

# $(p + 1)$ -ALGEBRA FOR A SUPER $p$ -BRANE: THE NAMBU BRACKET REFORMULATION

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We express the covariant actions of a super  $p$ -brane and the corresponding equations of motion, in flat and curved superspaces, in terms of the Nambu  $(p + 1)$ -brackets. These brackets make the  $(p + 1)$ -algebra structure of a super  $p$ -brane manifest. For the flat superspace, this reconstruction of the action also allows reformulating it in terms of two sets of differential forms.

## 1. INTRODUCTION

Recent studies reveal that M2-branes have a description in terms of a 3-algebra, a generalization of Lie algebras based on an antisymmetric triple product structure [1]. 3-algebra relations have played an important role in the construction of the worldvolume theories of multiple M2-branes, which have attracted considerable attention [1, 2]. Various aspects of the 3-algebra can be seen in [2] and the references therein. But the correspondence of the 2-algebra to the string theory and of the 3-algebra to the M-theory can be understood from the dimensions of the string worldsheet and membrane worldvolume. This implies that the description of the super  $p$ -brane theory may require a  $(p + 1)$ -algebra structure.

Here, we consider the important subject of the construction of worldvolume theories for multiple  $p$ -branes. Recently, this subject received much attention due to the discovery of its relation to the multiple algebras. These algebras are defined in terms of multiple commutators. Their classical approximation is given by the well-known Nambu multiple brackets. Therefore, to explicitly formulate the brane action in terms of the multiple algebras, the first step would be to rewrite the brane action in terms of the Nambu brackets. The Nambu  $n$ -brackets are a way for realizing the Lie  $n$ -algebra [3], which was developed by Filippov [4].

In Ref. [5], it was demonstrated that the (super-symmetric)  $p$ -brane action is invariant under  $(p + 1)$ -dimensional diffeomorphisms. In other words, there is an infinite-dimensional volume-preserving algebra of super

$p$ -branes. In this paper, we reformulate the super  $p$ -brane covariant action and the corresponding equations of motion, in the flat and curved superspaces, in terms of the Nambu  $(p + 1)$ -brackets. Because the Nambu  $(p + 1)$ -brackets are generators of the  $(p + 1)$ -dimensional diffeomorphisms, this reformulation reveals the above symmetry more explicitly. However, this reformulation represents the super  $p$ -branes based on the  $(p + 1)$ -algebra.

In fact, there are some advantages in reformulating the membrane theory in terms of the 3-algebra. The same advantages also appear in reformulating the  $p$ -brane theory in terms of the  $(p + 1)$ -algebra. In addition, this reconstruction may provide a method for quantizing the theory. Beside, for the flat superspace, this reconstruction of the action also allows reformulating it in terms of two sets of differential forms.

This paper is organized as follows. In Sec. 2, we reconstruct a covariant,  $(p + 1)$ -algebra based action for a super  $p$ -brane in flat superspace. In Sec. 3, we reformulate the super  $p$ -brane action in curved superspace in terms of the  $(p + 1)$ -algebra. In Sec. 4, quantizability of the theory is discussed. Section 5 is devoted to the conclusions.

## 2. THE SUPER $p$ -BRANE IN FLAT SUPERSPACE, BASED ON THE $(p + 1)$ -ALGEBRA

### 2.1. The action

For the  $(p + 1)$ -algebra description of a super  $p$ -brane propagating in the  $D$ -dimensional flat spacetime, we begin with the known action

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$$S_p = T_p \int d^{p+1} \sigma (\mathcal{L}_1 + \mathcal{L}_2), \tag{1}$$

$$\begin{aligned} \mathcal{L}_1 &= \frac{1}{2(p+1)!} \phi^{-1} \langle \Pi^{\mu_1}, \Pi^{\mu_2}, \dots, \Pi^{\mu_{p+1}} \rangle \times \\ &\times \langle \Pi_{\mu_1}, \Pi_{\mu_2}, \dots, \Pi_{\mu_{p+1}} \rangle - \frac{1}{2} \phi, \\ \mathcal{L}_2 &= -\frac{2}{(p+1)!} \epsilon^{i_1 \dots i_{p+1}} B_{i_1 \dots i_{p+1}}. \end{aligned} \tag{2}$$

