

The main feature of the Randall–Sundrum model is the existence of two branes that differ significantly in the strength of the four-dimensional gravity. Indeed, the four-dimensional Planck masses, defined by the coupling constant of the massless four-dimensional graviton, are [3, 8]

$$M_{Pl}^2 = \frac{M^3}{k}\left(e^{2kL} - 1\right)$$

for the IR brane and

$$M_{Pl}^{*2} = \frac{M^3}{k}\left(1 - e^{-2kL}\right)$$

for the UV brane (the terms "UV brane" and "IR brane" are used in the same sense as in the Randall–Sundrum model: the UV brane is the one with the stronger gravity, and the IR brane, on which the fields of the Standard Model are assumed to live, is the brane with the weaker gravity), where M is the five-dimensional Planck mass, k is the inverse anti de Sitter radius, L is the size of the extra dimension, $M \approx k \sim 1$ TeV and $kL \approx 36$. Obviously, for this choice of the parameters, $M_{Pl} \sim 10^{16}$ TeV, whereas $M_{Pl}^{*} \sim 1$ TeV. Therefore, the branes are strongly different from the gravitational standpoint, which leads to very different physics on the branes. A question arises whether it is possible to construct a stabilized brane world model with branes having comparable (or even equal) strengths of the effective four-dimensional gravity.

In this short paper, we discuss a stabilized brane world model based on the background solution presented in [9, 10]. Although the solution is quite well known, we found that it has an interesting property

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when applied to compact extra dimension. Namely, it allows obtaining any values of the four-dimensional Planck masses with respect to each other, retaining the main advantages of warped brane world models: strong five-dimensional gravity and the solution to the hierarchy problem. In other words, there can be two IR branes or even one IR brane and one brane with the gravity much weaker than that on the IR brane.

2. THE MODEL

To begin, we let denote coordinates in the five-dimensional space–time $E = M_4 \times S^1$,

$$\{x^N\} \equiv \{x^\mu, y\}, \quad N = 0,1,2,3,4, \quad \mu = 0,1,2,3,$$

with $x^4 \equiv y$, $-L \le y \le L$, being the coordinate parameterizing the fifth dimension with points $-y$ and y identified. The branes are located at the points $y = 0$ and $y = L$.

The action of the stabilized brane world model can be written as

$$\begin{aligned} S = \int d^4x \int\limits_{-L}^{L} dy \sqrt{-g} \times \\ \times \left[2M^3 R - \frac{1}{2} g^{MN} \partial_M \phi \partial_N \phi - V(\phi) \right] - \\ - \int\limits_{y=0} \sqrt{-\tilde{g}} \lambda_1(\phi)\, d^4x - \int\limits_{y=L} \sqrt{-\tilde{g}} \lambda_2(\phi)\, d^4x, \quad (1) \end{aligned}$$

where $V(\phi)$ is a bulk scalar field potential and $\lambda_i(\phi)$, $i = 1,2$, are the brane scalar field potentials, $\tilde{g} = \det \tilde{g}_{\mu\nu}$, and $\tilde{g}_{\mu\nu}$ is the metric induced on the brane. The signature of the metric g_{MN} is chosen to be $(-,+,+,+,+)$.

The standard ansatz for the metric and the scalar field, which preserves the Poincaré invariance in any four-dimensional subspace $y = \text{const}$, is given by

$$\begin{aligned} ds^2 = e^{-2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 \equiv \\ \equiv \gamma_{MN}(y)\, dx^M dx^N, \quad \phi(x,y) = \phi(y), \quad (2) \end{aligned}$$

where $\eta_{\mu\nu}$ is the flat Minkowski metric. For this ansatz, the Einstein and the scalar field equations derived from action (1) reduce to the system

$$\begin{aligned} &\frac{dV}{d\phi} + \frac{d\lambda_1}{d\phi}\delta(y) + \frac{d\lambda_2}{d\phi}\delta(y-L) = -4A'\phi' + \phi'', \\ &12M^3(A')^2 + \frac{1}{2}(V - \frac{1}{2}(\phi')^2) = 0, \\ &\frac{1}{2}\left(\frac{1}{2}(\phi')^2 + V + \lambda_1\delta(y) + \lambda_2\delta(y-L)\right) = \\ &\qquad = -2M^3\left(-3A'' + 6(A')^2\right). \end{aligned} \quad (3)$$

We consider the special class of potentials that can be represented as

$$V(\phi) = \frac{1}{8}\left(\frac{dW}{d\phi}\right)^2 - \frac{1}{24M^3} W^2(\phi).$$

Then the solutions of the first-order differential equations

$$\begin{aligned} \phi'(y) &= \operatorname{sign}(y) \frac{1}{2} \frac{dW}{d\phi}, \\ A'(y) &= \operatorname{sign}(y) \frac{1}{24M^3} W(\phi) \end{aligned} \quad (4)$$

solve Eqs. (3) in the bulk [6, 11].

