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TIME-ODD CORRELATION IN A NEUTRON REFLECTOMETRY EXPERIMENT

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We find that neutron transmission of a magnetic systems with the noncollinear magnetization contains a time-odd correlation. The neutron reflection from such a system violates detailed balance. Time-odd correlation is shown to violate T-invariance even in the presence of an irreversibility produced by losses and discribed by imaginary part in the neutron–matter interaction.

1. INTRODUCTION

The principles of invariance of physical processes under discrete transformations such as spatial inversion (P-invariance), time reversal (T-invariance), charge conjugation (C-invariance), and their products like CP and CPT, and the principle of detailed balance, sometimes called "reciprocity", are now a standard topic of textbooks. All these principles are mainly considered with respect to elementary particles and elementary scattering processes, and a large field of research is devoted to the search of a violation of these principles.

The detailed balance, like unitarity and energy conservation, was never checked because the reign of these principles is unquestionable. The P- and T-invariance were questioned because of experimental observation of P- and T-odd correlations. For instance, a P-odd correlation, like $\mathbf{p}_e \cdot \mathbf{s}$, was observed in neutron β-decay. This means that the numbers of electrons (with momenta $\mathbf{p}_e$) emitting along and opposite the neutron spin $\mathbf{s}$ are different. This correlation is P-odd because the momentum $\mathbf{p}_e$ changes its sign under spatial inversion, while the axial vector $\mathbf{s}$ does not. Therefore, the product $\mathbf{p}_e \cdot \mathbf{s}$ also changes sign. At the same time, this correlation is T-even because both $\mathbf{p}_e$ and $\mathbf{s}$ change their signs under time reversal, and their product does not.

A T-invariance violation was initially observed in K-meson decays, and great efforts, still without success, are now focused on the search for the neutron electric dipole moment, which is equivalent to the search for the correlation $\mathbf{s} \cdot \mathbf{E}$, where $\mathbf{E}$ is an external electric field.

In this paper, we concentrate on neutron optics and show that T-odd correlations can easily be discovered there, and hence the question of whether these correlations are an evidence of the T-violation naturally arises.

In fact, the T-invariance violation can be expected in neutron optics. This is because neutron interaction with matter is described by an optical potential, which contains an imaginary part due to losses, and the imaginary part like friction is considered as indication of time irreversibility.

A Hamiltonian with an imaginary part is not Hermitian. Under time reversal, which is described by a unitary operator times the complex conjugation operator, the imaginary part changes its sign. Normally, the imaginary part of an optical neutron–matter potential is negative and describes neutron losses due to absorption. After time reversal, it becomes positive, which means the creation of neutrons. It is clear that these two Hamiltonians are fundamentally different. Nevertheless, as we prove in what follows, this does not mean a violation of the T-invariance.

The detailed balance is a special fundamental principle. When it is not satisfied, principle of maximal entropy at equilibrium is violated. The detailed balance is a litmus paper for judgement whether one scattering or transport model or another is correct. For instance, if we consider gas flowing along a tube and suppose that reflection of atoms from the tube walls proceeds according to some model indicatrix [1], then we must be care-

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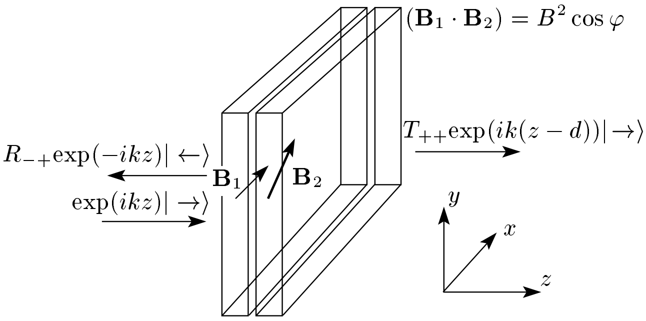

Fig. 1. Reflection and transmission of a magnetic mirror of thickness d, consisting of two films magnetized in the x, y coordinate plane, which is parallel to their interfaces. The external magnetic field is zero. Magnetic fields $\mathbf{B}_{1,2}$ of the films are at an angle φ to each other. The incident neutron going from the left and polarized along the normal to the films (the z axis) can be reflected, e.g., with a spin flip (R_{-+} is the reflection amplitude) or transmitted, e.g., without spin flip (T_{++} is the transmission amplitude)

ful about the model. We should guarantee that if we fill the tube with an isotropically distributed atomic gas, then reflection from the walls according to the model does not create fluxes in the tube.

In this paper, however, we show that the well-established laws of the interaction neutron–matter can lead to a violation of detailed balance. It looks as if some magnetic systems submerged into an isotropically distributed equilibrium neutron gas at a temperature T decrease its entropy by $-|\Delta S|$ and cool the gas by the amount $-T\Delta S$ without any work! However this cooling cannot be observed because submersion of the system into the neutron gas creates many new degrees of freedom, which increase the entropy by considerably larger amount.

Below, we consider reflection and transmission of a magnetic mirror shown in Fig. 1. The mirror of the total thickness d contains two magnetic layers. Their magnetizations are parallel to the interfaces (the x, y coordinate plane) and make an angle φ to each other. The outside field is supposed to be zero. We show that the neutron transmission matrix contains a term proportional to the time-odd correlation

$$\mathbf{s} \cdot [\mathbf{B}_1 \times \mathbf{B}_2]. \tag{1}$$

Observation of such a correlation can be interpreted as a T-invariance violation.

The problem of T-odd correlations in cross sections of polarized neutrons in the presence of corkscrew-like magnetic fields and fluctuations was first discussed in [2–5]. We show that the T-odd term in the transmission matrix does not imply a violation of the T-invariance even in presence of absorption.

In the next section, we show how correlation (1) appears in the transmission of the system shown in Fig. 1. In Sec. 3, reflection of the system in Fig. 1 is discussed. The reflection matrix is shown not to contain T-odd terms, but it contains terms that violate the detailed balance principle. In Sec. 4, we discuss a T-odd correlation appearing in the interaction of neutrons with a mirror having helicoidal magnetization, and show that the violation of the detailed balance principle in this case is very prominent. In Sec. 5, we discuss the concept of the T-invariance as applied to neutron reflectometry, and show why this invariance is not violated even if there is absorption in the system. In Sec. 6, we summarize our results.

