

# “ELASTIC” FLUCTUATION-INDUCED EFFECTS IN SMECTIC WETTING FILMS

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The Li–Kardar field theory approach is generalized to wetting smectic films and the “elastic” fluctuation-induced interaction is obtained between the external flat bounding surface and distorted IA (isotropic liquid–smectic A) interface acting as an “internal” (bulk) boundary of the wetting smectic film under the assumption that the IA interface is essentially “softer” than the surface smectic layer. This field theory approach allows calculating the fluctuation-induced corrections in Hamiltonians of the so-called “correlated” liquids confined by two surfaces in the case where one of the bounding surfaces is “rough” and with different types of surface smectic layer anchoring. We obtain that in practice, the account of thermal displacements of the smectic layers in a wetting smectic film reduces to the addition of two contributions to the IA interface Hamiltonian. The first, so-called local contribution describes the long-range thermal “elastic” repulsion of the fluctuating IA interface from the flat bounding surface. The second, so-called nonlocal contribution is connected with the occurrence of an “elastic” fluctuation-induced correction to the stiffness of the IA interface. An analytic expression for this correction is obtained.

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## 1. INTRODUCTION

It is known that just above the bulk isotropic liquid–smectic A (IA) phase transition temperature, close to the external surface bounding an isotropic liquid phase of a smectic liquid crystal (LC), smectic layering is observed [1–7]. Smectic layering is a special case of smectic wetting when the growth of the wetting smectic film (WSF) thickness proceeds via a series of discrete layering transitions. Smectic layering is observed close to the external bounding surface (free surface or solid substrate) and occurs above the bulk IA phase transitions located far from the triple isotropic liquid–smectic A–nematic (INA) point. In constructing the interface model of smectic layering [8], the question of the influence of thermal displacements of smectic layers on the IA interface Hamiltonian naturally occurred to us. This paper is devoted to the solution of this problem. We note that what is traditionally meant [8–16] by the IA interface is the boundary between isotropic liquid and smectic A phases. The IA interface acts as an “internal” (bulk) bounding surface of the WSF (see the Figure).

We emphasize that obtaining the “elastic” fluctuation-induced interaction between the IA interface and the external bounding surface has an independent interest because it solves the problem of thermal “elastic” fluctuation-induced effects (so-called thermal Casimir effects) in wetting smectic films.

The “elastic” fluctuation-induced contribution to the free energy density of a smectic film as a function of the equilibrium thickness of this film was first calculated in [9] within the “hydrodynamic” approach. However, this approach allowed considering only the case of strong anchoring of both surface smectic layers with both “smooth” smectic interfaces. We note that the result derived in [9] corresponds to the “elastic” fluctuation-induced interaction between unperturbed (flat) surfaces bounding the smectic film (see Sec. 5). Subsequently, the limit cases of a long-range Mikheev interaction [9] were obtained in [17].

Finally, in [18, 19], the general field theory approach has been developed. This approach allows calculating the fluctuation-induced corrections to the Hamiltonians of the so-called “correlated” liquids confined by two surfaces, in the case where one of the bounding surfaces is

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“rough” and with different types of surface smectic layer anchoring. The “correlated” liquid is a system with a long-range order due to a broken continuous symmetry. Thermal fluctuations in the “correlated” liquid are described by massless Goldstone modes. Possible examples are a superfluid or a liquid crystal. The surfaces bounding the “correlated” liquid naturally change its fluctuations in their vicinity.

The advantage of this field theory approach is that two types of boundary conditions are quite easily implemented. Boundaries of the first type correspond to the suppression of the fluid fluctuations. Such zero boundary conditions at the bounding surfaces (so-called Dirichlet boundary condition) correspond, for example, to strong anchoring for liquid crystals. Second-type boundaries correspond to the suppression of the normal gradients of fluctuations (so-called Neumann boundary conditions).

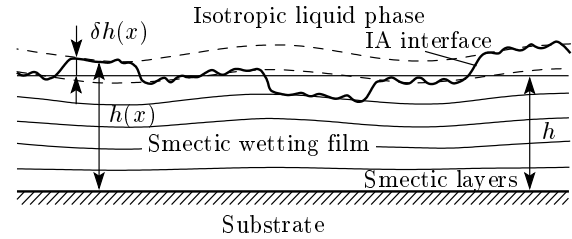
However, different types of boundary conditions were considered in [18, 19] only in the case of a superfluid liquid, in which phase fluctuations are the massless Goldstone modes and are described by a simple quadratic Hamiltonian. In particular, two cases of boundary conditions have been considered for this system. In the first case, the Dirichlet boundary conditions at both bounding surfaces are satisfied. In the second case, the simpler Dirichlet boundary condition is satisfied at a “rough” surface and the more complicated Neumann boundary condition is satisfied at a flat bounding surface.

It is important to note that only the simplest case of zero boundary conditions for thermal displacements of smectic layers at both bounding surfaces was considered for a smectic film in [18, 19]. As we show in what follows, for the problem of smectic wetting, on the contrary, the mixed boundary conditions are of interest and the more complicated Neumann boundary condition is assumed to be satisfied exactly at the “rough” bounding surface.

**2. FORMULATION OF THE PROBLEM. BOUNDARY CONDITIONS**

We develop the Li–Kardar formalism [18, 19] in the case of smectic wetting in the vicinity of a flat bounding surface and solve the problem of the effect of thermal fluctuations of smectic layers on the effective Hamiltonian of the IA interface (see the Figure).

The effective Hamiltonian of the IA interface, in the spirit of the known interface models [9, 10–16], without



Schematic picture of the wetting smectic film covering a flat bounding surface (substrate) with thermal displacements of smectic layers taken into account. The thick line represents the IA interface acting as a boundary between the isotropic and wetting smectic A phases. Local thickness of the WSF is determined as the local removal of the IA interface from the wetted surface. The thin straight line shows the equilibrium position of the IA interface

taking thermal fluctuations of the smectic layers into account, can be written in the general form (see [8])

$$H_{int}[h(\mathbf{x})] = \int_S d^2x \left\{ V_{int}(h(\mathbf{x})) + \frac{\gamma_{IA}}{2} (\nabla h(\mathbf{x}))^2 \right\}, \quad (1)$$

where  $h(\mathbf{x})$  is the local thickness of the WSF (see the Figure),  $V_{int}(h(\mathbf{x}))$  is the initial potential of the interaction of the IA interface with a flat bounding surface (substrate) and with smectic layers, without taking the influence of thermal fluctuations of the smectic layers into account,  $\gamma_{IA}$  is the initial stiffness of the IA interface, and  $S$  is the area of an external flat wetted surface (substrate).

To apply the field theory approach in [18, 19] to the WSF, we use the following assumptions. First, we suppose for simplicity that at the external flat bounding surface with the coordinate  $z = 0$ , the condition of strong anchoring of the surface smectic layer, i.e., the Dirichlet boundary condition is satisfied. Second, we suppose that the IA interface is essentially “softer” than the surface smectic layer and, accordingly, the condition

$$\gamma_{IA} \ll C_{33}\lambda_0 \quad (2)$$

is satisfied, where  $C_{33}$  is the compression modulus of smectic layers and  $\lambda_0$  is the De Gennes elastic “cross length” [20]. We consider static distortions of the smectic layers in WSF, which give rise the long-range fluctuation effects in the smectic film [9, 17], and neglect the density change in the system caused by deformation [21, §§ 44–46]. Condition (2) then allows neglecting

the influence of the IA interface on the smectic layers reaching the IA interface, i.e., allows supposing that condition of the equality of normal stresses [22, § 61], [21, §§ 44–46] in the WSF and isotropic phase at the IA interface is reduced to setting the normal gradients of fluctuations of the smectic layers reaching the IA interface to zero. Accordingly, the Neumann boundary condition (suppression of the normal gradients of fluctuations of smectic layers) is assumed to be satisfied at the IA interface. We note that our boundary conditions correspond to the case of limit values  $\gamma = \infty$  and  $\gamma = 0$  at the surfaces bounding the smectic film [9], or to the case of strong and weak coupling for smectic films in deriving the “pseudo-Casimir” contribution to the free energy of such films [17].

### 3. BASIC ASSUMPTIONS OF THE LI-KARDAR FIELD THEORY APPROACH

We specify two basic assumptions of the Li-Kardar field theory formalism [18, 19] applied to the problem of obtaining the fluctuation contribution to interface Hamiltonian (1), caused by thermal displacements of the smectic layers.

First, the fluctuation displacements of the smectic layers are described by bulk Grinstein-Pelcovits Hamiltonian [23], taken in the quadratic approximation:

$$H_0[u(z, \mathbf{x})] = \int_S d^2x \int dz \times \left\{ \frac{C_{33}}{2} [(\partial_z u(z, \mathbf{x}))^2 + \lambda_0^2 (\nabla^2 u(z, \mathbf{x}))^2] \right\}, \quad (3)$$

where  $u(z, \mathbf{x})$  is a nonuniform elastic thermal displacement of the smectic layer, which is at the point  $(z, \mathbf{x})$  because of the elastic deformation,  $\nabla$  is the gradient in the wetted surface plane, and  $\partial_z \equiv \partial/\partial z$ .

Second, the boundary conditions that must be satisfied by the elastic displacements  $u(z, \mathbf{x})$  at the flat external surface (substrate) and at the IA interface, i.e., at two surfaces bounding the smectic film, are regarded as perturbations acting on the unperturbed bulk system. The smectic film is thus modeled by the influence of these perturbations on the bulk smectic. The boundary conditions are imposed by inserting auxiliary fluctuation fields and using an integral representation for the  $\delta$ -function.

