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STRONG SUPPRESSION OF COULOMB CORRECTIONS TO THE e^+e^- PAIR PRODUCTION CROSS SECTION IN ULTRARELATIVISTIC NUCLEAR COLLISIONS

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The Coulomb corrections to the e^+e^- pair production cross section in ultrarelativistic nuclear collisions are calculated in the next-to-leading approximation with respect to the parameter $L = \ln \gamma_A \gamma_B$, where $\gamma_{A,B}$ are the Lorentz factors of colliding nuclei. We find considerable reduction of the Coulomb corrections even for large $\gamma_A \gamma_B$ due to the suppression of the e^+e^- pair production with the total energy of the order of a few electron masses in the rest frame of one of the nuclei. Our result explains why the deviation from the Born result was not observed in the SPS experiment [1, 2].

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Electron–positron pair production in ultrarelativistic nuclear collisions have been investigated intensively during almost two decades (see recent reviews [3, 4]). This process is important in the problem of beam lifetime and luminosity of hadron colliders. It is also a serious background for many experiments because of its large cross section. For heavy nuclei, the effect of higher-order terms (Coulomb corrections) of the perturbation theory with respect to the parameters $Z_A\alpha$ and $Z_B\alpha$ can be very important (Z_A and Z_B are the charge numbers of the nuclei A and B, and $\alpha \approx 1/137$ is the fine structure constant). However, no evidence of the Coulomb corrections has been found in the experiments in [1, 2]. This stimulated considerable theoretical interest in this process. In a series of theoretical works [5–7], it was found that the exact-in-$Z_{A,B}\alpha$ cross section coincides with that obtained in the Born approximation in the ultrarelativistic limit. This statement was regarded as an explanation of the experimental results [1, 2]. However, this conclusion contradicted the result obtained in Ref. [8] with the help of the Weizsäcker–Williams approximation in the leading logarithmic approximation. This contradiction was resolved in Ref. [9]. It was shown that the wrong conclusion in Refs. [5–7] regarding the absence of Coulomb corrections was due to the bad treatment of conditionally convergent integrals. The consistent approach in Ref. [9] results in the Coulomb corrections that coincide with those in Ref. [8]. Hence, the absence of the Coulomb corrections in the experiments in [1, 2] has remained unexplained.

In this paper, we calculate the Coulomb corrections to the e^+e^- pair production cross section in ultrarelativistic nuclear collisions in the next-to-leading approximation. We show that the account of the next-to-leading term leads to a strong suppression of the Coulomb corrections, which gives a natural explanation of the results obtained in the experiments [1, 2].

Because the nuclear mass is large compared to the electron mass, it is possible to treat the nuclei as sources of an external field and calculate the probability $P_n(b)$ of n-pair production at a fixed impact parameter b. It is convenient to introduce the average number $W(b)$ of produced pairs and the number-weighted cross section σ_T as

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$$W(b) = \sum_{n=1}^{\infty} n P_n(b),$$
$$\sigma_T = \int d^2 b\, W(b) = \sum_{n=1}^{\infty} n\sigma_n, \tag{1}$$

where

$$\sigma_n = \int d^2 b\, P_n(b)$$

is the n-pair production cross section. The cross section σ_T can be represented as

$$\sigma_T = \sigma^0 + \sigma^A + \sigma^B + \sigma^{AB}, \tag{2}$$

where

$$\sigma^0 \propto (Z_A\alpha)^2 (Z_B\alpha)^2$$

is the Born cross section, σ^A and σ^B are the Coulomb corrections with respect to nucleus A and B (containing the terms proportional to $(Z_B\alpha)^2(Z_A\alpha)^{2n}$ and $(Z_B\alpha)^{2n}(Z_A\alpha)^2$, $n \geqslant 2$), and σ^{AB} is the Coulomb correction with respect to both nuclei (containing the terms proportional to $(Z_B\alpha)^n(Z_A\alpha)^l$ with $n, l > 2$). The cross section σ^0 coincides with the Born cross section of one-pair production, which was calculated many years ago in Refs. [10, 11]. In the leading logarithmic approximation, the quantities $\sigma^{A,B} \propto L^2$ and $\sigma^{AB} \propto L$ were respectively obtained in Refs. [8, 9] and Ref. [12].