The Lagrangian  $\mathcal{L}_1$  is of the Schild type [6], i.e., an auxiliary scalar field  $\phi$  is introduced to take the square root of the Nambu–Goto action.  $\mathcal{L}_2$  is the Wess–Zumino Lagrangian. The degrees of freedom are the spacetime coordinates  $X^\mu$ , the Majorana spinor  $\theta$ , and the scalar field  $\phi$ . The indices  $\mu_1, \mu_2, \dots, \mu_{p+1} \in \{0, 1, \dots, D - 1\}$  belong to the spacetime, while  $i_1, i_2, \dots, i_{p+1} \in \{0, 1, \dots, p\}$  indicate the  $p + 1$  directions of the brane worldvolume. The worldvolume coordinates are  $\sigma^i$ . The Dirac matrices are denoted by  $\Gamma^\mu$ . The spacetime metric is

$$\eta_{\mu\nu} = \text{diag}(-1, 1, \dots, 1).$$

The brane tension is given by the constant  $T_p$ .

The variable  $\Pi_i^\mu$  is defined as

$$\Pi_i^\mu = \partial_i X^\mu - i \bar{\theta} \Gamma^\mu \partial_i \theta, \tag{3}$$

which is a supersymmetry invariant pull-back. In addition, we define

$$\begin{aligned} \langle \Pi^{\mu_1}, \Pi^{\mu_2}, \dots, \Pi^{\mu_{p+1}} \rangle &= \\ &= \epsilon^{i_1 i_2 \dots i_{p+1}} \Pi_{i_1}^{\mu_1} \Pi_{i_2}^{\mu_2} \dots \Pi_{i_{p+1}}^{\mu_{p+1}}, \end{aligned} \tag{4}$$

which is totally antisymmetric.

### 2.2. Equations of motion and symmetries

The equations of motion have been extracted in [5]. Because we want to express them in terms of the Nambu brackets, we write them explicitly. For the fields  $\phi$ ,  $X^\mu$ , and  $\theta$ , the equations of motion are

$$\begin{aligned} \phi - \sqrt{-g} &= 0, \\ \partial_i (\sqrt{-g} g^{ij} \Pi_j^\mu) - i \sqrt{-g} (-1)^{p(p+1)/2} \times \\ &\times \partial_i \bar{\theta} \Gamma^\mu \Gamma^{ij} \Gamma \partial_j \theta = 0, \\ [1 - (-1)^p \Gamma] \Gamma^i \partial_i \theta &= 0, \end{aligned} \tag{5}$$

where

$$g_{ij} = \Pi_i^\mu \Pi_j^\nu \eta_{\mu\nu} \tag{6}$$

is the induced metric. The determinant of this metric is denoted by  $g = \det g_{ij}$ , which is

$$g = \frac{1}{(p+1)!} \langle \Pi^{\mu_1}, \dots, \Pi^{\mu_{p+1}} \rangle \langle \Pi_{\mu_1}, \dots, \Pi_{\mu_{p+1}} \rangle. \tag{7}$$

In addition, the matrices  $\Gamma^i$ ,  $\Gamma^{ij}$ , and  $\Gamma$  are defined as

$$\begin{aligned} \Gamma^i &= g^{ij} \Gamma_\mu \Pi_j^\mu, \\ \Gamma^{ij} &= g^{ik} g^{jl} \Gamma_{\mu\nu} \Pi_k^\mu \Pi_l^\nu, \\ \Gamma &= \frac{(-1)^{(p-2)(p-5)/4}}{(p+1)! \sqrt{-g}} \Gamma_{\mu_1 \dots \mu_{p+1}} \langle \Pi^{\mu_1}, \dots, \Pi^{\mu_{p+1}} \rangle. \end{aligned} \tag{8}$$

The matrix  $\Gamma$  satisfies  $\Gamma^2 = 1$ . We see in what follows that the equations of motion have expressions in terms of the  $(p + 1)$ -algebra.