### 4. THE GENERAL EXPRESSION FOR THE “ELASTIC” FLUCTUATION-INDUCED CONTRIBUTION TO THE EFFECTIVE INTERFACE HAMILTONIAN

The general expression for the contribution to effective interface Hamiltonian (1) describing the “elastic” fluctuation-induced interaction between the surfaces bounding the WSF is obtained as follows.

We describe each point at the surfaces bounding the WSF by the three-dimensional radius vector

$$\mathbf{r}_1(\mathbf{x}) = (0, \mathbf{x}) \quad \text{and} \quad \mathbf{r}_2(\mathbf{y}) = (h + \delta h(\mathbf{y}), \mathbf{y}), \quad (4)$$

where  $\mathbf{x}, \mathbf{y}$  is the “internal” two-dimensional radius vector for each of the surfaces,  $\delta h(\mathbf{y})$  is the nonuniform thermal fluctuation distortion of the IA interface relative to its equilibrium position  $z = h$  ( $\int d^2y \delta h(\mathbf{y}) = 0$ ), and the local thickness of the WSF is accordingly represented in the form  $h(\mathbf{y}) = h + \delta h(\mathbf{y})$  (see the Figure).

We introduce an auxiliary fluctuation field  $\Omega_1(\mathbf{x})$  at the flat bounding surface and an auxiliary fluctuation field  $\Omega_2(\mathbf{y})$  at the IA interface. By analogy with [18, 19], the boundary conditions at the surfaces bounding the WSF (see Sec. 2) can be imposed through these auxiliary fields using the integral representation of  $\delta$ -functions. We therefore express the Dirichlet boundary condition at the flat bounding surface as

$$\delta(u(0, \mathbf{x})) = \int D\Omega_1(\mathbf{x}) \times \exp \left[ i \int d^2x \Omega_1(\mathbf{x}) u(\mathbf{r}_1(\mathbf{x})) \right], \quad (5)$$

and the Neumann boundary condition at the IA interface as

$$\delta(\nabla_{\mathbf{n}} u(h(\mathbf{y}), \mathbf{y})) = \int D\Omega_2(\mathbf{y}) \times \exp \left[ i \int d^2y \Omega_2(\mathbf{y}) (\nabla_{\mathbf{n}_2(\mathbf{y})} u(\mathbf{r}_2(\mathbf{y}))) \right], \quad (6)$$

where  $\nabla_{\mathbf{n}_2(\mathbf{y})}$  is the normal gradient of thermal displacements of the smectic layers at the IA interface at a point  $\mathbf{r}_2(\mathbf{y})$ .

With (3)–(6), in terms of functional integration over the thermal displacements of the smectic layers and over the auxiliary fields, the general expression for the “elastic” fluctuation-induced contribution  $H_{eff}$  to interface Hamiltonian (1) is given by

$$\begin{aligned} \exp \left[ -\frac{H_{eff}}{k_B T} \right] &= \int D\Omega_1(\mathbf{x}) D\Omega_2(\mathbf{y}) \times \\ &\times \left\{ \frac{1}{Z_0} \int Du(\mathbf{r}) \exp \left[ -\frac{H_0[u]}{k_B T} + \right. \right. \\ &\quad \left. \left. + i \int d^2x \Omega_1(\mathbf{x}) u(\mathbf{r}_1(\mathbf{x})) + \right. \right. \\ &\quad \left. \left. + i \int d^2y \Omega_2(\mathbf{y}) (\nabla_{\mathbf{n}_2(\mathbf{y})} u(\mathbf{r}_2(\mathbf{y}))) \right] \right\}, \quad (7) \end{aligned}$$

where

$$Z_0 = \int Du(\mathbf{r}) \exp \left[ -\frac{H_0[u]}{k_B T} \right]. \quad (8)$$

Expanding the expression in braces in the right-hand side of (7), in the terms proportional to  $i$ , we find

$$\begin{aligned} \exp \left[ -\frac{H_{eff}}{k_B T} \right] &\approx \int D\Omega_1(\mathbf{x}) D\Omega_2(\mathbf{y}) \frac{1}{Z_0} \int Du(\mathbf{r}) \times \\ &\times \exp \left[ -\frac{H_0[u]}{k_B T} \right] \left\{ 1 + i \int d^2x \Omega_1(\mathbf{x}) u(\mathbf{r}_1(\mathbf{x})) + \right. \\ &\quad \left. + i \int d^2y \Omega_2(\mathbf{y}) (\nabla_{\mathbf{n}_2(\mathbf{y})} u(\mathbf{r}_2(\mathbf{y}))) - \right. \\ &\quad \left. - \frac{1}{2} \left( \int d^2x \Omega_1(\mathbf{x}) u(\mathbf{r}_1(\mathbf{x})) + \right. \right. \\ &\quad \left. \left. + \int d^2y \Omega_2(\mathbf{y}) (\nabla_{\mathbf{n}_2(\mathbf{y})} u(\mathbf{r}_2(\mathbf{y}))) \right)^2 + \dots \right\}. \quad (9) \end{aligned}$$

The configuration integration in (9) over the elastic variables can thus be performed with the result

$$\begin{aligned} \exp \left[ -\frac{H_{eff}}{k_B T} \right] &= \int D\Omega_1(\mathbf{x}) D\Omega_2(\mathbf{y}) \times \\ &\times \exp \left[ -H_1[\Omega_1(\mathbf{x}), \Omega_2(\mathbf{y})] \right], \quad (10) \end{aligned}$$

where the effective Hamiltonian of the two-component field  $\Omega \equiv (\Omega_1(\mathbf{x}), \Omega_2(\mathbf{y}))$  is given by

$$\begin{aligned} H_1[\Omega] &= \frac{1}{2} \int d^2x \int d^2y \{ \Omega_1(\mathbf{x}) G(\mathbf{r}_1(\mathbf{y}) - \\ &- \mathbf{r}_1(\mathbf{x})) \Omega_1(\mathbf{y}) + \Omega_1(\mathbf{x}) (\nabla_{\mathbf{n}_2(\mathbf{y})} G(\mathbf{r}_2(\mathbf{y}) - \mathbf{r}_1(\mathbf{x}))) \Omega_2(\mathbf{y}) + \\ &+ \Omega_1(\mathbf{y}) (\nabla_{\mathbf{n}_2(\mathbf{x})} G(\mathbf{r}_2(\mathbf{x}) - \mathbf{r}_1(\mathbf{y}))) \Omega_2(\mathbf{x}) + \Omega_2(\mathbf{x}) \times \\ &\quad \times (\nabla_{\mathbf{n}_2(\mathbf{x})} \nabla_{\mathbf{n}_2(\mathbf{y})} G(\mathbf{r}_2(\mathbf{y}) - \mathbf{r}_2(\mathbf{x}))) \Omega_2(\mathbf{y}) \} \equiv \\ &\equiv \Omega M \Omega^T. \quad (11) \end{aligned}$$

Here,  $G(\mathbf{r}) = \langle u(0)u(\mathbf{r}) \rangle_0$  is the two-point correlation function in bulk smectic,

$$\langle \dots \rangle_0 = \frac{1}{Z_0} \int Du(\mathbf{r}) (\dots) \exp \left[ -\frac{H_0[u]}{k_B T} \right], \quad (12)$$

and the matrix  $M$  is a functional of the radius vectors  $\mathbf{r}_1(\mathbf{x})$  and  $\mathbf{r}_2(\mathbf{y})$ . In obtaining (10), we used that  $\langle u(\mathbf{r}_1) \dots u(\mathbf{r}_{2m}) u(\mathbf{r}_{2m+1}) \rangle_0 = 0$ .

Within the approach corresponding to the neglect of the bulk elastic anharmonic terms in (3), we suppose that

$$\nabla_{\mathbf{n}_2(\mathbf{y})} u(h(\mathbf{y}), \mathbf{y}) \approx \nabla_{z_y} u(z, \mathbf{y})|_{z=h(\mathbf{y})}. \quad (13)$$

In the considered case, the two-point correlation function in bulk smectic is defined as

$$\begin{aligned} G(\mathbf{y} - \mathbf{x}, z_y - z_x) &= \frac{k_B T}{C_{33}} \times \\ &\times \int \frac{d^2q}{(2\pi)^2} \frac{\exp(i \mathbf{q} \cdot (\mathbf{y} - \mathbf{x})) \exp(-\lambda_0 q^2 z)}{2\lambda_0 q^2}, \quad (14) \end{aligned}$$

where, following the choice in (4), we assume that  $z_y \geq z_x$  and set  $z = z_y - z_x$ . Accordingly, we find

$$\nabla_{z_y} G(\mathbf{y} - \mathbf{x}, z_y - z_x) \equiv \frac{\partial}{\partial z} G(\mathbf{y} - \mathbf{x}, z), \quad (15)$$

$$\nabla_{z_x} \nabla_{z_y} G(\mathbf{y} - \mathbf{x}, z_y - z_x) \equiv -\frac{\partial^2}{\partial z^2} G(\mathbf{y} - \mathbf{x}, z). \quad (16)$$