The leading logarithmic approximation for $W(b)$ provides the factorization of $P_n(b)$ [13–16], such that

$$P_n(b) = \frac{W^n(b)}{n!} e^{-W(b)}. \tag{3}$$

The function $W(b)$ was calculated in the Born approximation in Refs. [17–21] and with the Coulomb corrections taken into account in Refs. [22–25]. Using Eq. (3), the cross section σ_1 of one pair production can be represented as a sum of σ_T and the unitarity correction σ_{unit}

$$\sigma_1 = \sigma_T + \sigma_{unit},$$
$$\sigma_{unit} = -\int d^2 b\, W(b) \left(1 - e^{-W(b)}\right). \tag{4}$$

The existence of the unitarity correction was first recognized in Ref. [26] (see also review [3]). It was evaluated numerically in Refs. [20, 27]. The leading contribution to σ_1 is given by the term σ^0 in σ_T, Eq. (2), and is known with high accuracy [10, 11]. The terms σ^A and σ^B in σ_T also give important contributions to σ_1. In the leading logarithmic approximation, these terms were derived in Refs. [8, 9]. The last two contributions to σ_1, σ^{AB} and σ_{unit}, are rather small (see Refs. [12, 20]).

In this paper, we calculate the leading corrections to $\sigma^{A,B}$ (which are also the corrections to σ_1). We show that these corrections essentially diminish the magnitude of $\sigma^{A,B}$ even for the parameters of the LHC ($\gamma_A = \gamma_B \approx 3000$ and $Z_A = Z_B = 82$). It is convenient to calculate σ^A in the rest frame of the nucleus A, where the nucleus B has the Lorenz factor $\gamma = 2\gamma_A\gamma_B$ at $\gamma_{A,B} \gg 1$. We note that σ^A, being proportional to $(Z_B\alpha)^2$, can be directly calculated as the Coulomb corrections to σ_1 with respect to the parameter $Z_A\alpha$, and can therefore be represented as

$$\sigma^A = \int\limits_{2m}^{\infty} d\omega \int\limits_{(\omega/\gamma)^2}^{\infty} dQ^2 \left[\frac{dn_\perp(\omega, Q^2)}{d\omega\, dQ^2}\sigma_\perp(\omega, Q^2) + \right.$$
$$\left. + \frac{dn_\parallel(\omega, Q^2)}{d\omega\, dQ^2}\sigma_\parallel(\omega, Q^2)\right], \tag{5}$$

where

$$dn_\perp(\omega, Q^2) = \frac{Z_B^2\alpha}{\pi}\left(1 - \frac{(\omega/\gamma)^2}{Q^2}\right)\frac{d\omega}{\omega}\frac{dQ^2}{Q^2},$$
$$dn_\parallel(\omega, Q^2) = \frac{Z_B^2\alpha}{\pi}\frac{d\omega}{\omega}\frac{dQ^2}{Q^2} \tag{6}$$

are the numbers of virtual photons $\gamma^*_{\perp,\parallel}$ with the energy ω, the virtuality $-Q^2 < 0$, and the transverse and longitudinal polarizations. The quantities $\sigma_\perp(\omega, Q^2)$ and $\sigma_\parallel(\omega, Q^2)$ are the Coulomb corrections to the cross sections of the processes $\gamma^*_{\perp,\parallel} A \to e^+e^- A$.

We discuss the contributions to σ^A of different regions of the integration with respect to ω and Q^2.

The leading logarithmic contribution $\propto L^2$ comes from the integration of $\sigma_\perp$ over the region

$$\text{(I)} \quad m \ll \omega \ll m\gamma, \quad (\omega/\gamma)^2 \ll Q^2 \ll m^2. \tag{7}$$

The leading correction $\propto L$ comes from the following regions:

$$\text{(II)} \quad Q^2 \sim m^2, \quad m \ll \omega \ll \gamma m, \tag{8}$$
$$\text{(III)} \quad Q^2 \sim (\omega/\gamma)^2, \quad m \ll \omega \ll \gamma m, \tag{9}$$
$$\text{(IV)} \quad \omega \sim m, \quad (m/\gamma)^2 \ll Q^2 \ll m^2. \tag{10}$$