In addition to the worldvolume diffeomorphism invariance, the action is also invariant under the transformations

$$\delta \theta = \varepsilon, \quad \delta X^\mu = i \varepsilon \Gamma^\mu \theta, \quad \delta \phi = 0, \tag{9}$$

and

$$\begin{aligned} \delta_\kappa \theta &= [1 + (\phi/\sqrt{-g}) \Gamma] \kappa(\sigma), \quad \delta_\kappa X^\mu = i \bar{\theta} \Gamma^\mu \delta_\kappa \theta, \\ \delta_\kappa \phi &= 4i \phi g^{ij} \Pi_i^\mu \partial_j \bar{\theta} \Gamma_\mu \kappa(\sigma). \end{aligned} \tag{10}$$

The supersymmetry parameters  $\varepsilon$  and  $\kappa$  are spinors of the  $D$ -dimensional spacetime. The former is constant and the latter is local.

By removing the auxiliary field  $\phi$  through its equation of motion  $\phi = \sqrt{-g}$ , the Lagrangian  $\mathcal{L}_1$  reduces to the Nambu–Goto form

$$\mathcal{L}'_1 = -\sqrt{-\det(\Pi_i^\mu \Pi_j^\nu \eta_{\mu\nu})}. \tag{11}$$

This Lagrangian also has a Polyakov expression

$$\mathcal{L}''_1 = -\frac{1}{2} \sqrt{-h} [h^{ij} \Pi_i^\mu \Pi_j^\nu \eta_{\mu\nu} - (p - 1)], \tag{12}$$

where the independent auxiliary field  $h_{ij}$  is the intrinsic worldvolume metric with  $h = \det h_{ij}$ . This is a convenient alternative form for  $\mathcal{L}_1$ . The equation of motion for  $h_{ij}$ , extracted from (12), is

$$h_{ij} = \Pi_i^\mu \Pi_j^\nu \eta_{\mu\nu}.$$

After eliminating  $h_{ij}$  through its equation of motion, Lagrangian (12) also reduces to (11). Therefore, classically,  $\mathcal{L}_1$ ,  $\mathcal{L}'_1$ , and  $\mathcal{L}''_1$  are equivalent. However,  $\mathcal{L}_2$  and the form (12) of  $\mathcal{L}_1$  define the Green–Schwarz action for the super  $p$ -brane.

**2.3. The action based on the  $(p + 1)$ -algebra**

The Nambu  $(p + 1)$ -bracket of the variables  $\phi_1, \dots, \phi_{p+1}$  is defined by

$$\{\phi_1, \dots, \phi_{p+1}\}_{NB} = \epsilon^{i_1 \dots i_{p+1}} \partial_{i_1} \phi_1 \dots \partial_{i_{p+1}} \phi_{p+1}. \quad (13)$$