The quadratic form of the Hamiltonian  $H_1[\Omega]$  in (11) allows integrating over the auxiliary fields  $\Omega$  in (10) and then obtaining the general expression for the effective Hamiltonian that describes the additional ‘‘elastic’’ fluctuation-induced interaction between the IA interface and a flat surface bounding the WSF (cf. [18, 19, 24]):

$$\begin{aligned} H_{eff}[\mathbf{r}_1(\mathbf{x}), \mathbf{r}_2(\mathbf{y})] &= \\ &= \frac{k_B T}{2} \ln \text{Det} \left\{ \frac{M[\mathbf{r}_1(\mathbf{x}), \mathbf{r}_2(\mathbf{y})]}{\pi} \right\}. \quad (17) \end{aligned}$$

Here, with (13), in the case of bounding surfaces described by (4), the functional matrix  $M$  is deduced from (11) using (14)–(16):

$$M(\mathbf{x}, \mathbf{y}) = \frac{1}{2} \begin{pmatrix} G(\mathbf{y} - \mathbf{x}, 0) & \frac{\partial}{\partial z} G(\mathbf{y} - \mathbf{x}, h + \delta h(\mathbf{y})) \\ \frac{\partial}{\partial z} G(\mathbf{x} - \mathbf{y}, h + \delta h(\mathbf{x})) & -\frac{\partial^2}{\partial z^2} G(\mathbf{y} - \mathbf{x}, \delta h(\mathbf{y}) - \delta h(\mathbf{x})) \end{pmatrix}, \quad (18)$$

$$\frac{\partial^m}{\partial z^m} G(\mathbf{y} - \mathbf{x}, \phi(\mathbf{y}, \mathbf{x})) \equiv \frac{\partial^m}{\partial z^m} G(\mathbf{y} - \mathbf{x}, z)|_{z=\phi(\mathbf{y}, \mathbf{x})}.$$

**5.  $H_{eff}$  IN THE CASE OF SMALL DISTORTIONS OF THE IA INTERFACE**

In the case of small distortions  $\delta h(\mathbf{x})$  of the IA interface, the matrix  $M(\mathbf{x}, \mathbf{y})$  can be calculated approximately by expanding the correlation functions and their derivatives appearing in (18) in powers of  $\delta h(\mathbf{x})$ :

$$M(\mathbf{x}, \mathbf{y}) = M_0(\mathbf{x}, \mathbf{y}) + \delta M(\mathbf{x}, \mathbf{y}), \quad (19)$$

where

$$M_0(\mathbf{x}, \mathbf{y}) = \frac{1}{2} \times \begin{pmatrix} G(\mathbf{y} - \mathbf{x}, 0) & \frac{\partial}{\partial z} G(\mathbf{y} - \mathbf{x}, h) \\ \frac{\partial}{\partial z} G(\mathbf{x} - \mathbf{y}, h) & -\frac{\partial^2}{\partial z^2} G(\mathbf{y} - \mathbf{x}, 0) \end{pmatrix} \quad (20)$$

is the functional matrix for the flat bounding surfaces and  $\delta M(\mathbf{x}, \mathbf{y})$  is the correction caused by fluctuation displacements of the IA interface. The Fourier transform of the matrix  $M_0(\mathbf{x}, \mathbf{y})$  required for the calculations in what follows is given in Appendix A.

We note that the two-dimensional Fourier transform of the functional matrix  $\widetilde{M}$  can be represented as

$$\widetilde{M} = \widetilde{M}_0 + \widetilde{M}_0 \widetilde{M}_0^{-1} \delta \widetilde{M}, \quad (21)$$

where the tilde denotes the two-dimensional Fourier transform (see Appendix A).

In this case, effective Hamiltonian (17) can be decomposed as

$$H_{eff} = H_{flat} + H_{corr}, \quad (22)$$

where

$$H_{flat} = \frac{k_B T}{2} \ln \text{Det} \frac{\widetilde{M}_0}{\pi} \quad (23)$$

is the effective Hamiltonian describing the “elastic” fluctuation-induced interaction between the unperturbed (flat) IA interface and the flat external surface bounding the WSF, and

$$H_{corr} = \frac{k_B T}{2} \ln \text{Det} \left\{ 1 + \widetilde{M}_0^{-1} \delta \widetilde{M} \right\} \quad (24)$$

is the additional “elastic” fluctuation-induced contribution to  $H_{eff}$ , caused by thermal distortions of the IA interface.

We also note that using (14) and (A.1), (A.2), it is possible to illustrate the physical meaning of inequality (2). By analogy with obtaining  $H_{corr}$ , the field theory approach allows calculating the correlation function of thermal displacements of the smectic layers in bounded

systems, which is an independent problem. This use of the functional integration method [18, 19] for obtaining the correlation functions of fluctuating fields satisfying the Dirichlet boundary conditions has been considered in [25, 26]. Developing this method for boundary conditions (5) and (6) allows obtaining the leading contribution to the two-dimensional Fourier transform of the correlation function of thermal displacements of the smectic layers in WSF reaching the IA interface  $\langle u(\mathbf{x}, h(\mathbf{x}))u(\mathbf{y}, h(\mathbf{y})) \rangle$ :

$$\widetilde{G}_{IA}(\mathbf{q}) \approx \frac{k_B T}{C_{33} \lambda_0 q^2} \frac{1 - \exp(-2\lambda_0 q^2 h)}{1 + \exp(-2\lambda_0 q^2 h)}. \quad (25)$$

We introduce the Fourier transform of the nonuniform fluctuating values  $\delta h(\mathbf{x})$  given above:

$$\delta h(\mathbf{x}) = \sum_{\mathbf{q}}' h_{\mathbf{q}} \exp(i\mathbf{q} \cdot \mathbf{x}).$$

In the zeroth order of the interface potential up to a fixed point of the renormalization group procedure of eliminating fast fluctuations of the IA interface, i.e., up to  $q = q_{cap}$  [8, 27], we have

$$\langle h_{\mathbf{q}} h_{-\mathbf{q}} \rangle_0 \approx \frac{k_B T}{\gamma_{IA} q^2}. \quad (26)$$

In limit (2), it follows from (25) and (26) for  $q > q_C = 1/\sqrt{\lambda_0 h}$  that

$$\widetilde{G}_{IA}(\mathbf{q}) \approx \frac{k_B T}{C_{33} \lambda_0 q^2} \ll \langle h_{\mathbf{q}} h_{-\mathbf{q}} \rangle_0. \quad (27)$$

It is well known that the amplitudes of the interfacial potential  $V_{int}(h)$  determine the value of  $q_{cap}$  [8, 27]. It can be shown that the inequality  $\widetilde{G}_{IA}(\mathbf{q}) \ll \langle h_{\mathbf{q}} h_{-\mathbf{q}} \rangle_0$  is also satisfied for  $q < q_C$  under the assumption that the conditions analogous to (2) hold for the second derivatives of the interfacial potential  $V_{int}(h)$  and the gap  $C_{33}/h$  of correlator (25).

Hence, the condition of “softness” of the IA interface expressed by inequality (2) and leading to inequality (27) means, in particular, that fluctuations of the thermal capillary mode  $\delta h$  are dominant at the IA interface. This in turn means that in the smectic wetting problem, the roughening fluctuations of the IA interface  $\psi(\mathbf{x}) = \delta h(\mathbf{x}) - u(\mathbf{x}, h(\mathbf{x}))$  [8, 9] are actually reduced to the fluctuations of the mode  $\delta h$ . Capillary fluctuations of the IA interface are “dangerous”. This indicates that the roughening fluctuations of the IA interface should be understood as the thermal capillary displacements of the IA interface.

After evaluating the determinant of  $\widetilde{M}_0$  (see Appendix A for the details), we obtain the following  $h$ -dependent contribution to  $H_{flat}$ :

$$H_{flat}(h) = S \frac{k_B T}{2} \int \frac{d^2 k}{(2\pi)^2} \times \ln \left[ 1 + \exp(-2\lambda_0 k^2 h) \right] = SV_{Mikh}(h), \quad (28)$$

where

$$V_{Mikh}(h) = \frac{k_B T}{32\pi} \frac{\zeta(2)}{\lambda_0 h} \quad (29)$$

and  $\zeta_R(2) = \pi^2/6$  is a value of the Riemann zeta function. Expression (29) is a long-range “repulsive” contribution to the interaction potential  $V_{int}(h)$  between the unperturbed (flat) IA interface and the external flat surface bounding the WSF (more accurately, to the density of the free energy of the WSF with the equilibrium thickness  $h$ ).

We note that the long-range “repulsive” potential  $V_{Mikh}(h)$  coincides with the limit of the “hydrodynamic” Mikheev interaction (in the limit  $\gamma_{IA} \ll C_{33}\lambda_0$ ,  $\gamma_{ext} \gg C_{33}\lambda_0$ , where  $\gamma_{ext}$  is the stiffness of the external boundary of the WSF; see Secs. 2, 7) arising due to dimensional screening of the elastic smectic modes [9], which confirms the correctness of the boundary conditions imposed in Sec. 2. We also note that the potential  $V_{Mikh}(h)$  coincides with the contribution to the free energy density of the smectic film obtained in [17] for asymmetric boundary conditions at the surfaces bounding this film. In turn, the asymmetric boundary conditions agree with the boundary conditions imposed for the WSF in Sec. 2.

### 6. OBTAINING $H_{corr}$ . LOCAL AND NONLOCAL CORRECTIONS

The additional contribution to the effective Hamiltonian caused by thermal displacements of the IA interface can be evaluated approximately by expanding in powers of the fluctuation displacements  $\delta h(\mathbf{x})$  of the IA interface.

The evaluation of  $H_{corr}$  from Eq. (24) is sufficiently tedious. For this reason, the details of the calculation are given in Appendix B. The result is in (B.20).