We consider a linear function $W(\phi)$ as suggested in papers [9, 10]:

$$W(\phi) = \alpha\phi, \quad V = \frac{\alpha^2}{8} - \frac{\alpha^2}{24M^3}\phi^2. \quad (5)$$

Using equations (4), we obtain the corresponding background solution

$$\begin{aligned} \phi &= \frac{\alpha}{2}|y| - \frac{\alpha L_1}{4}, \\ A &= \frac{\alpha^2}{96M^3}\left[\left(|y| - \frac{L_1}{2}\right)^2 + C\right], \end{aligned} \quad (6)$$

where L_1 and C are integration constants treated as parameters.

For the equations of motion to be also valid on the branes, we can take the brane potentials $\lambda_i(\phi)$, $i = 1,2$, in the form

$$\lambda_1(\phi) = W(\phi) + \beta_1^2(\phi - \phi_1)^2, \quad (7)$$

$$\lambda_2(\phi) = -W(\phi) + \beta_2^2(\phi - \phi_2)^2. \quad (8)$$

It is easy to verify that the equations of motion are satisfied if

$$\phi|_{y=0} = \phi_1, \quad (9)$$

$$\phi|_{y=L} = \phi_2, \quad (10)$$

which means that

$$L_1 = -\frac{4\phi_1}{\alpha}, \tag{11}$$

$$L = \frac{2(\phi_2 - \phi_1)}{\alpha} \tag{12}$$

(we suppose that $\phi_1 < 0$, i.e., $L_1 > 0$). Thus, we see that the size of the extra dimension is fixed. The parameters α, $\phi_{1,2}$, and $\beta_{1,2}$ of the potentials, when made dimensionless by the fundamental five-dimensional energy scale M of the theory, do not contain a hierarchical difference. We note again that the fixation of the size of the extra dimension is caused by the boundary conditions on the branes, in contrast to the case discussed in [5], where the size of the extra dimension is determined by the minimum of an effective four-dimensional scalar field potential.

We suppose that we live on the brane at $y = L$ and $L \le L_1$. In order to have Galilean four-dimensional coordinates on this brane [3, 8], we choose the warp factor such that

$$e^{-2A}|_{y=L} = 1,$$

i.e.,

$$C = -\left(L - \frac{L_1}{2}\right)^2.$$

Because the wave function of the massless tensor graviton in the extra dimension is proportional to e^{-2A} [7], a standard technique (see, e.g., [4] for the details) gives an expression for the four-dimensional Planck mass on our brane

$$M_{Pl}^2 = M^3 \int\limits_{-L}^{L} e^{-2A} dy = M^3 2\exp\left\{\frac{\alpha^2(2L-L_1)^2}{192M^3}\right\} \times$$
$$\times \int\limits_0^L \exp\left\{-\frac{\alpha^2}{48M^3}\left(y - \frac{L_1}{2}\right)^2\right\} dy =$$
$$= M^3 2\exp\left\{\frac{\alpha^2(2L-L_1)^2}{192M^3}\right\} \times$$
$$\times \int\limits_{-L_1/2}^{(2L-L_1)/2} \exp\left\{-\frac{\alpha^2}{48M^3}y^2\right\} dy \approx$$
$$\approx M^3 2\exp\left\{\frac{\alpha^2(2L-L_1)^2}{192M^3}\right\} \int\limits_{-\infty}^{\infty} \exp\left\{-\frac{\alpha^2}{48M^3}y^2\right\} dy =$$
$$= M^3 \frac{2\sqrt{48\pi M^3}}{\alpha} \exp\left\{\frac{\alpha^2(2L-L_1)^2}{192M^3}\right\} \tag{13}$$

and

$$M_{Pl} \approx M\frac{5M^{5/4}}{\sqrt{\alpha}} \exp\left\{\frac{\alpha^2(2L-L_1)^2}{384M^3}\right\}. \tag{14}$$

We suppose that all fundamental parameters of the theory lie in the TeV range. To solve the hierarchy problem, i.e., to have $M_{Pl} \sim 10^{16}$ TeV, we should take

$$\frac{\alpha(2L-L_1)}{M^{3/2}} = \frac{4\phi_2}{M^{3/2}} \approx 120. \tag{15}$$

With these values of the parameters (and if L and L_1 are of the same order), the approximation used in (13) is very good. We also note that four-dimensional Planck mass (14) depends mainly on ϕ_2, not ϕ_1.