Therefore, in terms of the Nambu brackets, Eq. (4) takes the form

$$\begin{aligned} \langle \Pi^{\mu_1}, \Pi^{\mu_2}, \dots, \Pi^{\mu_{p+1}} \rangle &= \{X^{\mu_1}, X^{\mu_2}, \dots, X^{\mu_{p+1}}\}_{NB} - \\ &- i(p + 1) \bar{\theta}_\alpha \{(\Gamma^{[\mu_1} \theta]^\alpha}, X^{\mu_2}, \dots, X^{\mu_{p+1}}]\}_{NB} + \\ &+ \frac{p(p+1)}{2} \bar{\theta}_\alpha \bar{\theta}_\beta \{(\Gamma^{[\mu_1} \theta]^\alpha}, (\Gamma^{\mu_2} \theta)^\beta, X^{\mu_3}, \dots, X^{\mu_{p+1}}]\}_{NB} - \\ &- \frac{ip(p^2-1)}{6} \bar{\theta}_\alpha \bar{\theta}_\beta \bar{\theta}_\gamma \{(\Gamma^{[\mu_1} \theta]^\alpha}, (\Gamma^{\mu_2} \theta)^\beta, (\Gamma^{\mu_3} \theta)^\gamma, X^{\mu_4}, \dots, \\ &\dots, X^{\mu_{p+1}}]\}_{NB} + \dots + i^{p+1} (-1)^{(p+1)(p+2)/2} \bar{\theta}_{\alpha_1} \times \\ &\times \bar{\theta}_{\alpha_2} \dots \bar{\theta}_{\alpha_{p+1}} \{(\Gamma^{\mu_1} \theta)^{\alpha_1}, (\Gamma^{\mu_2} \theta)^{\alpha_2}, \dots, \\ &\dots, (\Gamma^{\mu_{p+1}} \theta)^{\alpha_{p+1}}\}_{NB} = \sum_{n=0}^{p+1} \left[ \binom{p+1}{n} \times \right. \\ &\times i^n (-1)^{n(n+1)/2} \times \\ &\times \bar{\theta}_{\alpha_1} \bar{\theta}_{\alpha_2} \dots \bar{\theta}_{\alpha_n} \{(\Gamma^{[\mu_1} \theta]^{\alpha_1}}, (\Gamma^{\mu_2} \theta)^{\alpha_2}, \dots, \\ &\dots, (\Gamma^{\mu_n} \theta)^{\alpha_n}, X^{\mu_{n+1}}, \dots, X^{\mu_{p+1}}]\}_{NB} \left. \right], \quad (14) \end{aligned}$$

where the bracket  $[\mu_1, \dots, \mu_{p+1}]$  indicates the antisymmetrization of the indices.

Substituting Eq. (14) in equations of motion (5) and the Lagrangian  $\mathcal{L}_1$ , we obtain the  $(p + 1)$ -algebra expressions of them. The explicit form of  $\mathcal{L}_1$  is

$$\begin{aligned} \mathcal{L}_1 &= \frac{1}{2(p+1)!} \phi^{-1} \sum_{n=0}^{p+1} \sum_{m=0}^{p+1} \left[ \binom{p+1}{n} \binom{p+1}{m} \times \right. \\ &\times i^{m+n} (-1)^{(m+n)(m+n+1)/2} \bar{\theta}_{\alpha_1} \dots \bar{\theta}_{\alpha_n} \bar{\theta}_{\beta_1} \dots \bar{\theta}_{\beta_m} \times \\ &\times \{(\Gamma^{[\mu_1} \theta]^{\alpha_1}}, \dots, (\Gamma^{\mu_n} \theta)^{\alpha_n}, X^{\mu_{n+1}}, \dots, X^{\mu_{p+1}}]\}_{NB} \times \\ &\times \{(\Gamma^{[\mu_1} \theta]^{\beta_1}}, \dots, (\Gamma^{\mu_m} \theta)^{\beta_m}, X_{\mu_{m+1}}, \dots, X_{\mu_{p+1}}]\}_{NB} \left. \right] - \\ &- \frac{1}{2} \phi. \quad (15) \end{aligned}$$

In fact, the  $B$ -field can be expressed in terms of the  $X^\mu$  and  $\theta$  as [7]

$$\begin{aligned} B_{i_1 i_2 \dots i_{p+1}} &= \frac{1}{2} \eta \bar{\theta} \Gamma_{\mu_1 \dots \mu_p} \partial_{i_{p+1}} \theta \left[ \sum_{r=0}^p i^{r+1} \binom{p+1}{r+1} \times \right. \\ &\times (\bar{\theta} \Gamma^{\mu_1} \partial_{i_1} \theta) \dots (\bar{\theta} \Gamma^{\mu_r} \partial_{i_r} \theta) \Pi_{i_{r+1}}^{\mu_{r+1}} \dots \Pi_{i_p}^{\mu_p} \left. \right], \quad (16) \end{aligned}$$

where

$$\eta = (-1)^{(p-1)(p+6)/4}.$$

After eliminating the coefficients  $B_{i_1 i_2 \dots i_{p+1}}$ , the Lagrangian  $\mathcal{L}_2$  becomes

$$\begin{aligned} \mathcal{L}_2 &= -\frac{\eta}{(p+1)!} \epsilon^{i_1 \dots i_{p+1}} \bar{\theta} \Gamma_{\mu_1 \dots \mu_p} \partial_{i_{p+1}} \theta \times \\ &\times \left[ \sum_{r=0}^p i^{r+1} \binom{p+1}{r+1} (\bar{\theta} \Gamma^{\mu_1} \partial_{i_1} \theta) \dots \right. \\ &\dots (\bar{\theta} \Gamma^{\mu_r} \partial_{i_r} \theta) \Pi_{i_{r+1}}^{\mu_{r+1}} \dots \Pi_{i_p}^{\mu_p} \left. \right]. \quad (17) \end{aligned}$$