Integrating over the relative variables  $\mathbf{v}_1 - \mathbf{y}$  and  $\mathbf{v}_2 - \mathbf{x}$  in (B.20) and using the identity

$$\delta h(\mathbf{x})\delta h(\mathbf{y}) = (1/2) (\delta h^2(\mathbf{x}) + \delta h^2(\mathbf{y}) - (\delta h(\mathbf{y}) - \delta h(\mathbf{x}))^2),$$

it is possible to decompose  $H_{corr}$  into local and nonlocal contributions.

The local contribution is given by

$$H_{corr}^{(loc)} = \int d^2 y \delta h^2(\mathbf{y}) \left[ \frac{k_B T}{2} \int \frac{d^2 k}{(2\pi)^2} \times \left\{ -\frac{1}{4D_0(\mathbf{k})} \left( \frac{\partial}{\partial h} \tilde{G}(\mathbf{k}, h) \right) \left( \frac{\partial^3}{\partial h^3} \tilde{G}(\mathbf{k}, h) \right) - \left( \left( \frac{\partial}{\partial h} \tilde{G}(\mathbf{k}, h) \right)^2 - \tilde{G}(\mathbf{k}, 0) \left( \frac{\partial^2}{\partial z^2} \tilde{G}(\mathbf{k}, z) \right)_{z=0} \right) \times \frac{1}{16D_0^2(\mathbf{k})} \left( \frac{\partial^2}{\partial h^2} \tilde{G}(\mathbf{k}, h) \right)^2 \right\} \right]. \quad (30)$$

We note that the expression in square brackets in (30) is a fluctuation-induced correction to the gap of the mode  $\delta h$ . Inserting (A.1), (A.2), and (B.11) in (30) and extending the integration over  $\mathbf{k}$  from 0 to  $\infty$  (using the fast convergence of integrals due to the presence of a decreasing exponential), we find

$$H_{corr}^{(loc)} = \frac{k_B T}{16\pi} \frac{\zeta(2)}{\lambda_0 h^3} \int d^2 y \frac{\delta h^2(\mathbf{y})}{2}. \quad (31)$$

It is important that if we formally keep the term linear in  $\delta h(\mathbf{y})$  in  $A(\mathbf{k}, \mathbf{k})$  given by (B.19), then the following additional contribution to the local part of  $H_{corr}^{(loc)}$  formally appears:

$$-\frac{k_B T}{2} \int \frac{d^2 k}{(2\pi)^2} \frac{1}{2D_0(\mathbf{k})} \left( \frac{\partial}{\partial h} \tilde{G}(\mathbf{k}, h) \right) \times \left( \frac{\partial^2}{\partial h^2} \tilde{G}(\mathbf{k}, h) \right) \int d^2 y \delta h(\mathbf{y}) = -\frac{k_B T}{32\pi} \frac{\zeta(2)}{\lambda_0 h^2} \int d^2 y \delta h(\mathbf{y}). \quad (32)$$

Expressions (31) and (32) define first-order and second-order corrections of the  $\delta h(\mathbf{y})$ -expansion of the contribution to  $V_{int}(h(\mathbf{y}))$  following from taking the thermal displacements of smectic layers into account. Therefore, we can formally combine the essential thermal “elastic” corrections  $V_{Mikh}(h)$ ,  $V'_{Mikh}(h)\delta h(\mathbf{y})$ , and  $(1/2)V''_{Mikh}(h)\delta h^2(\mathbf{y})$  to  $V_{int}(h(\mathbf{y}))$  into the total “local” long-range thermal “elastic” potential of the repulsion of the distorted IA interface from the flat bounding surface and regard these corrections as the first terms of its expansion:

$$V_{Mikh}^{(loc)}(h + \delta h(\mathbf{y})) = \frac{k_B T}{32\pi} \frac{\zeta(2)}{\lambda_0 h(\mathbf{y})} \approx V_{Mikh}(h) + V'_{Mikh}(h)\delta h(\mathbf{y}) + \frac{1}{2}V''_{Mikh}(h)\delta h^2(\mathbf{y}). \quad (33)$$

Combining (31) and (32) with (28), we obtain the following expression for the corresponding local contribution to interface Hamiltonian (1):

$$H_{Mikh}^{(loc)}[h(\mathbf{y})] = \int d^2y V_{Mikh}^{(loc)}(h + \delta h(\mathbf{y})) = \int d^2y \frac{k_B T}{32\pi} \frac{\zeta(2)}{\lambda_0 h(\mathbf{y})}. \quad (34)$$

The nonlocal contribution to  $H_{corr}$  is also unwieldy and is therefore given in Appendix B (see (B.21)). It is obvious that in the case of weak nonlocality, when the expansion

$$\delta h(\mathbf{x}) \approx \delta h(\mathbf{y}) + (\nabla_{\mathbf{y}} \delta h(\mathbf{y}))(\mathbf{x} - \mathbf{y})$$

is valid, nonlocal contribution (B.21) describes the occurrence of an “elastic” fluctuation-induced correction  $\delta\gamma_{el}$  to the stiffness of the IA interface:

$$H_{corr}^{(nonloc)}[\delta h(\mathbf{y})] \approx \int d^2y \delta\gamma_{el} \frac{(\nabla \delta h(\mathbf{y}))^2}{2}, \quad (35)$$

where

$$\begin{aligned} \delta\gamma_{el} = & \frac{k_B T}{2} \int d^2x (\mathbf{y} - \mathbf{x})^2 \times \\ & \times \left[ \int \frac{k dk}{2\pi} J_0(k|\mathbf{y} - \mathbf{x}|) (\lambda_0 k^2)^3 \times \right. \\ & \times \int \frac{q dq}{2\pi} J_0(q|\mathbf{y} - \mathbf{x}|) \frac{1}{1 + \exp(-2\lambda_0 q^2 h)} \frac{2}{\lambda_0 q^2} + \\ & + \left( \int \frac{q dq}{2\pi} J_0(q|\mathbf{y} - \mathbf{x}|) \lambda_0 q^2 \frac{\exp(-2\lambda_0 q^2 h)}{1 + \exp(-2\lambda_0 q^2 h)} \right)^2 - \\ & - \int \frac{q dq}{2\pi} J_0(q|\mathbf{y} - \mathbf{x}|) (\lambda_0 q^2)^2 \frac{\exp(-\lambda_0 q^2 h)}{1 + \exp(-2\lambda_0 q^2 h)} \times \\ & \left. \times \int \frac{k dk}{2\pi} J_0(k|\mathbf{y} - \mathbf{x}|) \frac{\exp(-\lambda_0 k^2 h)}{1 + \exp(-2\lambda_0 k^2 h)} \right]. \quad (36) \end{aligned}$$

Expression (36) is a fluctuation correction to the stiffness of the capillary mode  $\delta h$ , that is, to the stiffness of the IA interface caused by thermal displacements of smectic layers in WSF. It is obvious that integrals over the wave vector in the first term in the square brackets in (36) are defined by the cutoff parameters of these integrals and the corresponding contribution to  $\delta\gamma_{el}$  is traditionally (see [27]) included into a redefined stiffness of the IA interface. But the other two terms in (36) give an  $h$ -dependent (dimensional) correction to the stiffness of the capillary mode  $\delta h$ , caused by thermal displacements of smectic layers in WSF. The integrals over the wave vector in these terms can be calculated approximately in view of their fast convergence because of the presence of the rapidly decreasing exponential  $\exp(-\lambda_0 q^2 h)$ . Indeed, considering that the  $q \lesssim q_C$  give the leading contribution to these integrals, it is possible to omit the decreasing exponentials in the denominators. Then these integrals reduce to

tabulated ones. Changing the variables as  $Q = q/q_C$ ,  $\rho = q_C |\mathbf{y} - \mathbf{x}|$ , we find the following expression for the  $h$ -dependent correction  $\delta\gamma_{el}(h)$  to the stiffness of the IA interface:

$$\begin{aligned} \delta\gamma_{el}(h) \approx & \frac{k_B T}{4\pi} \frac{1}{h^2} \times \\ & \times \int \rho^3 d\rho \left[ \left( \int Q^3 dQ \exp(-2Q^2) J_0(Q\rho) \right)^2 - \right. \\ & - \int Q_1^5 dQ_1 \exp(-Q_1^2) J_0(Q_1\rho) \times \\ & \left. \times \int Q_2 dQ_2 \exp(-Q_2^2) J_0(Q_2^2\rho) \right]. \quad (37) \end{aligned}$$

After some calculations, we have

$$\delta\gamma_{el}(h) \approx \frac{5}{64\pi} \frac{k_B T}{h^2}. \quad (38)$$

We note that even in the case of the simple boundary conditions for smectic films considered in [18, 19],  $\delta\gamma_{el}(h)$  was not calculated and the corresponding analytic result was not obtained.

## 7. CONCLUSION

In the absence of experimental data concerning the value of  $\gamma_{IA}$ , it is natural to assume [8] that

$$\gamma_{IA} \sim 10^{-1} \gamma_0 \sim 10^0 \text{ erg} \cdot \text{cm}^{-2}, \quad (39)$$

where  $\gamma_0$  is the stiffness of the free surface of WSF ( $\gamma_0 \approx 30 \text{ erg} \cdot \text{cm}^{-2}$  [18]). In particular, the case of rotator phases of the normal alkanes (paraffins) [28] confirms this estimation of  $\gamma_{IA}$ . For these phases, the tension of the isotropic liquid-layered rotator phase interface is much smaller (by almost an order of magnitude) than the tension of the free surface of molten alkanes, which is of the same characteristic value as  $\gamma_0$  [28].