2. NEUTRON TRANSMISSION OF THE TWO-LAYER MAGNETIC MIRROR AND DERIVATION OF Eq. (1)

The left-to-right transmission matrix $\hat{\boldsymbol{\tau}}_t$ for the system of two films depicted in Fig. 1 is representable as [6, 7]

$$\hat{\boldsymbol{\tau}}_t = \hat{\boldsymbol{\tau}}_2(\boldsymbol{\sigma}\cdot\mathbf{B}_2)[\hat{\mathbf{I}} - \hat{\boldsymbol{\rho}}_1(\boldsymbol{\sigma}\cdot\mathbf{B}_1)\hat{\boldsymbol{\rho}}_2(\boldsymbol{\sigma}\cdot\mathbf{B}_2)]^{-1} \times \times \hat{\boldsymbol{\tau}}_1(\boldsymbol{\sigma}\cdot\mathbf{B}_1), \tag{2}$$

where $\mathbf{B}_i$ are the magnetic fields inside the films, $\hat{\mathbf{I}}$ is the unit 2×2 matrix, and $\boldsymbol{\sigma}$ is the vector $(\sigma_x, \sigma_y, \sigma_z)$ of the Pauli matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{3}$$

Expression (2) is easily obtained if we imagine that the films are separated by an infinitesimal vacuum gap. The first right and the last left factors in (2) are transmissions of the separate films, and the middle factor accounts for multiple reflections between them.

The transmission matrix $\hat{\boldsymbol{\tau}}(\boldsymbol{\sigma}\cdot\mathbf{B})$ of a single film is

$$\hat{\boldsymbol{\tau}}(\boldsymbol{\sigma}\cdot\mathbf{B}) = \exp(i\hat{\mathbf{k}}'(\boldsymbol{\sigma}\cdot\mathbf{B})l) \times \times \frac{\hat{\mathbf{I}} - \hat{\mathbf{r}}^2(\boldsymbol{\sigma}\cdot\mathbf{B})}{\hat{\mathbf{I}} - \hat{\mathbf{r}}^2(\boldsymbol{\sigma}\cdot\mathbf{B})\exp(2i\hat{\mathbf{k}}'(\boldsymbol{\sigma}\cdot\mathbf{B})l)}, \tag{4}$$

where l is the film thickness,

$$\hat{\mathbf{k}}'(\boldsymbol{\sigma}\cdot\mathbf{B}) = \sqrt{(k^2-u)\hat{\mathbf{I}} - \boldsymbol{\sigma}\cdot\mathbf{B}}, \tag{5}$$

k is the wave number of the incident neutron, $u = u' - iu''$ is the optical potential of the film,

$$\hat{\mathbf{r}}(\boldsymbol{\sigma}\cdot\mathbf{B}) = \frac{\hat{\mathbf{k}} - \hat{\mathbf{k}}'(\boldsymbol{\sigma}\cdot\mathbf{B})}{\hat{\mathbf{k}} + \hat{\mathbf{k}}'(\boldsymbol{\sigma}\cdot\mathbf{B})} \tag{6}$$

is the reflection matrix of the interface between the vacuum and the film, and $\hat{\mathbf{k}} = k\hat{\mathbf{I}}$. The potential u is defined with the factor $2m/\hbar^2$, and the field $\mathbf{B}$ is defined with the factor $2\mu m/\hbar^2$ (m and μ are respectively the neutron mass and the absolute value of its magnetic moment).

The reflection matrix $\hat{\boldsymbol{\rho}}(\boldsymbol{\sigma}\cdot\mathbf{B})$ of the film is

$$\hat{\boldsymbol{\rho}}(\boldsymbol{\sigma}\cdot\mathbf{B}) = \hat{\mathbf{r}}(\boldsymbol{\sigma}\cdot\mathbf{B})\frac{\hat{\mathbf{I}} - \exp(2i\hat{\mathbf{k}}'(\boldsymbol{\sigma}\cdot\mathbf{B})l)}{\hat{\mathbf{I}} - \hat{\mathbf{r}}^2(\boldsymbol{\sigma}\cdot\mathbf{B})\exp(2i\hat{\mathbf{k}}'(\boldsymbol{\sigma}\cdot\mathbf{B})l)}. \tag{7}$$

We see that (4) and (7) transform into each other under the exchange

$$\hat{\mathbf{r}}(\boldsymbol{\sigma}\cdot\mathbf{B}) \leftrightarrow \exp(i\hat{\mathbf{k}}'(\boldsymbol{\sigma}\cdot\mathbf{B})l).$$

An arbitrary function $\hat{\mathbf{f}}(\boldsymbol{\sigma}\cdot\mathbf{B})$ of the matrix $(\boldsymbol{\sigma}\cdot\mathbf{B})$ is also a matrix, which is representable in the form

$$\hat{\mathbf{f}}(\boldsymbol{\sigma}\cdot\mathbf{B}) = \hat{\mathbf{I}}f^{(+)}(B) + \boldsymbol{\sigma}\cdot\mathbf{b}f^{(-)}(B), \tag{8}$$

where

$$f^{(\pm)}(B) = \frac{f(B) \pm f(-B)}{2}, \quad \mathbf{b} = \frac{\mathbf{B}}{B}. \tag{9}$$

Relation (8) is easily derived by expanding the function into the Taylor (or MacLaurent) series in power of $(\boldsymbol{\sigma}\cdot\mathbf{B})^n$ and taking into account that $(\boldsymbol{\sigma}\cdot\mathbf{B})^2 = \hat{\mathbf{I}}B^2$. Therefore, all even powers $(\boldsymbol{\sigma}\cdot\mathbf{B})^{2n}$ are scalars B^{2n}, they constitute $f^{(+)}(B)$, and all odd powers

$$(\boldsymbol{\sigma}\cdot\mathbf{B})^{2n-1} = (\boldsymbol{\sigma}\cdot\mathbf{B})B^{2n} = (\boldsymbol{\sigma}\cdot\mathbf{b})B^{2n+1}$$

constitute $\boldsymbol{\sigma}\cdot\mathbf{b}f^{(-)}(B)$.

With (8), we can easily find that

$$\hat{\mathbf{f}}(\boldsymbol{\sigma}\cdot\mathbf{B})\hat{\mathbf{f}}(-\boldsymbol{\sigma}\cdot\mathbf{B}) = f(B)f(-B),$$

and therefore

$$\frac{1}{\hat{\mathbf{I}} - \hat{\mathbf{f}}(\boldsymbol{\sigma}\cdot\mathbf{B})} = \frac{1}{N}[\hat{\mathbf{I}} - \hat{\mathbf{f}}(-\boldsymbol{\sigma}\cdot\mathbf{B})], \quad N = [1 - f(B)][1 - f(-B)]. \tag{10}$$