Similarly to  $\mathcal{L}_1$ , the Lagrangian  $\mathcal{L}_2$  in terms of the Nambu brackets has the expansion

$$\begin{aligned} \mathcal{L}_2 &= -\frac{1}{(p+1)!} \sum_{r=0}^p \sum_{m=0}^{p-r} \left[ \binom{p+1}{r+1} \binom{p-r}{m} \times \right. \\ &\times i^{p-m+1} (-1)^{K_{r,m}} \bar{\theta}_{\alpha_1} \dots \bar{\theta}_{\alpha_r} \bar{\theta}_{\alpha_{r+m+1}} \dots \bar{\theta}_{\alpha_p} \bar{\theta}_{\alpha_{p+1}} \times \\ &\times \{(\Gamma^{\mu_1} \theta)^{\alpha_1}, \dots, (\Gamma^{\mu_r} \theta)^{\alpha_r}, X^{\mu_{r+1}}, \dots, X^{\mu_{r+m}}, \\ &\quad (\Gamma^{\mu_{r+m+1}} \theta)^{\alpha_{r+m+1}}, \dots, (\Gamma^{\mu_p} \theta)^{\alpha_p}, \\ &\quad \left. (\Gamma_{\mu_1 \dots \mu_p} \theta)^{\alpha_{p+1}}\}_{NB} \right], \quad (18) \end{aligned}$$

where

$$\begin{aligned} K_{r,m} &= p + \frac{1}{4}(p-1)(p+6) + \\ &+ \frac{1}{2}[r(r-1) + (p-r-m)(p+r-m+1)]. \quad (19) \end{aligned}$$

Let  $Z^M = (X^\mu, \theta^\alpha)$  denote coordinates of the target space of the super  $p$ -brane. The worldvolume form  $B_{i_1 \dots i_{p+1}}$  is a pull-back, i. e.,

$$B_{i_1 \dots i_{p+1}} = \partial_{i_1} Z^{M_1} \dots \partial_{i_{p+1}} Z^{M_{p+1}} B_{M_{p+1} \dots M_1}, \quad (20)$$

where  $B_{M_{p+1} \dots M_1}$  are components of a  $(p + 1)$ -form potential in the superspace. Therefore, an other  $(p + 1)$ -algebra expression of  $\mathcal{L}_2$  is

$$\begin{aligned} \mathcal{L}_2 &= \\ &= -\frac{2}{(p+1)!} \{Z^{M_1}, \dots, Z^{M_{p+1}}\}_{NB} B_{M_{p+1} \dots M_1}. \quad (21) \end{aligned}$$

According to (15), (18), and (21), all the derivatives have been absorbed in the Nambu  $(p+1)$ -brackets. Hence, the  $(p + 1)$ -algebra structure is made manifest.

**2.4. The action in terms of differential forms**

The  $(p + 1)$ -algebra description of a super  $p$ -brane allows writing the action

$$S_p = S_p^{(1)} + S_p^{(2)}$$

in the language of differential forms,

$$\begin{aligned}
 S_p^{(1)} &= \frac{T_p}{2} \sum_{n=0}^{p+1} \sum_{m=0}^{p+1} \left\{ \binom{p+1}{n} \binom{p+1}{m} \times \right. \\
 &\times \left. i^{m+n} (-1)^{(m+n)(m+n+1)/2} \int_{WV} \phi^{-1} A_{(m,n)} \right\} - \\
 &\quad - \frac{T_p}{2} \int_{WV} d^{p+1} \sigma \phi, \tag{22} \\
 S_p^{(2)} &= -T_p \sum_{r=0}^p \sum_{m=0}^{p-r} \left\{ \binom{p+1}{r+1} \binom{p-r}{m} \times \right. \\
 &\times \left. i^{p-m+1} (-1)^{K_{r,m}} \int_{WV} C_{(r,m)} \right\}.
 \end{aligned}$$