For a typical bilayer smectic LC (see [7, 20, 29]), we have

$$\Delta H_{IA} \sim 10^8 \text{ erg} \cdot \text{cm}^{-3}, \quad (40)$$

$$C_{33} \sim 10^8 \text{ erg} \cdot \text{cm}^{-3}. \quad (41)$$

The fact that  $\Delta H_{IA}$  and  $C_{33}$  are of the same order indicates the noncritical character of the bulk IA transition. For the subsequent estimations, we use that (see [20])

$$\lambda_0 \sim d_0 \sim 10^{-7} \text{ cm}. \quad (42)$$

In this case, simple analysis shows that inequality (2) is satisfied in practice. For  $T_{IA} \approx 300 \text{ K}$ , correction (38) to the initial stiffness of the IA interface  $\gamma_{IA}$  turns out to be negligibly small ( $\delta\gamma_{el}(h) \ll \gamma_{IA}$ ) even for  $h \approx d_0$ :

$$\delta\gamma_{el}(h) \sim 10^{-2}-10^{-1} \text{ erg} \cdot \text{cm}^{-2}. \quad (43)$$

Thus, having generalized the field theory approach [18, 19] to the case of a WSF, we have obtained that under condition (2), which is satisfied in practice, and the condition of strong anchoring of the surface smectic layer and the external bounding surface, taking the elastic thermal displacements of the smectic layers in the WSF into account reduces to adding two contributions,  $H_{Mikh}^{(loc)}[h(\mathbf{y})]$  and  $H_{corr}^{(nonloc)}[h(\mathbf{y})]$ , to the effective Hamiltonian of the IA interface (1). This, accordingly, leads to adding the potential of long-range repulsion of the IA interface from the flat bounding surface  $V_{Mikh}^{(loc)}(h(\mathbf{y}))$  to the WSF interface potential  $V_{int}(h(\mathbf{y}))$  and to the occurrence of the correction  $\delta\gamma_{el}(h)$  to the stiffness of the IA interface.

These conclusions are also valid in the case of wetting by the smectic A phase of a free surface of a smectic LC under the condition of strong anchoring of this bounding surface and the adjoining smectic layer and with the condition  $\gamma_0 > C_{33}\lambda_0$ , which is necessary for preserving the repulsive character of the Mikheev interaction [9], imposed in addition to (2). The first condition is apparently always satisfied, by particular, by virtue of clearly distinguishable steps at the surface of a drop of smectic A [30, 31]. Fulfillment of the second condition is also confirmed experimentally (see [32]).

It is interesting to note that “elastic” fluctuation-induced long-range repulsion (33) is most dangerous in the limit of large  $h$  ( $h \gtrsim 10d_0$  [8]) and provides the complete smectic wetting in the absence of layering transitions independently of the sign of the long-range van der Waals interaction constant. We recall that the so-called “oscillatory” regular regime of smectic wetting is typical of the bulk IA transition in close vicinity of a triple INA point [3, 5, 6, 34, 35] and occurs just above the temperature of such bulk IA-transitions, when the potential of the pinning of the IA interface at the positions of the smectic layers in  $V_{int}(h)$  is not sufficient and layering transitions are absent [8]. In this regime, the WSF equilibrium thickness grows continuously, weakly oscillating relative to its average temperature dependence [8]. The growth of the WSF thickness is directly registered by ellipsometric study of the free surface of the smectic LC above the bulk IA transition [4, 5, 33] or verified by fitting the X-ray reflectivity from the WSF using the model mass density profile [3, 6, 34, 35]. Thus, for the IA transitions mentioned above, the smectic wetting is found to be complete within the experimental accuracy [5, 34]. In this case, in the limit of large WSF thickness, the deviation of the temperature positions of the layering tran-

sitions from equidistant on a logarithmic temperature scale in the layering regime or the deviation of the temperature positions of the inflection points of the WSF thickness temperature dependence in the “oscillatory” regime is experimentally observed [4, 5, 33]. These deviations correspond to a decrease in the temperature intervals between these positions [4, 5, 33] and confirm the occurrence in the system of the long-range repulsion of the IA interface from the external wetted surface in addition to the short-range repulsion. Moreover, the temperature dependence of the average WSF thickness in the limit of large  $h$  does not coincide with either the logarithmic or the power law caused by the long-range van der Waals interaction temperature dependences [4, 5, 33].

Unfortunately, the temperature dependence of the average WSF thickness still was not fitted using both the logarithmic and power-law temperature dependences caused by long-range interaction (33). Such a study of the WSF for the bulk IA transitions in the close vicinity of a triple INA point or in the case of a large finite WSF thickness in the smectic layering regime (which is typical, e.g., of the homologues of the  $\bar{n}.O.\bar{6}$ -series with  $n \geq 18$  [4, 5]), using the results in [8], would be an additional confirmation of the occurrence of long-range interaction (33) in the system and a direct experimental observation of the “elastic” fluctuation-induced long-range repulsion of the IA interface from the external wetted surface.

Presently, because of the absence in [4, 5, 33] of the temperature dependence of the WSF thickness tabulated data for different smectic homologues and the WSF thickness tabulated data for  $h \gtrsim 10d_0$  in particular, only some qualitative estimations confirming this statement can be made. Using the results in [8] and isolating the smooth part  $V_0(h)$  of the interfacial potential  $V_{int}(h)$ , it is not difficult to derive an equation for the temperature dependence of the average WSF thickness  $h_0(t)$  taking interaction (33) into account ( $V'_{0h} = 0$ ):

$$t = A \exp\left(-\frac{h_0}{\xi_C}\right) + \frac{1}{\Delta H_{IA}} \frac{k_B T \zeta_R(2)}{32\pi\lambda_0 h_0^2}, \quad (44)$$

where  $t = (T - T_{IA})/T_{IA}$  is the reduced deviation of the temperature  $T$  from the temperature of the bulk IA phase transition ( $t \geq 0$ ),  $\xi_C$  is the correlation length in the bulk smectic A phase, and  $A$  is the reduced amplitude of the short-range repulsive interaction ( $A > 0$ ). Equation (44) determines the inverse dependence of  $h_0(t)$ , and hence plotting  $h_0(t)$  with the given values of the constants is trivial [8]. Choosing three points from seven points of the WSF thickness temperature dependence presented in Fig. 4 in [4] ( $h \gtrsim 10d_0$ ) for



compound  $\overline{18.O.6}$  ( $T_{IA} = 359.88$  K [4]) within the accuracy acceptable in [4],  $\{(t_1 = 0.000152778, h_1 = 26 \cdot 10^{-7}$  cm),  $(t_2 = 0.0000694444, h_2 = 30 \cdot 10^{-7}$  cm),  $(t_3 = 0.0000263889, h_3 = 34 \cdot 10^{-7}$  cm) $\}$ , we obtain that these points are described by Eq. (44) with

$$A \approx 0.0455 \quad \text{and} \quad \xi_C \approx 4.5 \cdot 10^{-7} \text{ cm}, \quad (45)$$

which is in agreement with the estimations  $A \sim 10^{-2} - 10^{-1}$  and  $q_0 \xi_C \approx 10$  obtained in [8] in the analysis of the data in [1–7, 33–35], where  $q_0 = 2\pi/d_0$  is the wave number of the bulk smectic lattice. We note that from the analysis of Eq. (44), it is easy to obtain the estimation  $h \gtrsim 10d_0$  for the WSF thickness, for which it is necessary to take the long-range “elastic” fluctuation-induced interaction (33) into account in addition to the short-range interaction in the WSF interfacial potential.

We also note that the conclusion in Sec. 5 that the roughening fluctuations of the IA interface should be understood as fluctuations of the thermal capillary mode  $\delta h$ , allows us to define the Fourier transform of the two-point correlator of the relative thermal displacements of the IA interface  $\langle \psi_{\mathbf{k}} \psi_{-\mathbf{k}} \rangle$  within the interface model discussed:

$$\begin{aligned} \langle \psi_{\mathbf{k}} \psi_{-\mathbf{k}} \rangle &\approx \langle h_{\mathbf{k}} h_{-\mathbf{k}} \rangle \approx \\ &\approx \frac{k_B T}{V''_{int}(h) + V''_{Mikh}(h) + (\gamma_{IA} + \delta\gamma_{el}(h))k^2}. \end{aligned} \quad (46)$$

This Fourier transform (46) of the two-point correlator, in particular, defines the intensity of light scattered by a free LC surface in the presence of WSF in the experiments similar to those in [36] and structure factors related to the reduced  $X$ -ray reflectivity both from the free LC surface in the presence of WSF (see, e.g., [1–3, 8, 28, 35]) and from the WSF wetting a specially treated solid substrate (see [6, 7]). We take into account that in the case of large WSF thickness in the regime without layering transitions just above the bulk IA-transitions in a close vicinity of the triple INA point, the terms  $V''_{Mikh}(h)$  and  $(\gamma_{IA} + \delta\gamma_{el}(h))k^2$  become the leading terms in the denominator of the correlator  $\langle h_{\mathbf{k}} h_{-\mathbf{k}} \rangle$ . In this case, from the simultaneous fitting of the intensity of light scattered by the relative capillary displacements of the IA interface similar to [36] and of the  $X$ -ray reflectivity from the WSF similar to [1–3, 6, 8, 28, 35] for the same LC compound, it would be possible to experimentally determine the value of  $\gamma_{IA}$  and to derive the dependences  $V''_{Mikh}(h)$  and  $\delta\gamma_{el}(h)$  for comparison with the results obtained in this paper. Such experiments would also be the direct study of the “elastic” fluctuation-induced effects in the

WSF, but it is important for this study that expressions for the structural factors used in [1, 2] have to be calculated more accurately [8]. In addition, with similar research goals, similar experiments may be carried out for the films of different correlated liquids.