For two arbitrary functions $\hat{\mathbf{f}}(\boldsymbol{\sigma}\cdot\mathbf{A})$ and $\hat{\mathbf{g}}(\boldsymbol{\sigma}\cdot\mathbf{B})$ and arbitrary vectors $\mathbf{A}$ and $\mathbf{B}$, the relation

$$[\hat{\mathbf{I}} - \hat{\mathbf{f}}(\boldsymbol{\sigma}\cdot\mathbf{A})\hat{\mathbf{g}}(\boldsymbol{\sigma}\cdot\mathbf{B})]^{-1} = \frac{1}{N}[\hat{\mathbf{I}} - \hat{\mathbf{g}}(-\boldsymbol{\sigma}\cdot\mathbf{B})\hat{\mathbf{f}}(-\boldsymbol{\sigma}\cdot\mathbf{A})] \tag{11}$$

holds, where

$$N\hat{\mathbf{I}} = [\hat{\mathbf{I}} - \hat{\mathbf{f}}(\boldsymbol{\sigma}\cdot\mathbf{A})\hat{\mathbf{g}}(\boldsymbol{\sigma}\cdot\mathbf{B})][\hat{\mathbf{I}} - \hat{\mathbf{g}}(-\boldsymbol{\sigma}\cdot\mathbf{B})\hat{\mathbf{f}}(-\boldsymbol{\sigma}\cdot\mathbf{A})],$$

and hence

$$N = 1 - 2[f^{(+)}(A)g^{(+)}(B) + f^{(-)}(A)g^{(-)}(B)(\mathbf{a}\cdot\mathbf{b})] + f(A)f(-A)g(B)g(-B). \tag{12}$$

Here,

$$\mathbf{a} = \frac{\mathbf{A}}{|\mathbf{A}|}, \quad \mathbf{b} = \frac{\mathbf{B}}{|\mathbf{b}|},$$

and we use the well-known relation

$$(\boldsymbol{\sigma}\cdot\mathbf{a})(\boldsymbol{\sigma}\cdot\mathbf{b}) = (\mathbf{a}\cdot\mathbf{b}) + i(\boldsymbol{\sigma}\cdot[\mathbf{a}\times\mathbf{b}]). \tag{13}$$

With account of (11), the matrix of the total transmission amplitude $\hat{\boldsymbol{\tau}}_t$ in (2) takes the form

$$\hat{\boldsymbol{\tau}}_t = \frac{1}{N}\hat{\boldsymbol{\tau}}_2(\boldsymbol{\sigma}\cdot\mathbf{B}_2)\left[\hat{\mathbf{I}} - \hat{\boldsymbol{\rho}}_2(-\boldsymbol{\sigma}\cdot\mathbf{B}_2)\hat{\boldsymbol{\rho}}_1(-\boldsymbol{\sigma}\cdot\mathbf{B}_1)\right] \times \hat{\boldsymbol{\tau}}_1(\boldsymbol{\sigma}\cdot\mathbf{B}_1). \tag{14}$$

Therefore, it has the structure

$$\hat{\boldsymbol{\tau}}_t = \frac{1}{N}\left[\hat{\boldsymbol{\tau}}_2(\boldsymbol{\sigma}\cdot\mathbf{B}_2)\hat{\boldsymbol{\tau}}_1(\boldsymbol{\sigma}\cdot\mathbf{B}_1) - \hat{\mathbf{F}}_2(\boldsymbol{\sigma}\cdot\mathbf{B}_2)\hat{\mathbf{F}}_1(\boldsymbol{\sigma}\cdot\mathbf{B}_1)\right] \equiv \frac{1}{N}[\hat{\boldsymbol{\tau}}_{t1} - \hat{\boldsymbol{\tau}}_{t2}], \tag{15}$$

where we set

$$\begin{aligned}\hat{\mathbf{F}}_2(\boldsymbol{\sigma}\cdot\mathbf{B}_2) &= \hat{\boldsymbol{\tau}}_2(\boldsymbol{\sigma}\cdot\mathbf{B}_2)\hat{\boldsymbol{\rho}}_2(-\boldsymbol{\sigma}\cdot\mathbf{B}_2),\\ \hat{\mathbf{F}}_1(\boldsymbol{\sigma}\cdot\mathbf{B}_1) &= \hat{\boldsymbol{\rho}}_1(\boldsymbol{\sigma}\cdot\mathbf{B}_1)\hat{\boldsymbol{\tau}}_1(-\boldsymbol{\sigma}\cdot\mathbf{B}_1).\end{aligned} \tag{16}$$

The first term in (15) with (8) and (13) taken into account is

$$\begin{aligned}\hat{\boldsymbol{\tau}}_{t1} &= \hat{\boldsymbol{\tau}}_2(\boldsymbol{\sigma}\cdot\mathbf{B}_2)\hat{\boldsymbol{\tau}}_1(\boldsymbol{\sigma}\cdot\mathbf{B}_1) = \\ &= \left[\tau_1^{(+)}\tau_2^{(+)} + \tau_1^{(-)}\tau_2^{(-)}(\mathbf{b}_1\cdot\mathbf{b}_2)\right]\hat{\mathbf{I}} + \\ &+ \tau_1^{(-)}\tau_2^{(+)}\boldsymbol{\sigma}\cdot\mathbf{b}_1 + \tau_2^{(-)}\tau_1^{(+)}\boldsymbol{\sigma}\cdot\mathbf{b}_2 + \\ &+ i\tau_1^{(-)}\tau_2^{(-)}(\boldsymbol{\sigma}\cdot[\mathbf{b}_2\times\mathbf{b}_1]),\end{aligned} \tag{17}$$

where the last term contains correlation (1), and the second term $\hat{\boldsymbol{\tau}}_{t2}$ in (15) has the same structure with the replacement of $\hat{\boldsymbol{\tau}}$ by $\hat{\mathbf{F}}$.

In what follows, we take $|\mathbf{B}_1| = |\mathbf{B}_2| = B$ for simplicity. If we choose the x axis along $\mathbf{B}_1$, then

$$\boldsymbol{\sigma}\mathbf{B}_1 = \sigma_x B, \quad \boldsymbol{\sigma}\mathbf{B}_2 = \sigma_x B\exp(i\varphi\sigma_z),$$

and Eq. (2) with account of (17) becomes

$$\hat{\boldsymbol{\tau}}_t = C_0\hat{\mathbf{I}} + C_x\sigma_x[\hat{\mathbf{I}} + \alpha\exp(i\varphi\sigma_z)] + iC_z\sigma_z, \tag{18}$$

where all the C and α are some scalar functions of k. It follows from this expression that if the quantization axis for incident neutrons is chosen along z, then the transmission probability without a spin flip is

$$T(\pm \to \pm) = |\langle \pm|\hat{\boldsymbol{\tau}}_t|\pm\rangle|^2 = = |C_0|^2 + |C_z|^2 \pm 2\,\mathrm{Im}[C_0 C_z^*], \quad (19)$$

where $\mathrm{Im}[x]$, and later $\mathrm{Re}[x]$, denote imaginary and real parts of x, and the $|\pm\rangle$ states are

$$|+\rangle = \begin{pmatrix}1\\0\end{pmatrix}, \quad |-\rangle = \begin{pmatrix}0\\1\end{pmatrix}. \quad (20)$$

If we know nothing about the magnetic structure of the mirror, then the difference of transmissions for different spin directions along the normal, which coincides with the direction of the neutron wave vector $\mathbf{k}$, can be interpreted as a P-odd correlation $\mathbf{k}\cdot\mathbf{s}$. In our case, we know the magnetic structure, and hence we see that the difference of transmissions is due to not a P-odd but a T-odd correlation $(\mathbf{s}\cdot[\mathbf{b}_1 \times \mathbf{b}_2])$, and the question arises as to whether this correlation is a manifestation of the T-irreversibility of the Schrödinger equation for such a magnetic fields configuration.