The differential (p + 1)-forms are defined by

$$\begin{aligned}
 A_{(m,n)} &= \frac{1}{(p+1)!} \times \\
 &\times \{Y_{[\mu_1}, \dots, Y_{\mu_m}, X_{\mu_{m+1}}, \dots, X_{\mu_{p+1}}]\}_{NB} \times \\
 &\times (dY^{\mu_1} \wedge \dots \wedge dY^{\mu_n} \wedge dX^{\mu_{n+1}} \wedge \dots \\
 &\quad \dots \wedge dX^{\mu_{p+1}})|_{WV}, \tag{23} \\
 C_{(r,m)} &= \frac{1}{(p+1)!} (dY^{\mu_1} \wedge \dots \wedge dY^{\mu_r} \wedge \\
 &\wedge dX^{\mu_{r+1}} \wedge \dots \wedge dX^{\mu_{r+m}} \wedge dY^{\mu_{r+m+1}} \wedge \dots \\
 &\quad \dots \wedge dY^{\mu_p} \wedge dZ_{\mu_1 \dots \mu_p})|_{WV},
 \end{aligned}$$

where the restriction  $|_{WV}$  means the pull-back of the wedge products on the worldvolume of the super p-brane, e. g.,

$$dX^\mu|_{WV} = \partial_i X^\mu d\sigma^i.$$

The variable  $Y^\mu$  and the antisymmetric tensor  $Z_{\mu_1 \dots \mu_p}$  are given by

$$\begin{aligned}
 Y^\mu &= \bar{\theta} \Gamma^\mu \theta, \\
 Z_{\mu_1 \dots \mu_p} &= \bar{\theta} \Gamma_{\mu_1 \dots \mu_p} \theta. \tag{24}
 \end{aligned}$$

These wedge products define differential (p + 1)-forms. The components of these forms explicitly have been given in terms of the coordinates  $\{X^\mu\} \cup \{\theta^\alpha\}$ . These forms do not have the pure bosonic part, i. e., they vanish in the absence of the  $\theta^\alpha$ . Hence, they exist only for super branes.

Because

$$\{X^\mu(\tau; \sigma^1, \dots, \sigma^p)\} \cup \{\theta^\alpha(\tau; \sigma^1, \dots, \sigma^p)\}$$

are coordinates of the worldvolume of the super p-brane in superspace, the actions  $S_p^{(1)}$  and  $S_p^{(2)}$  imply that the super p-brane is coupled to the potential forms

$$\{A_{(m,n)}|m, n = 0, 1, \dots, p + 1\}$$

and

$$\{C_{(r,m)}|m = 0, 1, \dots, p - r; r = 0, 1, \dots, p\}.$$

In fact, only reformulating the super p-brane action on the basis of the (p + 1)-algebra reveals these differential forms.

### 3. THE SUPER p-BRANE IN A CURVED SUPERSPACE

We assume the target space of the super p-brane to be a curved supermanifold with  $E_M^A(Z)$  as its corresponding supervielbeins. The  $A = a, \alpha$  are the tangent-space indices. Then the super p-brane action is given by

$$\begin{aligned}
 I_p &= -T_p \int d^{p+1} \sigma \left( \sqrt{-\det(E_i^a E_j^b \eta_{ab})} + \right. \\
 &\left. + \frac{2}{(p+1)!} \epsilon^{i_1 \dots i_{p+1}} E_{i_1}^{A_1} \dots E_{i_{p+1}}^{A_{p+1}} B_{A_{p+1} \dots A_1} \right), \tag{25}
 \end{aligned}$$

where

$$E_i^A = \partial_i Z^M E_M^A \tag{26}$$

is the pull-back of the supervielbeins  $E_M^A$ . The field  $B_{A_{p+1} \dots A_1}(Z)$  is a superspace (p + 1)-form potential. We note that due to the  $\kappa$ -symmetry of the action, only special values of p and D are allowed (see Ref. [8] and the references therein).