To summarize, we note that the considered functional-integral method allows simplifying the calculation of the fluctuation-induced interactions in the bounded systems and carrying out these calculations in the same way for various correlated liquids with quite easily implemented boundary conditions on “rough” bounding surfaces.

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## APPENDIX A

In this appendix, we give details of the derivation of the effective Hamiltonian  $H_{flat}$  describing the “elastic” fluctuation-induced interaction between the unperturbed (flat) IA interface and a flat external surface bounding the WSF, defined by (23). It is obvious that these calculations require finding the determinant of  $\widetilde{M}_0$ . For obtaining the Fourier transform of the matrix  $M_0(\mathbf{x}, \mathbf{y})$ , we use (14) to define the two-dimensional Fourier transformation of the correlation functions occurring in (20) and find their Fourier transforms as

$$\begin{aligned} \widetilde{G}(\mathbf{q}, 0) &= \int d^2\Upsilon \exp(-i\mathbf{q} \cdot \Upsilon) \widetilde{G}(\Upsilon, 0) = \\ &= \frac{k_B T}{C_{33}} \frac{1}{2\lambda_0 q^2}, \end{aligned} \quad (A.1)$$

$$\begin{aligned} \widetilde{G}(\mathbf{q}, h) &= \int d^2\Upsilon \exp(-i\mathbf{q} \cdot \Upsilon) \widetilde{G}(\Upsilon, h) = \\ &= \frac{k_B T}{C_{33}} \frac{\exp(-\lambda_0 q^2 h)}{2\lambda_0 q^2}, \end{aligned} \quad (A.2)$$

where  $\Upsilon = \mathbf{y} - \mathbf{x}$  and the tilde specifies the two-dimensional Fourier transformation.

Using (A.1) and (A.2), we then obtain the Fourier-transformed matrix  $M_0$ :

$$\begin{aligned} \widetilde{M}_0(\mathbf{k}) &= \\ &= \frac{1}{2} \begin{pmatrix} \widetilde{G}(\mathbf{k}, 0) & \frac{\partial}{\partial h} \widetilde{G}(\mathbf{k}, h) \\ \frac{\partial}{\partial h} \widetilde{G}(\mathbf{k}, h) & -\frac{\partial^2}{\partial z^2} \widetilde{G}(\mathbf{k}, z)|_{z=0} \end{pmatrix}. \end{aligned} \quad (A.3)$$

For simplicity, we pass to the discrete wave vector notation  $\mathbf{k}_i$ . Each block in expression (A.3) should

then be understood as an infinite-dimensional matrix, namely:

$$\widetilde{M}_0(\mathbf{k}) = \frac{1}{2} \begin{pmatrix} \widetilde{G}(\mathbf{k}_1, 0) & 0 & \dots & \frac{\partial}{\partial h} \widetilde{G}(\mathbf{k}_1, h) & 0 & \dots \\ 0 & \widetilde{G}(\mathbf{k}_2, 0) & \dots & 0 & \frac{\partial}{\partial h} \widetilde{G}(\mathbf{k}_2, h) & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial}{\partial h} \widetilde{G}(\mathbf{k}_1, h) & 0 & \dots & -\frac{\partial^2}{\partial z^2} \widetilde{G}(\mathbf{k}_1, z)|_{z=0} & 0 & \dots \\ 0 & \frac{\partial}{\partial h} \widetilde{G}(\mathbf{k}_2, h) & \dots & 0 & -\frac{\partial^2}{\partial z^2} \widetilde{G}(\mathbf{k}_2, z)|_{z=0} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix} \quad (\text{A.4})$$

To simplify the definition of  $H_{flat}$ , after an even number of rearrangements and substitution of the Fourier transforms of correlation functions (A.1) and (A.2), the matrix  $\widetilde{M}_0(\mathbf{k})$  takes the simple form

$$\widetilde{M}_0(\mathbf{k}) = \frac{1}{2} \frac{k_B T}{C_{33}} \times \begin{pmatrix} \dots & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2\lambda_0 k_i^2} & -\frac{1}{2} \exp(-\lambda_0 k_i^2 h) & 0 & 0 & 0 \\ 0 & -\frac{1}{2} \exp(-\lambda_0 k_i^2 h) & -\frac{\lambda_0 k_i^2}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2\lambda_0 k_{i+1}^2} & -\frac{1}{2} \exp(-\lambda_0 k_{i+1}^2 h) & 0 \\ 0 & 0 & 0 & -\frac{1}{2} \exp(-\lambda_0 k_{i+1}^2 h) & -\frac{\lambda_0 k_{i+1}^2}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \dots \end{pmatrix}. \quad (\text{A.5})$$

Then the determinant of  $\widetilde{M}_0$  is easily calculated as

$$\text{Det } \widetilde{M}_0 = \prod_i \left( \frac{1}{2} \frac{k_B T}{C_{33}} \right)^2 \times \frac{1}{4} \left[ 1 + \exp(-2\lambda_0 k_i^2 h) \right]. \quad (\text{A.6})$$

Substituting (A.6) in (23) and passing from summation over  $\mathbf{k}_i$  to integration, it is easy to find the dimensional  $h$ -dependent contribution to  $H_{flat}$ , given by expression (28).

### APPENDIX B

In this appendix, we give details of the derivation of the effective Hamiltonian  $H_{corr}$  defined by (24). This Hamiltonian is an additional contribution to the effective

Hamiltonian, caused by thermal displacements of the IA interface.

To obtain an explicit expression for the matrix  $\delta M(\mathbf{x}, \mathbf{y})$ , we expand the correlation functions and their derivatives appearing in (18) in powers of small  $\delta h(\mathbf{x})$  through the second order:

$$G(\mathbf{y}-\mathbf{x}, h+\delta h(\mathbf{y})) = G(\mathbf{y}-\mathbf{x}, h) + \frac{\partial G}{\partial z}(\mathbf{y}-\mathbf{x}, h) \delta h(\mathbf{y}) + \frac{1}{2} \frac{\partial^2 G}{\partial z^2}(\mathbf{y}-\mathbf{x}, h) \delta h^2(\mathbf{y}), \quad (\text{B.1})$$

$$\begin{aligned} \frac{\partial}{\partial z} G(\mathbf{y}-\mathbf{x}, h+\delta h(\mathbf{y})) &= \frac{\partial}{\partial z} G(\mathbf{y}-\mathbf{x}, h) + \\ &+ \frac{\partial^2}{\partial z^2} G(\mathbf{y}-\mathbf{x}, h) \delta h(\mathbf{y}) + \\ &+ \frac{1}{2} \frac{\partial^3}{\partial z^3} G(\mathbf{y}-\mathbf{x}, h) \delta h^2(\mathbf{y}), \quad (\text{B.2}) \end{aligned}$$

$$\frac{\partial^2}{\partial z^2} G(\mathbf{y} - \mathbf{x}, \delta h(\mathbf{y}) - \delta h(\mathbf{x})) = \frac{\partial^2}{\partial z^2} G(\mathbf{y} - \mathbf{x}, 0) + \frac{1}{2} \frac{\partial^4}{\partial z^4} G(\mathbf{y} - \mathbf{x}, 0) (\delta h(\mathbf{y}) - \delta h(\mathbf{x}))^2. \quad (\text{B.3})$$

In (B.3), the term linear in  $(\delta h(\mathbf{y}) - \delta h(\mathbf{x}))$  is identically equal to zero, as can be easily verified using representation (14).

By analogy with [18, 19], after taking the two-dimensional Fourier transformation for the expansions of correlation functions (B.1)–(B.3), we obtain

$$\delta \widetilde{M}(\mathbf{k}, \mathbf{q}) = \frac{1}{2} \begin{pmatrix} 0 & A(\mathbf{k}, \mathbf{q}) \\ A(\mathbf{q}, \mathbf{k}) & B(\mathbf{k}, \mathbf{q}) \end{pmatrix}, \quad (\text{B.4})$$

where

$$A(\mathbf{k}, \mathbf{q}) = \int d^2x \int d^2y \exp(-i\mathbf{k} \cdot \mathbf{y}) \times \exp(i\mathbf{q} \cdot \mathbf{x}) \frac{\partial^2}{\partial h^2} G(\mathbf{y} - \mathbf{x}, h) \delta h(\mathbf{y}) + \frac{1}{2} \int d^2x \int d^2y \exp(-i\mathbf{k} \cdot \mathbf{y}) \exp(i\mathbf{q} \cdot \mathbf{x}) \times \frac{\partial^3}{\partial h^3} G(\mathbf{y} - \mathbf{x}, h) \delta h^2(\mathbf{y}), \quad (\text{B.5})$$

$$B(\mathbf{k}, \mathbf{q}) = -\frac{1}{2} \int d^2x \int d^2y \exp(-i\mathbf{k} \cdot \mathbf{y}) \exp(i\mathbf{q} \cdot \mathbf{x}) \times \left( \frac{\partial^4}{\partial z^4} G(\mathbf{y} - \mathbf{x}, z) \right)_{z=0} (\delta h(\mathbf{y}) - \delta h(\mathbf{x}))^2. \quad (\text{B.6})$$