In fact it does not violate T-invariance, because T-inversion includes exchange of initial and final states. Therefore transmission from left to right after T-inversion is replaced by transmission in opposite direction, which leads to transposition of magnetic fields and additional change of sign of the product $[\mathbf{b}_1 \times \mathbf{b}_2]$. Moreover, as will be shown later, the T-invariance is not violated even in presence of losses in matter.

It is important to notice that transmission without spin flip depends on angle φ, and the difference $T(+ \to +) - T(- \to -) \propto \sin\varphi$ changes sign when $\varphi \to -\varphi$. Therefore left rotation in nature is distinguished from right rotation, which can be considered as violation of space parity. However, space parity is also not violated. Distinguishing of two rotations occurs because of dynamics. Interaction of neutron with magnetic field $|\mu|\boldsymbol{\sigma}\cdot\mathbf{B}$ leads to counterclockwise precession of the neutron spin around the field. Because of that only counterclockwise rotating radio frequency field turns the neutron spin in spin flipper. Therefore it is not surprising that counterclockwise turn of the magnetic field in the second film acts differently than clockwise turn. By the way this fact gives an opportunity to communicate to a distant galaxy what do we mean clockwise and counterclockwise rotation. Even if the distant galaxy is composed of antimatter, the observers at the distant galaxy can use the neutron (or antineutron) experiment to see neutron precession in their magnetic field and direction of this precession will be exactly counterclockwise.

Transmissions with spin flip of the system depicted in Fig. 1,

$$T(\pm \to \mp) = |\langle \mp|\hat{\boldsymbol{\tau}}_t|\pm\rangle|^2 = = |C_x|^2\Big(1 + |\alpha|^2 + 2\,\mathrm{Re}[\alpha\exp(\pm i\varphi)]\Big), \quad (21)$$

can also be different. However, if the two films in the mirror are identical and differ only by the magnetization direction, then $\alpha = 1$, and the spin-flip transmissions are identical.

3. NEUTRON REFLECTION FROM THE TWO LAYER MAGNETIC MIRROR

Here we prove that the reflectivity of the mirror shown in Fig. 1 does not contain T-odd terms, but violates the detailed balance principle.

The amplitude of the reflection from the left is represented by the expression

$$\hat{\boldsymbol{\rho}}_t = \hat{\boldsymbol{\rho}}_1(\boldsymbol{\sigma}\cdot\mathbf{B}_1) + \hat{\boldsymbol{\tau}}_1(\boldsymbol{\sigma}\cdot\mathbf{B}_1)\hat{\boldsymbol{\rho}}_2(\boldsymbol{\sigma}\cdot\mathbf{B}_2)\times \times[I - \hat{\boldsymbol{\rho}}_1(\boldsymbol{\sigma}\cdot\mathbf{B}_1)\hat{\boldsymbol{\rho}}_2(\boldsymbol{\sigma}\cdot\mathbf{B}_2)]^{-1}\hat{\boldsymbol{\tau}}_1(\boldsymbol{\sigma}\cdot\mathbf{B}_1), \quad (22)$$

where (11) can be used to write

$$\hat{\boldsymbol{\rho}}_t = \hat{\boldsymbol{\rho}}_1(\boldsymbol{\sigma}\cdot\mathbf{B}_1) + \frac{1}{N}[\hat{\boldsymbol{\tau}}_1(\boldsymbol{\sigma}\cdot\mathbf{B}_1)\hat{\boldsymbol{\rho}}_2(\boldsymbol{\sigma}\cdot\mathbf{B}_2)\times \times[1 - \hat{\boldsymbol{\rho}}_2(-\boldsymbol{\sigma}\cdot\mathbf{B}_2)\hat{\boldsymbol{\rho}}_1(-\boldsymbol{\sigma}\cdot\mathbf{B}_1)]\hat{\boldsymbol{\tau}}_1(\boldsymbol{\sigma}\cdot\mathbf{B}_1)], \quad (23)$$

or

$$\hat{\boldsymbol{\rho}}_t = \hat{\boldsymbol{\rho}}_1(\boldsymbol{\sigma}\cdot\mathbf{B}_1) - \frac{\rho_2(B_2)\rho_2(-B_2)}{N}\times \times\hat{\boldsymbol{\tau}}_1^2(\boldsymbol{\sigma}\cdot\mathbf{B}_1)\hat{\boldsymbol{\rho}}_1(-\boldsymbol{\sigma}\cdot\mathbf{B}_1) + + \frac{1}{N}\hat{\boldsymbol{\tau}}_1(\boldsymbol{\sigma}\cdot\mathbf{B}_1)\hat{\boldsymbol{\rho}}_2(\boldsymbol{\sigma}\cdot\mathbf{B}_2)\hat{\boldsymbol{\tau}}_1(\boldsymbol{\sigma}\cdot\mathbf{B}_1). \quad (24)$$

Using (8), this expression is reduced to the form

$$\hat{\boldsymbol{\rho}}_t = D_0\hat{\mathbf{I}} + D_1(\boldsymbol{\sigma}\cdot\mathbf{b}_1) + D_2(\boldsymbol{\sigma}\cdot\mathbf{b}_2) = = D_0\hat{\mathbf{I}} + D_1\sigma_x(1 + \beta\exp(i\varphi\sigma_z)], \quad (25)$$

where all the D and $\beta = D_2/D_1$ are some scalar functions of k. It follows from this equation that the no-spin-flip reflectivities for both incident polarizations are equal to each other,

$$R(+ \to +) = R(- \to -) = |\langle \pm|\hat{\boldsymbol{\rho}}_t|\pm\rangle|^2 = |D_0|^2, \quad (26)$$

while the spin-flip reflectivities $R(\pm \to \mp)$ are different,

$$R(\pm \to \mp) = |\langle \mp | \hat{\boldsymbol{\rho}}_t | \pm \rangle|^2 = \\ = |D_1|^2 \Big(1 + |\beta|^2 + 2\,\mathrm{Re}[\beta \exp(\pm i\varphi)]\Big), \quad (27)$$

which means that

$$R(+ \to -) \neq R(- \to +),$$

and their dependence on the angle φ is such that

$$R(\pm \to \mp, \varphi) = R(\mp \to \pm, -\varphi). \quad (28)$$

.

The reflectivity does not contain T-odd correlations, but violates the detailed balance principle. Of course, it is not evident from (27) that the detailed balance is violated. It is not violated if β is a real number. But numerical calculations presented in Fig. 2 demonstrate that the two spin-flip reflectivities are actually different.