We now discuss some properties of this solution. On the second brane,

$$\exp(-2A)|_{y=0} = \exp\left\{-\frac{\alpha^2}{48M^3}\left[L\left(L_1 - L\right)\right]\right\} < 1$$

for $L < L_1$. We compare this behavior of the warp factor with that in the RS1 model. In the RS1 model (as well as in the stabilized case [6]), if we live on the IR brane, there exists the UV brane, where gravity is much stronger. In the case of background solution (6), the gravity on our brane is weak in comparison with the bulk gravity strength, but the gravity on the brane at $y = 0$ is even weaker. Thus, we have no UV brane as a physical object in this model. In the RS1 model, the exponent $A(y)$ takes its smallest value at the point where the UV brane is located, whereas in the case of solution (6), the analogous point $y = L_1/2$ lies in the bulk, i.e., the massless tensor graviton, whose wave function in the extra dimension is proportional to e^{-2A}, is localized in the bulk (see the Figure for the case $L_1 = L$). This can lead to interesting consequences for the effects of shadow matter (localized on the mirror brane) on our brane and in the case of universal extra dimensions.

It is not difficult to calculate the four-dimensional Planck mass on the brane at $y = 0$. For this, we pass to Galilean four-dimensional coordinates on that brane, which means that the integration constant in solution (6) for $A(y)$ should be taken such that $A(y)|_{y=0} = 0$, i.e.,

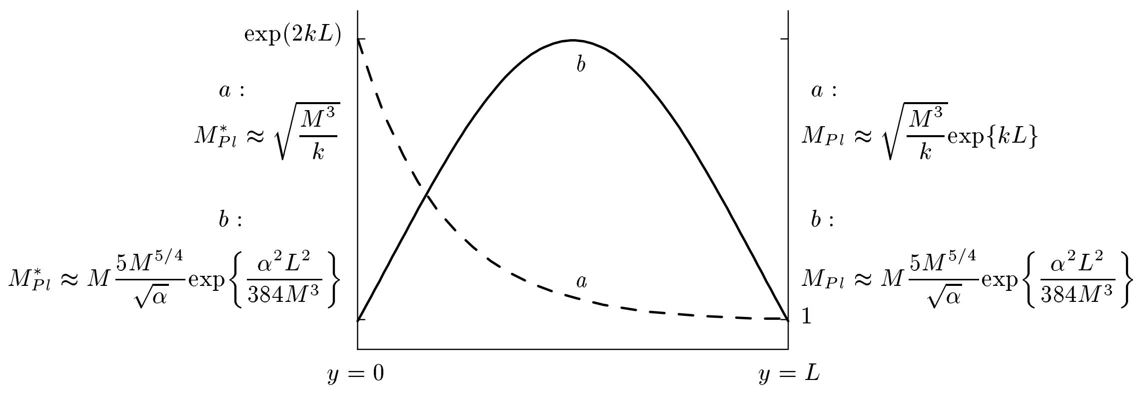
$$A = \frac{\alpha^2}{96M^3}\left[\left(y - \frac{L_1}{2}\right)^2 - \frac{L_1^2}{4}\right]. \tag{16}$$

Carrying out the calculations analogous to those presented above, we obtain

$$M_{Pl}^* \approx M\frac{5M^{5/4}}{\sqrt{\alpha}} \exp\left\{\frac{\alpha^2 L_1^2}{384M^3}\right\}, \tag{17}$$

whence

$$M_{Pl}^* = M_{Pl} \exp\left\{\frac{\alpha^2 L\left(L_1 - L\right)}{96M^3}\right\}. \tag{18}$$



Warp factors in the Randall–Sundrum model (dashed line, a) and in the model under discussion (solid line, b). The branes are located at the points $y = 0$, $y = L$ of the extra dimension. The formulas for the four-dimensional Planck masses in Galilean coordinates on each brane for both cases are presented (in case b, we consider $L_1 = L$). All the parameters are supposed to lie in the TeV range; the hierarchy problem is solved due to the exponential factors $\exp(kL) \approx \exp\left\{\dfrac{\alpha^2L^2}{384M^3}\right\} \sim 10^{16}$

We note that M_{Pl} and M^*_{Pl} were calculated in four-dimensional coordinates, which are Galilean on the respective branes at $y = L$ and $y = 0$, not in a four-dimensional coordinate system common for both branes.