For simplicity of the calculation, we set

$$W = 1 + \widetilde{M}_0^{-1} \delta \widetilde{M}. \quad (\text{B.7})$$

Then  $H_{corr}$  is written as

$$H_{corr} = \frac{k_B T}{2} \ln \text{Det } W. \quad (\text{B.8})$$

Inverting the matrix  $\widetilde{M}_0$  using (A.4) results in

$$\widetilde{M}_0^{-1}(\mathbf{k}) = \frac{1}{2 D_0(\mathbf{k})} \times \begin{pmatrix} -\frac{\partial^2}{\partial z^2} \widetilde{G}(\mathbf{k}, z)|_{z=0} & -\frac{\partial}{\partial h} \widetilde{G}(\mathbf{k}, h) \\ -\frac{\partial}{\partial h} \widetilde{G}(\mathbf{k}, h) & \widetilde{G}(\mathbf{k}, 0) \end{pmatrix}, \quad (\text{B.9})$$

where

$$D_0(\mathbf{k}) = \frac{1}{4} \left( -\frac{\partial^2}{\partial z^2} \widetilde{G}(\mathbf{k}, z)|_{z=0} \widetilde{G}(\mathbf{k}, 0) - \left( \frac{\partial}{\partial h} \widetilde{G}(\mathbf{k}, h) \right)^2 \right), \quad (\text{B.10})$$

and substitution of (A.1) and (A.2) in (B.10) yields

$$D_0(\mathbf{k}) = -\left( \frac{1}{2} \frac{k_B T}{C_{33}} \right)^2 \frac{1}{4} (1 + \exp(-2\lambda_0 k^2 h)). \quad (\text{B.11})$$

Substituting (B.4) and (B.11) in (B.7), we obtain

$$W(\mathbf{k}, \mathbf{q}) = \begin{pmatrix} 1 - a \frac{\partial}{\partial h} \widetilde{G}(\mathbf{k}, h) A(\mathbf{k}, \mathbf{q}) & -a \frac{\partial^2}{\partial z^2} \widetilde{G}(\mathbf{k}, z)|_{z=0} A(\mathbf{k}, \mathbf{q}) - a \frac{\partial}{\partial h} \widetilde{G}(\mathbf{k}, h) B(\mathbf{k}, \mathbf{q}) \\ a \widetilde{G}(\mathbf{k}, 0) A(\mathbf{k}, \mathbf{q}) & 1 - a \frac{\partial}{\partial h} \widetilde{G}(\mathbf{k}, h) A(\mathbf{k}, \mathbf{q}) + a \widetilde{G}(\mathbf{k}, 0) B(\mathbf{k}, \mathbf{q}) \end{pmatrix}, \quad (\text{B.12})$$

$$a = \frac{1}{4 D_0(\mathbf{k})},$$

where discrete representation of this matrix and its elements in wave vectors  $\mathbf{k}_i$  are given below.

In the discrete wave vector notation  $\mathbf{k}_i (\mathbf{k}_i = \mathbf{q}_i)$ , each block in the matrix  $\delta \widetilde{M}(\mathbf{k}, \mathbf{q})$  in (B.4) should also be understood as an infinite-dimensional matrix, namely:

$$\delta \widetilde{M}(\mathbf{k}, \mathbf{q}) = \frac{1}{2} \begin{pmatrix} 0 & 0 & \dots & A(\mathbf{k}_1, \mathbf{k}_1) & A(\mathbf{k}_1, \mathbf{q}_2) & \dots \\ 0 & 0 & \dots & A(\mathbf{k}_2, \mathbf{q}_1) & A(\mathbf{k}_2, \mathbf{k}_2) & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ A(\mathbf{q}_1, \mathbf{q}_1) & A(\mathbf{q}_1, \mathbf{k}_2) & \dots & B(\mathbf{k}_1, \mathbf{k}_1) & B(\mathbf{k}_1, \mathbf{q}_2) & \dots \\ A(\mathbf{q}_2, \mathbf{k}_1) & A(\mathbf{q}_2, \mathbf{q}_2) & \dots & B(\mathbf{k}_2, \mathbf{q}_1) & B(\mathbf{k}_2, \mathbf{k}_2) & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}. \quad (\text{B.13})$$

In the discrete wave vector notation  $\mathbf{k}_i$ , from (A.9) we obtain

$$\widetilde{M}_0^{-1}(\mathbf{k}) = \frac{1}{2 \prod_i D_0(\mathbf{k}_i)} \times \begin{pmatrix} -\frac{\partial^2}{\partial z^2} \widetilde{G}(\mathbf{k}_1, z)_{z=0} & 0 & \dots & -\frac{\partial}{\partial h} \widetilde{G}(\mathbf{k}_1, h) & 0 & \dots \\ 0 & -\frac{\partial^2}{\partial z^2} \widetilde{G}(\mathbf{k}_2, z)_{z=0} & \dots & 0 & -\frac{\partial}{\partial h} \widetilde{G}(\mathbf{k}_2, h) & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -\frac{\partial}{\partial h} \widetilde{G}(\mathbf{k}_1, h) & 0 & \dots & \widetilde{G}(\mathbf{k}_1, 0) & 0 & \dots \\ 0 & -\frac{\partial}{\partial h} \widetilde{G}(\mathbf{k}_2, h) & \dots & 0 & \widetilde{G}(\mathbf{k}_2, 0) & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}. \quad (\text{B.14})$$

After substitution of (B.13) and (B.14) in (B.7), we find in the discrete wave vector notation  $\mathbf{k}_i$  that each block of the matrix  $W(\mathbf{k}, \mathbf{q})$  is given by

$$W_{11}(\mathbf{k}, \mathbf{q}) = \begin{bmatrix} 1 - \frac{1}{4 D_0(\mathbf{k}_1)} \frac{\partial}{\partial h} \widetilde{G}(\mathbf{k}_1, h) A(\mathbf{k}_1, \mathbf{k}_1) & -\frac{1}{4 D_0(\mathbf{k}_1)} \frac{\partial}{\partial h} \widetilde{G}(\mathbf{k}_1, h) A(\mathbf{k}_1, \mathbf{q}_2) & \dots \\ -\frac{1}{4 D_0(\mathbf{k}_2)} \frac{\partial}{\partial h} \widetilde{G}(\mathbf{k}_2, h) A(\mathbf{k}_2, \mathbf{q}_1) & 1 - \frac{1}{4 D_0(\mathbf{k}_2)} \frac{\partial}{\partial h} \widetilde{G}(\mathbf{k}_2, h) A(\mathbf{k}_2, \mathbf{k}_2) & \dots \\ \vdots & \vdots & \vdots \end{bmatrix}, \quad (\text{B.15})$$

$$W_{12}(\mathbf{k}, \mathbf{q}) = \begin{bmatrix} -\frac{c_{11}}{4 D_0(\mathbf{k}_1)} & -\frac{c_{12}}{4 D_0(\mathbf{k}_1)} & \dots \\ -\frac{c_{21}}{4 D_0(\mathbf{k}_2)} & -\frac{c_{22}}{4 D_0(\mathbf{k}_2)} & \dots \\ \vdots & \vdots & \vdots \end{bmatrix}, \quad (\text{B.16})$$

$$c_{11} = \frac{\partial^2}{\partial z^2} \widetilde{G}(\mathbf{k}_1, z)_{z=0} A(\mathbf{k}_1, \mathbf{k}_1) + \frac{\partial}{\partial h} \widetilde{G}(\mathbf{k}_1, h) B(\mathbf{k}_1, \mathbf{k}_1),$$

$$c_{12} = \frac{\partial^2}{\partial z^2} \widetilde{G}(\mathbf{k}_1, z)_{z=0} A(\mathbf{k}_1, \mathbf{q}_2) + \frac{\partial}{\partial h} \widetilde{G}(\mathbf{k}_1, h) B(\mathbf{k}_1, \mathbf{q}_2),$$

$$c_{21} = \frac{\partial^2}{\partial z^2} \widetilde{G}(\mathbf{k}_2, z)_{z=0} A(\mathbf{k}_2, \mathbf{q}_1) + \frac{\partial}{\partial h} \widetilde{G}(\mathbf{k}_2, h) B(\mathbf{k}_2, \mathbf{q}_1),$$

$$c_{22} = \frac{\partial^2}{\partial z^2} \widetilde{G}(\mathbf{k}_2, z)_{z=0} A(\mathbf{k}_2, \mathbf{k}_2) + \frac{\partial}{\partial h} \widetilde{G}(\mathbf{k}_2, h) B(\mathbf{k}_2, \mathbf{k}_2),$$

$$W_{21}(\mathbf{k}, \mathbf{q}) = \begin{bmatrix} \frac{1}{4 D_0(\mathbf{k}_1)} \widetilde{G}(\mathbf{k}_1, 0) A(\mathbf{k}_1, \mathbf{k}_1) & \frac{1}{4 D_0(\mathbf{k}_1)} \widetilde{G}(\mathbf{k}_1, 0) A(\mathbf{k}_1, \mathbf{q}_2) & \dots \\ \frac{1}{4 D_0(\mathbf{k}_2)} \widetilde{G}(\mathbf{k}_2, 0) A(\mathbf{k}_2, \mathbf{q}_1) & \frac{1}{4 D_0(\mathbf{k}_2)} \widetilde{G}(\mathbf{k}_2, 0) A(\mathbf{k}_2, \mathbf{k}_2) & \dots \\ \vdots & \vdots & \vdots \end{bmatrix}, \quad (\text{B.17})$$