In Fig. 2, we show the results of numerical calculation of reflectivities and transmissivities of the mirror in Fig. 1 consisting of two identical Co layers of the thickness 25 nm magnetized to $B = 1$ T, when angle between their magnetizations is $\varphi = \pm\pi/2$. For identical layers, the factor α in (21) is equal to unity in accordance with (17). Therefore, the probabilities of transmission with a spin flip are identical and their dependence on φ is proportional to $\cos^2\varphi$.

The difference of the two spin-flip reflectivities implies violation of the detailed balance principle, because it creates a cycle current in phase space, which diminishes the entropy. We discuss this effect in the next section, where the violation of the detailed balance is seen more clearly.

4. NEUTRON REFLECTION AND TRANSMISSION OF A MAGNETIC MIRROR WITH HELICOIDAL MAGNETIZATION

The false effect of the time and parity violation is seen especially well in the case of a neutron reflection from a magnetic mirror magnetized helicoidally [8] around a vector $\mathbf{q}$ that is directed along the z axis parallel to the normal to the mirror interface. The neutron wave function in a helicoidal field was found in [9], and the reflection and transmission of helicoidal mirrors were calculated in [8, 10]. In Fig. 3, we show the reflectivities with and without a spin flip for polarizations of the incident neutron along and opposite the z axis. Outside the mirror, the magnetic field is absent. The analogous transmission probabilities are shown in Fig. 4. We see that there is again a time-odd correlation $\boldsymbol{\sigma}\cdot\mathbf{q}$, which can also be interpreted as a parity-odd correlation of spin with the incident neutron momentum $\boldsymbol{\sigma}\cdot\hat{\mathbf{k}}$. The resonant spin-flip reflectivity for the $|-\rangle$ polarization increases with the mirror thickness, and becomes almost total. Such a reflectivity violates the detailed balance principle, and the violation is especially well seen in this example. Indeed, we imagine that a vessel with ideal walls is homogeneously filled with a gas of unpolarized neutrons. If we split the vessel into two parts as shown in Fig. 5, inserting a helicoidal mirror, then all the neutrons from the left part I go through the mirror to the right part II, and become completely polarized along the z axis. Indeed, the neutrons in the $|+\rangle$ state go directly through the mirror, and cannot return, while the neutrons in the $|-\rangle$ state are reflected from the mirror with a spin flip, and after the reflection from ideal walls of the vessel, go again to the mirror and through it to part II. Therefore, it looks as if all the neutrons from part I gathered in part II in the single state $|+\rangle$, which strongly decreases the entropy.

However, such a spilling over the mirror from left to right is actually compensated by the opposite flux from II to I, because the neutrons in the $|-\rangle$ state in the right part can go through the mirror, while neutrons in the $|+\rangle$ state are reflected from the mirror with a spin flip to the $|-\rangle$ state. Therefore, the neutrons in both parts remain isotropic and in the unpolarized state, although a cycle occurs in the phase space. This cycle has 6 steps listed below, where the last 7th step is the same as the first one.

1. The neutron in the $|+\rangle$ state goes through the mirror (M) from the left to the right part of the vessel.
2. Then it reflects from the right wall of the vessel and goes to M.
3. It reflects from M to the right with a spin flip to the state $|-\rangle$.
4. Then it reflects from the right wall of the vessel and, being in the $|-\rangle$ state, goes through M from the right to the left part of the vessel.
5. Then it reflects from the left wall of the vessel and goes back to M.
6. It reflects from M to the left with a spin flip to the $|+\rangle$ state.
7. Then it reflects from the left wall of the vessel and, being in the $|+\rangle$ state, goes through M.

If there were the same cycle but with the initial state $|-\rangle$, and a neutron could go through both cycles with the same probability, then we could say that both cycles were equally well populated, and the entropy of

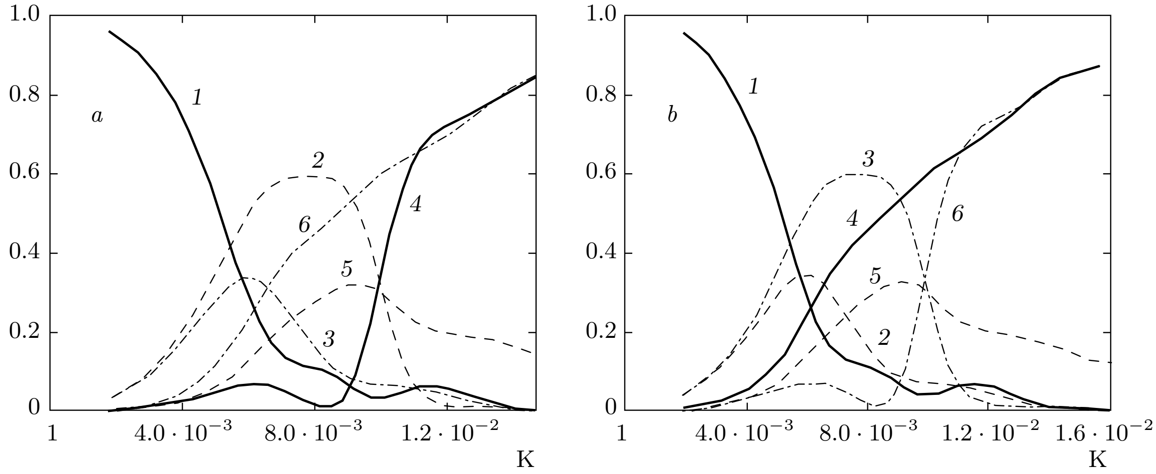

Fig. 2. The calculated reflection $R(i \to j) = |\langle j|\hat{\boldsymbol{\rho}}_t|i\rangle|^2$ and transmission $T(i \to j) = |\langle j|\hat{\boldsymbol{\tau}}_t|i\rangle|^2$, $(i, j = \pm)$, coefficients of two-layer mirror of the same thickness when the angle φ between magnetizations is $\pi/2$ (a), and $-\pi/2$ (b). The curves correspond to *1* — $R(+ \to +) = R(- \to -)$, *2* — $R(- \to +)$, *3* — $R(+ \to -)$, *4* — $T(+ \to +)$, *5* — $T(+ \to -) = T(- \to +)$, *6* — $T(- \to -)$. The change of the sign of φ leads to the exchange $R(\pm \to \mp) \to R(\mp \to \pm)$ of spin-flip curves in reflectivities and of no-spin-flip $T(\pm \to \pm) \to T(\mp \to \mp)$ curves in transmissivities