A quite peculiar case is $L = L_1$. For this choice of the parameters, the warp factor has equal values on both branes, and we have two equal branes from the gravitational standpoint! Simultaneously, the hierarchy problem is also solved in this case because of the quadratic behavior of the function $A(y)$ and the corresponding behavior of the warp factor (see the Figure). The peculiar feature of the model is that the hierarchy problem appears to be solved for both branes, contrarily to the case of the RS1 model, in which the hierarchy problem is solved only for brane at $y = L$.

The branes in this case are not only gravitationally equivalent but also have the same negative tension (energy density), which is characteristic of the IR brane in the RS1 model. If not only the gravitational constants on both branes are of the same order but also the properties of the shadow matter are analogous to those of the ordinary matter on our brane, the shadow matter, in principle, may account for a part of dark matter. Indeed, there are some indications that not all the dark matter is collisionless (see, e. g., short review [12]). There may also be processes of interaction between ordinary and shadow matter through the Kaluza–Klein gravitons, which is of interest for the collider phenomenology. However, such possibilities rely on the unknown properties of the shadow matter and are nothing but assumptions, and justifying them requires a more detailed and thorough investigation.

We now discuss the stability of this background solution under small fluctuations of the fields. For this, we consider the linearized theory. The physical degrees of freedom in five-dimensional brane world models stabilized by the scalar field were described in [7] in the general case of a stabilizing scalar field potential. It was shown that if a background solution of field equations (3) exists in the form given in (2) and if the size of the extra dimension is fixed by boundary conditions on the branes, then the tensor sector of Kaluza–Klein excitations does not contain tachyons or fields with the wrong sign of the kinetic term (ghosts). As regards the scalar sector, it does not contain tachyons, ghosts, and massless (from the four-dimensional point of view) modes if [7]

$$\begin{aligned} \left.\left(\frac{1}{2}\,\frac{d^2\lambda_1}{d\phi^2} - \frac{\phi''}{\phi'}\right)\right|_{y=0+\epsilon} &> 0, \\ \left.\left(\frac{1}{2}\,\frac{d^2\lambda_2}{d\phi^2} + \frac{\phi''}{\phi'}\right)\right|_{y=L-\epsilon} &> 0. \end{aligned} \qquad (19)$$

We note that conditions (19) do not involve the bulk potential $V(\phi)$, and it may be unbounded from below as in Eq. (5).

It is easy to find that for the scalar field configuration satisfying Eq. (4) and potentials given by (7) and (8), conditions (19) reduce to $\beta^2_{1,2} > 0$ and are satisfied. Therefore, the model under consideration is indeed stable, at least perturbatively.

It is also worth mentioning that in the case of gravitationally equivalent branes, there exists another interesting possibility to stabilize the size of the extra dimension. There is a good reason to guess that the branes are equivalent initially, i.e., the scalar field potentials on the branes are of the same form. Namely, the (fine-tuned) brane potentials can be chosen to be nonpolynomial,

$$\lambda_{1,2}(\phi) = -3\left(\frac{\rho\alpha^2}{4}\right)^{1/3} + \frac{\rho}{\phi^2} \tag{20}$$

with $\rho > 0$. From the boundary conditions on the branes, which follow from Eqs. (3), we then easily obtain

$$L = L_1 = 4\left(\frac{2\rho}{\alpha^4}\right)^{1/3}. \tag{21}$$

Stability conditions (19) are satisfed for the choice of the potentials in (20).

As regards the wave functions, coupling constants, and masses of the tensor and scalar Kaluza–Klein modes, it seems that it is impossible to solve the corresponding equations of motion analytically in the case of background solution (6) (except for the tensor zero mode, which is proportional to e^{-2A}). But it is quite obvious that masses of the lowest excitations are of the order of $1/L$ (because the model is stabilized, there is no massless radion), and the corresponding coupling constants should also be expressed through the fundamental parameters of the theory. Hence, all these parameters should lie in the TeV energy range, as it usually happens in the brane world models.

3. CONCLUSION

In this paper, we have discussed a stabilized brane world model allowing a solution to the hierarchy problem on both branes, contrary to the case of the Randall–Sundrum-type models, in which a solution to the hierarchy problem can be obtained only for one brane. The stability of the model under fluctuations of metric and scalar field is provided by conditions (19), which are valid for any brane world model with the action of form (1), stabilized by a scalar field [7]. We also show that if the branes are assumed to have an equal structure, namely, the brane potential to be the same on both branes, then there also exists a solution with a fixed size of the extra dimension. In this case, the four-dimensional Planck masses on both branes appear to be equal.

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