$$W_{22}(\mathbf{k}, \mathbf{q}) = \begin{bmatrix} 1 + \frac{s_{11}}{4D_0(\mathbf{k}_1)} & \frac{s_{12}}{4D_0(\mathbf{k}_1)} & \dots \\ \frac{s_{21}}{4D_0(\mathbf{k}_2)} & 1 + \frac{s_{22}}{4D_0(\mathbf{k}_2)} & \dots \\ \vdots & \vdots & \vdots \end{bmatrix}, \quad (\text{B.18})$$

$$s_{11} = -\frac{\partial}{\partial h} \tilde{G}(\mathbf{k}_1, h) A(\mathbf{k}_1, \mathbf{k}_1) + \tilde{G}(\mathbf{k}_1, 0) B(\mathbf{k}_1, \mathbf{k}_1),$$

$$s_{12} = -\frac{\partial}{\partial h} \tilde{G}(\mathbf{k}_1, h) A(\mathbf{k}_1, \mathbf{q}_2) + \tilde{G}(\mathbf{k}_1, 0) B(\mathbf{k}_1, \mathbf{q}_2),$$

$$s_{21} = -\frac{\partial}{\partial h} \tilde{G}(\mathbf{k}_2, h) A(\mathbf{k}_2, \mathbf{q}_1) + \tilde{G}(\mathbf{k}_2, 0) B(\mathbf{k}_2, \mathbf{q}_1),$$

$$s_{22} = -\frac{\partial}{\partial h} \tilde{G}(\mathbf{k}_2, h) A(\mathbf{k}_2, \mathbf{k}_2) + \tilde{G}(\mathbf{k}_2, 0) B(\mathbf{k}_2, \mathbf{k}_2).$$

For the calculations in what follows, we note that due to the condition  $\int d^2x \delta h(\mathbf{x}) = 0$ , the term linear in  $\delta h(\mathbf{x})$  disappears in  $A(\mathbf{k}, \mathbf{k})$ . That is essential for the chosen quadratic approximation. Using (B.12) and (B.15)–(B.18), we can also obtain the expansion of  $\text{Det}W$  through the second order in small fluctuation displacements of the IA interface  $\delta h(\mathbf{x})$ :

$$\begin{aligned} \text{Det} W &= \\ &= 1 + \sum_{\mathbf{k}} \left\{ \frac{\tilde{G}(\mathbf{k}, 0) B(\mathbf{k}, \mathbf{k})}{4D_0(\mathbf{k})} - 2 \frac{\frac{\partial}{\partial h} \tilde{G}(\mathbf{k}, h) A(\mathbf{k}, \mathbf{k})}{4D_0(\mathbf{k})} \right\} - \\ &- \sum_{\mathbf{k}, \mathbf{q} (\mathbf{q} \neq \mathbf{k})} \left\{ \frac{\frac{\partial}{\partial h} \tilde{G}(\mathbf{k}, h) A(\mathbf{k}, \mathbf{q})}{4D_0(\mathbf{k})} \frac{\frac{\partial}{\partial h} \tilde{G}(\mathbf{q}, h) A(\mathbf{q}, \mathbf{k})}{4D_0(\mathbf{q})} \right\} + \\ &+ \sum_{\mathbf{k}, \mathbf{q} (\mathbf{q} \neq \mathbf{k})} \left\{ \frac{\tilde{G}(\mathbf{k}, 0) A(\mathbf{k}, \mathbf{q})}{4D_0(\mathbf{k})} \times \right. \\ &\quad \left. \times \frac{\frac{\partial^2}{\partial z^2} \tilde{G}(\mathbf{q}, z)_{z=0} A(\mathbf{q}, \mathbf{k})}{4D_0(\mathbf{q})} \right\}. \quad (\text{B.19}) \end{aligned}$$

Substituting (B.19) in (B.8), expanding the logarithm in  $\delta h(\mathbf{x})$  through the second order, passing from summation over  $\mathbf{k}_i$  to integration, and using expressions (B.5) and (B.6), we obtain

$$\begin{aligned} H_{corr} &= \frac{k_B T}{2} \left[ \int \frac{d^2k}{(2\pi)^2} \iint d^2x d^2y \times \right. \\ &\times \left\{ -\frac{\tilde{G}(\mathbf{k}, 0)}{4D_0(\mathbf{k})} \exp(-i\mathbf{k} \cdot (\mathbf{y} - \mathbf{x})) \left( \frac{\partial^4}{\partial z^4} G(\mathbf{y} - \mathbf{x}, z) \right)_{z=0} \times \right. \\ &\quad \times \frac{(\delta h(\mathbf{y}) - \delta h(\mathbf{x}))^2}{2} - 2 \frac{\frac{\partial}{\partial h} \tilde{G}(\mathbf{k}, h)}{4D_0(\mathbf{k})} \times \\ &\quad \times \exp(-i\mathbf{k} \cdot (\mathbf{y} - \mathbf{x})) \frac{\partial^3}{\partial h^3} G(\mathbf{y} - \mathbf{x}, h) \frac{\delta h^2(\mathbf{y})}{2} \left. \right\} - \\ &- \int \frac{d^2k}{(2\pi)^2} \int \frac{d^2q}{(2\pi)^2} \left\{ \frac{\partial}{\partial h} \tilde{G}(\mathbf{k}, h) \frac{\partial}{\partial h} \tilde{G}(\mathbf{q}, h) - \right. \\ &\quad \left. - \tilde{G}(\mathbf{k}, 0) \left( \frac{\partial^2}{\partial z^2} \tilde{G}(\mathbf{q}, z) \right)_{z=0} \right\} \frac{1}{16D_0(\mathbf{k})D_0(\mathbf{q})} \times \\ &\times \iint d^2y d^2v_1 \exp(i\mathbf{q} \cdot \mathbf{y} - i\mathbf{k} \cdot \mathbf{v}_1) \frac{\partial^2}{\partial h^2} G(\mathbf{v}_1 - \mathbf{y}, h) \delta h(\mathbf{y}) \times \\ &\quad \times \iint d^2x d^2v_2 \exp(i\mathbf{k} \cdot \mathbf{x} - i\mathbf{q} \cdot \mathbf{v}_2) \times \\ &\quad \times \frac{\partial^2}{\partial h^2} G(\mathbf{v}_2 - \mathbf{x}, h) \delta h(\mathbf{x}) \left. \right]. \quad (\text{B.20}) \end{aligned}$$

After performing the integration over the relative variables  $\mathbf{v}_1 - \mathbf{y}$  and  $\mathbf{v}_2 - \mathbf{x}$  in the general expression for  $H_{corr}$  in (B.9) and using the identity

$$\delta h(\mathbf{x}) \delta h(\mathbf{y}) = \frac{1}{2} (\delta h^2(\mathbf{x}) + \delta h^2(\mathbf{y}) - (\delta h(\mathbf{y}) - \delta h(\mathbf{x}))^2),$$

it is possible to decompose  $H_{corr}$  into the local and nonlocal contributions discussed in the main text.

In particular, the nonlocal contribution to  $H_{corr}$  is given by

$$\begin{aligned} H_{corr}^{(nonloc)} &= \frac{k_B T}{2} \iint d^2x d^2y \frac{(\delta h(\mathbf{y}) - \delta h(\mathbf{x}))^2}{2} \times \\ &\times \left[ -\left( \frac{\partial^4}{\partial z^4} G(\mathbf{y} - \mathbf{x}, z) \right)_{z=0} \Phi_1(\mathbf{y} - \mathbf{x}) + \Phi_2^2(\mathbf{y} - \mathbf{x}) - \right. \\ &\quad \left. - \Phi_3(\mathbf{y} - \mathbf{x}) \Phi_4(\mathbf{y} - \mathbf{x}) \right], \quad (\text{B.21}) \end{aligned}$$

where

$$\begin{aligned}
\Phi_1(\mathbf{y} - \mathbf{x}) &= \int \frac{d^2q}{(2\pi)^2} \frac{\tilde{G}(\mathbf{q}, 0)}{4D_0(\mathbf{q})} \times \\
&\quad \times \exp(-i\mathbf{q} \cdot (\mathbf{y} - \mathbf{x})), \\
\Phi_2(\mathbf{y} - \mathbf{x}) &= \int \frac{d^2q}{(2\pi)^2} \frac{\partial}{\partial h} \tilde{G}(\mathbf{q}, h) \times \\
&\quad \times \frac{\partial^2}{\partial h^2} \tilde{G}(\mathbf{q}, h) \frac{\exp(i\mathbf{q} \cdot (\mathbf{y} - \mathbf{x}))}{4D_0(\mathbf{q})}, \\
\Phi_3(\mathbf{y} - \mathbf{x}) &= \int \frac{d^2q}{(2\pi)^2} \frac{\partial^2}{\partial h^2} \tilde{G}(\mathbf{q}, h) \times \\
&\quad \times \left( \frac{\partial^2}{\partial z^2} \tilde{G}(\mathbf{q}, z) \right)_{z=0} \frac{\exp(i\mathbf{q} \cdot (\mathbf{y} - \mathbf{x}))}{4D_0(\mathbf{q})}, \\
\Phi_4(\mathbf{y} - \mathbf{x}) &= \int \frac{d^2k}{(2\pi)^2} \tilde{G}(\mathbf{k}, 0) \frac{\partial^2}{\partial h^2} \tilde{G}(\mathbf{k}, h) \times \\
&\quad \times \frac{\exp(-i\mathbf{k} \cdot (\mathbf{y} - \mathbf{x}))}{4D_0(\mathbf{q})}.
\end{aligned} \tag{B.22}$$

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