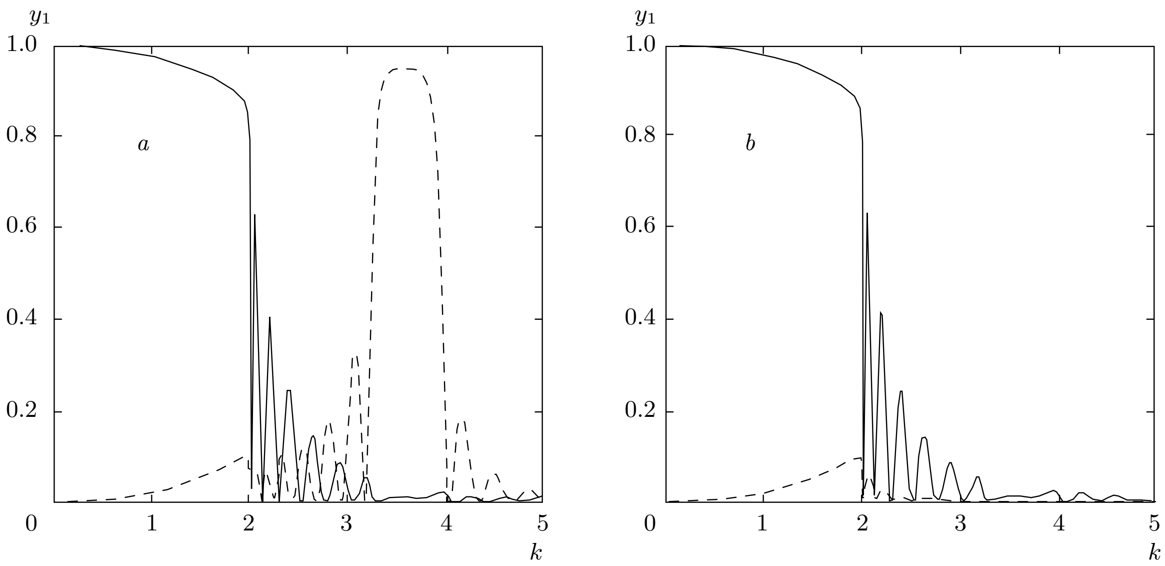

Fig. 3. Reflectivity of a helicoidal magnetic mirror with (dashed line) and without (solid line) spin flip for the incident polarization opposite (a) and along (b) the z axis [4]. We see that above the total reflection range, there is a well-pronounced peak of almost total reflection with a spin flip, when the incident neutron is polarized against the z axis

the isotropically distributed unpolarized neutron gas would be maximal. Because only one cycle is populated in our case and the other is not, the entropy of the neutron gas is not maximal. If the helicoid mirror is thin, and the reflection with a spin flip is not total, then the opposite cycle exists, but the population of two cycles is not equal, and therefore the entropy is not maximal again. The same happens with the mirror shown in Fig. 1. In that case, the population of two cycles and the value of the entropy depend on the angle φ between the two magnetizations.

A decrease in the entropy in both cases is created by the vector, around which the magnetic field turns. Such a turn violates the space balance, and therefore the equilibrium state of the neutron gas does not have the maximal entropy.

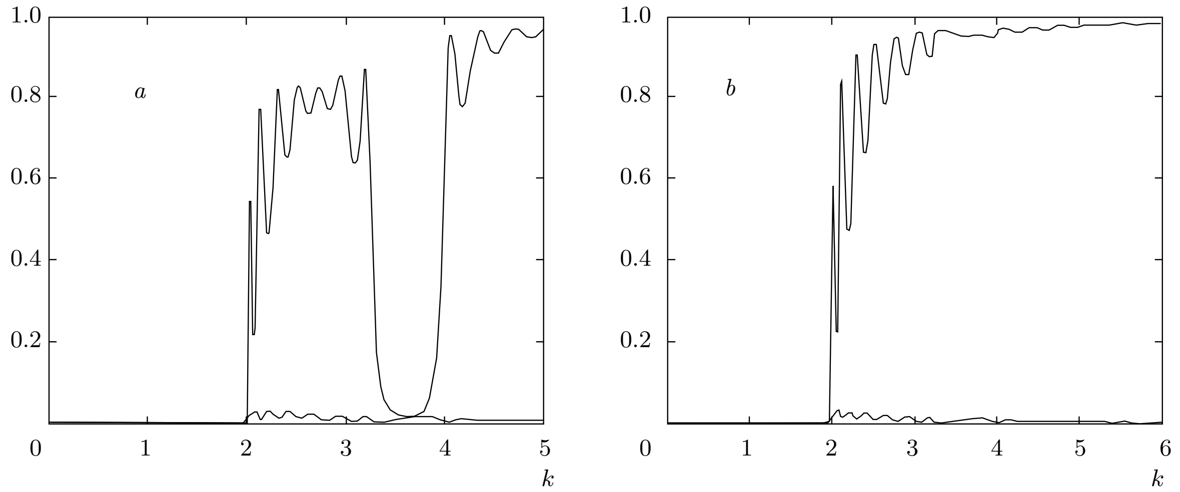

Fig. 4. Transmission probabilities of a helicoidal magnetic mirror with (dashed line) and without (solid line) spin flip for the incident polarization opposite (left) and along (right) the z axis [4]. We see that above the total reflection range, there is a well-pronounced dip in transmission of neutrons initially polarized against the z axis

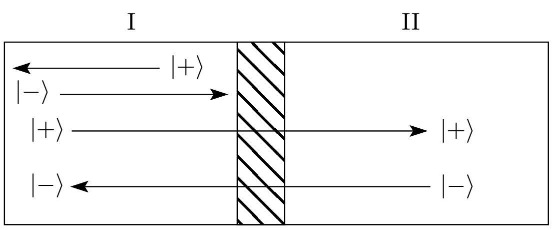

Fig. 5. Illustration of violation of the detailed balance principle

At the end of this section, we mention the paper [11] (see also [4]), where diffraction on crystals with helicoidal magnetization was considered. It was found that diffraction contains peaks, which in the perturbation theory are proportional to

$$I \propto \delta(\boldsymbol{\kappa} - \boldsymbol{\kappa}_B \pm \mathbf{q}), \tag{29}$$

where $\boldsymbol{\kappa} = \mathbf{k}_i - \mathbf{k}_f$ is the momentum transferred to the crystal by the neutron with the initial and final momenta $\mathbf{k}_{i,f}$, $\boldsymbol{\kappa}_B$ is a vector of the reciprocal lattice of the crystal, and $\mathbf{q}$ is the helicoidal vector similar to ours in helicoidal mirrors. In (29), we see some interesting discrepancy. The argument of the delta function contains a combination of polar vectors $\boldsymbol{\kappa}$ and $\boldsymbol{\kappa}_B$ and of the axial vector $\mathbf{q}$. This is inconsistent. The additional peak appearing because of the helical magnetization is related not to the crystal structure but to the width of the Bragg peak. If the energy width of the peak is Δu, then the position of the helical magnetic peak is shifted from the Bragg position κ_B of the nonmagnetic crystal by $\Delta\kappa = \sqrt{\Delta u + \mathbf{q}^2}$.

But this discrepancy does not devaluate the importance of [11]. Its main value is in the proof that additional magnetic peaks exist near Brag peaks, resulting from scattering with one-directional spin flip. In that respect, reflection from single crystals with helical magnetization does also violate the detailed balance principle.