We note that the cross section $\sigma_\parallel$ gives a logarithmically enhanced contribution only in region II. Therefore, because we keep the terms proportional to L^2 or L, we can write σ^A as

$$\sigma^A = \sigma^A_{as} + \delta\sigma^A,$$
$$\sigma^A_{as} = \int\limits_{2m}^{\infty} d\omega \int\limits_{(\omega/\gamma)^2}^{\infty} dQ^2 \left[\frac{dn_\perp(\omega, Q^2)}{d\omega\, dQ^2}\sigma_\perp(\infty, Q^2) + \right.$$
$$\left. + \frac{dn_\parallel(\omega, Q^2)}{d\omega dQ^2}\sigma_\parallel(\infty, Q^2)\right], \tag{11}$$

$$\delta\sigma^A = \int\limits_{2m}^{\infty} d\omega \int\limits_{(\omega/\gamma)^2}^{\infty} dQ^2 \frac{dn_\perp(\omega, Q^2)}{d\omega\, dQ^2} \delta\sigma_\perp(\omega, Q^2), \quad (12)$$
$$\delta\sigma_\perp(\omega, Q^2) = \sigma_\perp(\omega, Q^2) - \sigma_\perp(\infty, Q^2).$$

The quantities $\sigma_{\perp,\parallel}(\infty, Q^2)$ can be calculated in the semiclassical approximation. Following the method described in detail in Ref. [28], we obtain

$$\sigma_{\perp,\parallel}(\infty, Q^2) = \frac{\alpha}{\omega} \operatorname{Re} \int d\varepsilon \operatorname{Sp} \int d\mathbf{r}_1 d\mathbf{r}_2 e^{-i\mathbf{k}\cdot\mathbf{r}} \times$$
$$\times \left[\left(2\mathbf{e}\cdot\mathbf{p}_2 + \hat{k}\hat{e}\right) D_-\right] \left[\left(2\mathbf{e}^*\cdot\mathbf{p}_1 - \hat{k}\hat{e}^*\right) D_+\right], \quad (13)$$
$$D_- = D(\mathbf{r}_2, \mathbf{r}_1|\varepsilon), \quad D_+ = D(\mathbf{r}_1, \mathbf{r}_2|\varepsilon - \omega),$$

where $D(\mathbf{r}_2, \mathbf{r}_1|\varepsilon)$ is the semiclassical Green's function of the squared Dirac equation, $e = (0,1,0,0)$ for $\sigma_\perp$ and $e = (0,0,0,Q/\omega)$ for $\sigma_\parallel$ in the frame where $\mathbf{k}$ is directed along the z axis. Using the explicit expressions for the Green's functions in Ref. [28], we obtain these cross sections as

$$\sigma_\perp(\infty, Q^2) =$$
$$= \alpha N \int\limits_0^1 dy \frac{1 + 2\,(1 - 2y\bar{y})\,(1 + y\bar{y}Q^2/m^2)}{(1 + y\bar{y}Q^2/m^2)^2},$$
$$\sigma_\parallel(\infty, Q^2) = 4\alpha N \int\limits_0^1 dy \frac{y^2\bar{y}^2 Q^2/m^2}{(1 + y\bar{y}Q^2/m^2)^2}, \quad (14)$$
$$N = -\frac{4(Z_A\alpha)^2}{3m^2} \operatorname{Re}\left[\psi(1 + iZ_A\alpha) - \psi(1)\right],$$
$$\bar{y} = 1 - y,$$

where
$$\psi(x) = \frac{d\ln\Gamma(x)}{dx}.$$

These formulas agree with the result in Ref. [29] if the missing factor $y\bar{y}$ in $\sigma_\parallel$ pointed out in Ref. [30] is taken into account. Substituting Eq. (14) in Eq. (11) and taking the integrals over ω and Q^2, we obtain

$$\sigma^A_{as} = \frac{7(Z_B\alpha)^2 N}{3\pi}\left[L^2 + \frac{20}{21}L\right] \quad (15)$$

in the logarithmic accuracy. We recall that

$$L = \ln(\gamma_A\gamma_B) = \ln(\gamma/2).$$

Result (15) is in agreement with those obtained in Refs. [30, 31].

We now pass to the contribution $\delta\sigma^A$, Eq. (12), which has not been considered previously. In Ref. [31], it was conjectured that the term $\delta\sigma^A$ can be safely omitted. We show below that this guess is completely wrong. The function $\delta\sigma_\perp(\omega, Q^2)$ in the integrand provides the convergence of the integral over ω in the region $\omega \sim m$. The logarithmically enhanced contribution is given by the region $(m/\gamma)^2 \ll Q^2 \ll m^2$ of the integration over Q^2. Because $Q^2 \ll m^2$ in this region, we can substitute $\delta\sigma_\perp(\omega, Q^2) \to \delta\sigma_\perp(\omega, 0)$ in Eq. (12). Then we take the integral over Q^2 and obtain

$$\delta\sigma^A = \frac{7(Z_B\alpha)^2 N\, G(Z_A\alpha)}{3\pi} L,$$
$$G(Z_A\alpha) = 2\int\limits_{2m}^{\infty} \frac{d\omega}{\omega}\left[\frac{\sigma_\perp(\omega, 0)}{\sigma_\perp(\infty, 0)} - 1\right]. \quad (16)$$