5. ANALYSIS OF THE TIME INVARIANCE

Here, we first discuss the question whether the T-odd term in (1) actually manifests a T-invariance violation. Next, we analyze the principle of T-invariance in the case of the neutron scattering on a nonmagnetic system described by an optical potential with an imaginary part.

5.1. T-invariance with term (1)

The left-to-right transmission probability without spin flip can be represented by the function

$$\overrightarrow{W} = Q_0 + (\mathbf{s} \cdot [\mathbf{b}_1 \times \mathbf{b}_2])Q_1, \tag{30}$$

where $Q_{0,1}$ are some scalar functions of k, and $\mathbf{s}$ is a unit vector directed along the incident neutron polarization; it can be either parallel or antiparallel to $[\mathbf{b}_1 \times \mathbf{b}_2]$. Hence,

$$\overrightarrow{W(\pm)} = Q_0 \pm |[\mathbf{b}_1 \times \mathbf{b}_2]|Q_1. \tag{31}$$

The right-to-left transmission is

$$\overleftarrow{W} = Q_0 + (\mathbf{s}\cdot[\mathbf{b}_2\times\mathbf{b}_1])Q_1 = \\ = Q_0 - (\mathbf{s}\cdot[\mathbf{b}_1\times\mathbf{b}_2])Q_1, \quad (32)$$

because the order of films met by the neutron at transmission changes. Hence,

$$\overleftarrow{W(+)} \neq \overrightarrow{W(+)}, \quad (33)$$

which is a manifestation of a violation of the detailed balance principle.

After the time reversal transformation, not only $\mathbf{s}$ and $\mathbf{b}_{1,2}$ change sign but also the initial and final states are permuted. Therefore, $\overrightarrow{W(+)}$ is transformed to $\overleftarrow{W(-)}$, but

$$\overleftarrow{W(-)} = \overrightarrow{W(+)}, \quad (34)$$

and this proves that the T-invariance is not violated.

5.2. T-invariance of the neutron scattering on absorbing potentials

We consider the simplest case of the neutron scattering on a nonmagnetic one-dimensional potential $u(x)$ that is nonzero in an interval $0 \le x \le d$ and contains an imaginary part. The neutron wave function outside the potential is

$$\psi(x,t) = \exp(-i\omega t)\times \\ \times\Big[\Theta(x<0)\Big(\exp(ikx)+\rho(k)\exp(-ikx)\Big) + \\ + \Theta(x>d)\tau(k)\exp(ik(x-d))\Big], \quad (35)$$

where $\Theta(x)$ is a step function equal to unity when the inequality in its argument is satisfied, and to zero otherwise, and $\rho(k)$ and $\tau(k)$ are the reflection and transmission amplitudes, which are complex function of the incident wave number k. The wave function is a solution of the Schrödinger equation

$$\left(i\frac{\partial}{\partial t}+\frac{\partial^2}{\partial x^2}-u(x)\right)\psi(x,t)=0. \quad (36)$$

If we make the transformation

$$t\to -t, \quad (37)$$

then the equation for $\psi(x,-t)$ changes its form compared with (36). To restore its form, we have to make a complex conjugation, after which we obtain

$$\left(i\frac{\partial}{\partial t}+\frac{\partial^2}{\partial x^2}-u^*(x)\right)\psi^*(x,-t)=0. \quad (38)$$

However, we must be careful here. A potential that has an imaginary part changes after complex conjugation, and therefore we cannot be sure that the function $\psi^*(x,-t)$ remains a solution of (38). Instead of (38) we must write

$$\left(i\frac{\partial}{\partial t}+\frac{\partial^2}{\partial x^2}-u^*(x)\right)\Psi(x,t)=0, \quad (39)$$

and check whether $\Psi(x,t)=\psi^*(x,-t)$. We prove that this equality is true in the case of a rectangular potential, and claim that there are no reasons to doubt its validity for other potentials.

In fact, we can deal with stationary equations, writing

$$\Psi(x,t)=\exp(-i\omega t)\Phi(x)$$

and

$$\psi(x,t)=\exp(-i\omega t)\phi(x),$$

and our goal is then to show that a solution $\Phi(x)$ of the equation

$$\left(k^2+\frac{\partial^2}{\partial x^2}-u^*(x)\right)\Phi(x)=0, \quad (40)$$

coincides with $\phi^*(x)$.

In the case of a rectangular barrier potential of height $u=u'-iu''$ and width d, the wave function on the full x axis is [12]

$$\phi(x,u)=\Theta(x<0)\Big(\exp(ikx)+R(k,u)\exp(-ikx)\Big)+ \\ +\Theta(0<x<d)\frac{[1+r(k,u)]\exp(ik'(u)d)}{1-r^2(k,u)\exp(2ik'(u)d)}\times \\ \times[\exp(ik'(u)(x-d))-r(k,u)\exp(-ik'(u)(x-d))]+ \\ +\Theta(x>d)T(k,u)\exp(ik(x-d)), \quad (41)$$

where

$$R(k,u)=\frac{r(k,u)[1-\exp(2ik'(u)d)]}{1-r^2(k,u)\exp(2ik'(u)d)}, \\ T(k,u)=\frac{\exp(ik'(u)d)[1-r^2(k,u)]}{1-r^2(k,u)\exp(2ik'(u)d)}, \quad (42)$$

$$r(k,u)=\frac{k-k'(u)}{k+k'(u)},\quad k'(u)=\sqrt{k^2-u}. \quad (43)$$

We here suppose that $k^2>u'$, and everywhere we explicitly indicate the dependence on the complex potential u. The function $\phi^*(x,u)$ is

$$\begin{aligned}
&\phi^*(x,u) = \\
&= \Theta(x<0)\Big(\exp(-ikx) + R^*(k,u)\exp(ikx)\Big) + \\
&+ \Theta(0<x<d)\frac{[1+r^*(k,u)]\exp(-ik'^*(u)d)}{1-r^{*2}(k,u)\exp(-2ik'^*(u)d)} \times \\
&\times [\exp(-ik'^*(u)(x-d)) - r^*(k,u) \times \\
&\times \exp(ik'^*(u)(x-d))] + \\
&+ \Theta(x>d)T^*(k,u)\exp(-ik(x-d)). \quad (44)
\end{aligned}$$

It describes the interference of two waves incident from the left and right with the respective amplitudes $R^*(k)$ and $T^*(k)$. We show that it coincides with the solution $\Phi(x)$ in (40) containing these two incident waves.