The quantity
$$\sigma_\perp(\omega, 0) \equiv \sigma_{\gamma A}(\omega)$$
is the Coulomb corrections to the e^+e^- pair production cross section by a real photon in the Coulomb field, and

$$\sigma_\perp(\infty, 0) = \frac{7\alpha N}{3}.$$

Taking the sum of Eqs. (15) and (16), we finally obtain σ^A in the next-to-leading approximation

$$\sigma^A = \frac{7(Z_B\alpha)^2 N}{3\pi}\left[L^2 + \left(G(Z_A\alpha) + \frac{20}{21}\right)L\right]. \quad (17)$$

To calculate the function $G(Z_A\alpha)$, it is necessary to know the magnitude of the Coulomb corrections $\sigma_{\gamma A}(\omega)$ in the energy region where the produced e^+e^- pair is not ultrarelativistic. The formal expression for it, exact in $Z_A\alpha$ and ω, was derived in Ref. [32]. This expression has a very complicated form causing severe difficulties in computations. The difficulties increase as ω increases, and the numerical results in Refs. [32, 33] were obtained only for $\omega < 5$ MeV. In a series of later publications [34–37] (see also reviews [38, 39]), the magnitude of $\sigma_{\gamma A}(\omega)$ has been obtained for higher values of ω and several Z_A. In the high-energy region $\omega \gg m$, the analysis is greatly simplified. As a result, a rather simple form of the Coulomb corrections was obtained in [40, 41] in the leading approximation with respect to m/ω and in [28] in the next-to-leading approximation. In Ref. [42], a simple formula that correctly reproduces the low-energy results and the high-energy limit was suggested. This "bridging" expression has high accuracy at intermediate energies and differs from the exact result for $\sigma_{\gamma A}(\omega)$ only in the region close to the threshold $\omega = 2m$. For our purpose, this difference is not important because the ratio $\sigma_{\gamma A}(\omega)/\sigma_{\gamma A}(\infty)$ can be neglected in comparison with unity in this region.

The function $G(Z\alpha)$ is shown in Fig. 1. It can be seen that $G(Z\alpha)$ varies slowly from -6.6 for $Z = 1$

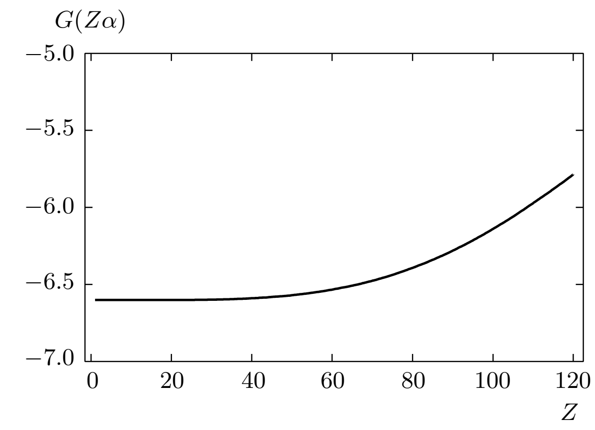

Fig. 1. The dependence of $G(Z\alpha)$ on Z

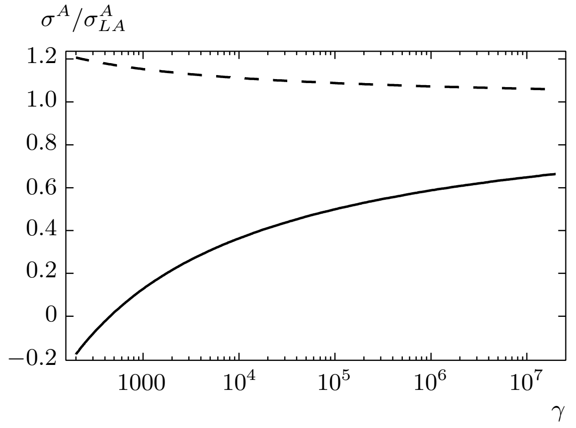

Fig. 2. The ratio σ^A/σ^A_{LA} (solid curve) as a function of γ for $Z_A = 82$. Here, $\sigma^A_{LA} = 7(Z_B\alpha)^2NL^2/3\pi$ is the Coulomb corrections calculated in the leading logarithmic approximation. The dashed curve shows the ratio $\sigma^A_{as}/\sigma^A_{LA}$

to -6.14 for $Z = 100$, being large for all interesting values of Z. A large value of G leads to a big difference between σ^A in Eq. (17) and its leading logarithmic approximation

$$\sigma^A_{LA} = \frac{7(Z_B\alpha)^2NL^2}{3\pi}$$

even for very large γ. This statement is illustrated in Fig. 2, where the ratio σ^A/σ^A_{LA} is shown as a function of γ (solid curve). If we omit the contribution $\delta\sigma^A$ and use σ^A_{as} in Eq. (15) as an approximation to σ^A, then the contribution of the term linear in L becomes much less important (see the dashed curve in Fig. 2). We note that for the Pb–Pb collisions at the LHC, $\gamma \approx 1.8\cdot 10^7$ and $\sigma^A/\sigma^A_{LA} \approx 0.66$. For Au–Au collisions at RHIC,

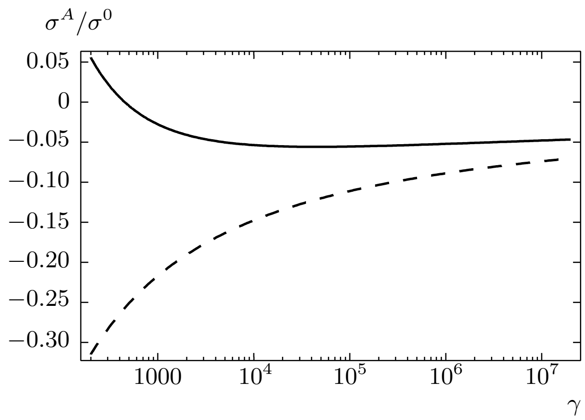

Fig. 3. The ratio σ^A/σ^0 (solid curve) as a function of γ for $Z_A = 82$. The dashed curve shows the ratio σ^A_{LA}/σ^0

$\gamma \approx 2.3\cdot 10^4$ and $\sigma^A/\sigma^A_{LA} \approx 0.42$. In the experiments at SPS [1, 2], the Lorentz factor was $\gamma \approx 200$. Naturally, we cannot use result (17) obtained in the logarithmic approximation in the region $\gamma \lesssim 500$, where the logarithmic correction to σ^A becomes larger than the leading term σ^A_{LA}. However, we can claim that due to the strong compensation between the leading term and the correction, the Coulomb corrections σ^A are much smaller than σ^A_{LA} for $\gamma \lesssim 500$. Therefore, this naturally explains why there was no evidence of the Coulomb corrections in the experiments [1, 2].

We now discuss the importance of the Coulomb corrections σ^A in comparison with the Born cross section σ^0. The ratio σ^A/σ^0 is shown in Fig. 3. In the next-to-leading approximation for σ^A, this ratio (solid curve) is small ($\lesssim 5\,\%$), while the same ratio obtained with σ^A approximated by σ^A_{LA} reaches 20 % at $\gamma \sim 1000$.

In Ref. [43], the Coulomb corrections were calculated using the light-front approach. The author claimed that the results in [43] include all next-to-leading terms ($\propto L$). However, the region where the e^+e^- pair has the energy of a few electron masses evidently cannot be correctly described by the light-front approach (see, e.g., the condition after Eq. (3) in Ref. [5]). Because our derivation shows that just this region gives the largest contribution to the next-to-leading term, the statement in [43] is incorrect.

To summarize, we have calculated the Coulomb corrections σ^A to the e^+e^- pair production in the next-to-leading logarithmic approximation. After the account of the next-to-leading term, the magnitude of σ^A becomes small in comparison with the Born cross section, in contrast to the leading term σ_{LA}^{A}. The large difference between our result and the previously suggested one has a simple explanation. The previous result was based on the use of the high-energy asymptotic form of the Coulomb corrections to the photoproduction cross section instead of the exact Coulomb corrections. But the exact Coulomb corrections are strongly suppressed in the rather wide region $2m < \omega \lesssim 20m$. We note that our results, combined with σ^{AB} in [12], complete the calculation of the linear-in-L terms in the number-weighted cross section σ_T.

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