The wave incident from the left gives a solution $\Phi_l(x)$, the wave incident from the right gives a solution $\Phi_r(x)$, and the total solution is equal to $\Phi_l(x)+\Phi_r(x)$. Using the general approach in [13, 14] for the incident wave $R^*(k,u)\exp(ikx)$, we obtain

$$\begin{aligned}
&\Phi_l(x,u^*) = \Theta(x<0)R^*(k,u) \times \\
&\times \Big(\exp(ikx) + R(k,u^*)\exp(-ikx)\Big) + \\
&+ \Theta(0<x<d)R^*(k,u)\frac{[1+r(k,u^*)]\exp(ik'(u^*)d)}{1-r^2(k,u^*)\exp(2ik'(u^*)d)} \times \\
&\times [\exp(ik'(u^*)(x-d)) - r(k,u^*) \times \\
&\times \exp(-ik'(u^*)(x-d))] + \\
&+ \Theta(x>d)R^*(k,u)T(k,u^*)\exp(ik(x-d)), \quad (45)
\end{aligned}$$

and for the incident wave $T^*(k)\exp(-ik(x-d))$, the wave function is

$$\begin{aligned}
&\Phi_r(x,u^*) = \Theta(x<0)T^*(k,u)T(k,u^*)\exp(-ikx) + \\
&+ \Theta(0<x<d)T^*(k,u)\frac{[1+r(k,u^*)]\exp(ik'(u^*)d)}{1-r^2(k,u^*)\exp(2ik'(u^*)d)} \times \\
&\times [\exp(-ik'(u^*)x) - r(k,u^*)\exp(ik'(u^*)x)] + \\
&+ \Theta(x>d)T^*(k,u)\,[T(k,u^*) \times \\
&\times \exp(-ik(x-d)) + R(k,u^*)\exp(ik(x-d))]. \quad (46)
\end{aligned}$$

It follows from (43) and (42) for $k^2>u'$ that $k'(u^*) = k'^*(u)$ and $r(u^*) = r^*(u)$, but $R(k,u^*) \neq R^*(k,u)$ and $T(k,u^*) \neq T^*(k,u)$.

It is easy to verify by simple algebra that the sum of terms in the interval $0<x<d$ from (45) and (46) is equal to the middle term in (44). This is shown in the next equation, where K' denotes $k'(u^*)$, and the dependence on k and u is omitted in the other terms:

$$\begin{aligned}
&R^*\frac{[1+r^*]\exp(iK'd)}{1-r^{*2}\exp(2iK'd)} \times \\
&\times [\exp(iK'(x-d)) - r^*\exp(-iK'(x-d))] + \\
&+ T^*\frac{[1+r^*]\exp(iK'd)}{1-r^{*2}\exp(2iK'd)} \times \\
&\times [\exp(-iK'x) - r^*\exp(iK'x)] = \\
&= \frac{r^*(1-\exp(-2iK'd))}{1-r^{*2}\exp(-2iK'd)}\,\frac{[1+r^*]\exp(iK'd)}{1-r^{*2}\exp(2iK'd)} \times \\
&\times [\exp(iK'(x-d)) - r^*\exp(-iK'(x-d))] + \\
&+ \frac{\exp(-iK'd)(1-r^{*2})}{1-r^{*2}\exp(-2iK'd)}\,\frac{[1+r^*]}{1-r^{*2}\exp(2iK'd)} \times \\
&\times [\exp(-iK'(x-d)) - \exp(2iK'd)r^* \times \\
&\times \exp(iK'(x-d))] = \exp(iK'(x-d)) \times \\
&\times \frac{r^*[1+r^*]\exp(-iK'd)}{(1-r^{*2}\exp(2iK'd))(1-r^{*2}\exp(-2iK'd))} \times \\
&\times \Big[\exp(2iK'd) - 1 - \exp(2iK'd)(1-r^{*2})\Big] + \\
&+ \exp(-iK'(x-d)) \times \\
&\times \frac{[1+r^*]\exp(-iK'd)}{(1-r^{*2}\exp(2iK'd))(1-r^{*2}\exp(-2iK'd))} \times \\
&\times \Big[(1-\exp(2iK'd))r^{*2} + (1-r^{*2})\Big] = \\
&= \frac{[1+r^*]\exp(-ik'^*d)}{1-r^{*2}\exp(-2ik'^*d)} \times \\
&\times \Big[\exp(-ik'^*(x-d)) - r^*\exp(ik'^*(x-d))\Big]. \quad (47)
\end{aligned}$$

It can be verified similarly that the sum of amplitudes of two outgoing waves at $x<0$ is equal to

$$R^*(k,u)R(k,u^*) + T^*(k,u)T(k,u^*) = 1. \quad (48)$$

The right outgoing wave at $x>d$ vanishes. Its amplitude is

$$\begin{aligned}
&R^*(k,u)T(k,u^*) + T^*(k,u)R(k,u^*) = \\
&= 2\,\mathrm{Re}(R^*(k,u)T(k,u^*)) = 0, \quad (49)
\end{aligned}$$

which shows that the phases of the amplitudes $R(k,u)$ and $T(k,u^*)$ differ by $\pi/2$.

Therefore, we see that $\Phi(x) = \phi^*(x)$, i.e., scattering of a scalar particle on a complex potential is time reversible. We have checked this for a simple rectangular potential, but there are no reasons to expect the result to be different for more complex potentials.

We considered the case $k^2>u'$ above. If $k^2<u'$, then $k'(u) = ik''(u)$, where $k''(u) = \sqrt{u-k^2}$, and

$$r(k,u) = \frac{k-ik''(u)}{k+ik''(u)}, \quad r(k,u^*) = \frac{1}{r^*(k,u)}. \quad (50)$$

Nevertheless, it can be proved again that $\Phi(x) = \phi^*(x)$, i. e., scattering on a complex potential is time reversible. The proof is a good exercise for the readers, and we do not therefore present it here.

It can be shown similarly that scattering of a spinor particle on an arbitrary magnetic potential is time reversible, even if the nuclear optical potential of the neutron-matter interaction contains an imaginary part.

6. CONCLUSION

Using simple examples, we have shown, in a simple scheme of a neutron reflectometry experiment, how a T-odd correlation can appear that can be interpreted as the T- or P-parity violation, although it does not violate T- and P-invariances. The experiment to check the theoretical predictions can be easily realized with two magnetic films of different coercivities evaporated upon nonmagnetic substrate. After magnetization to saturation of the high-coercivity film, the external magnetic field can be decreased and the sample rotated through an angle φ. The result is a system close to the one shown in Fig. 1.

At the same time, it is shown that if the space contains a couple of noncollinear magnetic fields, then the scattering of neutrons from this couple does not satisfy the detailed balance principle. This means that the neutron gas in the presence of two magnetic mirrors with noncollinear magnetizations has an equilibrium with an entropy that is not absolutely maximal.

We found some interesting features in considering neutron scattering on a noncollinear magnetic system. We can expect to find interesting features in considering a three-layer magnetic system with noncomplanar magnetic fields $\mathbf{B}_1$, $\mathbf{B}_2$, and $\mathbf{B}_3$. In this system, we can expect T-odd correlation like

$$(\mathbf{B}_1 \cdot [\mathbf{B}_2 \times \mathbf{B}_3]), \tag{51}$$

which at the same time violate reciprocity at transmission, but this subject, will be discussed elsewhere [11].

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