In this action, the (p + 1)-algebra can also be introduced. Because

$$\begin{aligned}
 \det(E_i^a E_j^b \eta_{ab}) &= \frac{1}{(p+1)!} \langle E^{a_1}, \dots, E^{a_{p+1}} \rangle \times \\
 &\quad \times \langle E_{a_1}, \dots, E_{a_{p+1}} \rangle, \tag{27} \\
 \langle E^{a_1}, \dots, E^{a_{p+1}} \rangle &= \epsilon^{i_1 \dots i_{p+1}} E_{i_1}^{a_1} \dots E_{i_{p+1}}^{a_{p+1}},
 \end{aligned}$$

the action (22) can be reformulated in terms of the Nambu (p + 1)-brackets

$$\begin{aligned}
 I_p &= -T_p \int d^{p+1} \sigma \times \\
 &\times \left\{ \left( -\frac{1}{(p+1)!} E_{M_1}^{a_1} \dots E_{M_{p+1}}^{a_{p+1}} E_{N_1}^{b_1} \dots E_{N_{p+1}}^{b_{p+1}} \times \right. \right. \\
 &\quad \times \left. \left. \{Z^{M_1}, \dots, Z^{M_{p+1}}\}_{NB} \times \right. \right. \\
 &\quad \times \left. \left. \{Z^{N_1}, \dots, Z^{N_{p+1}}\}_{NB} \eta_{a_1 b_1} \dots \eta_{a_{p+1} b_{p+1}} \right)^{1/2} + \right. \\
 &\left. + \frac{2}{(p+1)!} E_{M_1}^{A_1} \dots E_{M_{p+1}}^{A_{p+1}} \{Z^{M_1}, \dots, Z^{M_{p+1}}\}_{NB} \times \right. \\
 &\quad \left. \times B_{A_{p+1} \dots A_1} \right\}. \tag{28}
 \end{aligned}$$

The novelty of this reformulation is the appearance of the (p + 1)-algebra.

#### 4. A NOTE ON THE QUANTIZATION OF THE REFORMULATED ACTIONS

Due to the intrinsic nonlinearities, quantization of  $p$ -branes is a difficult problem. There are different quantum mechanical approaches associated with the quantum dynamics of  $p$ -branes based on viewing  $p$ -branes as gauge theories of volume-preserving diffeomorphisms. In other words, several quantum mechanical methods for  $p$ -branes are proposed based on the role that the volume-preserving diffeomorphism group has in the physics of these extended objects.

The other experience is the quantum Nambu brackets. They describe the quantum behavior of systems equivalently to the standard Hamiltonian quantization. For example, they serve to guide quantization of more general even-dimensional topological branes [9].

Thus, by appropriately replacing the classical Nambu brackets with the quantum Nambu brackets, one may achieve the quantization of the reformulated actions in this paper. This is not straightforward. It seems the status of quantizability of the reformulated actions is that they are not quantizable for  $p \neq 1$ , for the same reason that a quantum membrane theory has yet to be formulated.

#### 5. CONCLUSIONS

In the first part of this paper, we expressed the super  $p$ -brane action and the corresponding equations of motion, in the flat superspace, in terms of the Nambu  $(p+1)$ -brackets. In the second part, for a super  $p$ -brane that lives in a curved superspace, we obtained the Nambu  $(p+1)$ -bracket expression of the action. This reformulation is another language for describing the super  $p$ -branes and gives a new insight into the branes. It may provide a way for quantizing the  $p$ -branes.

In both the above cases, all derivatives appeared through the Nambu  $(p+1)$ -brackets, and hence the  $(p+1)$ -algebra structure for the super  $p$ -brane theory was made manifest. This is related to the fact that the (supersymmetric)  $p$ -brane action is invariant under the  $(p+1)$ -dimensional diffeomorphisms and the Nambu  $(p+1)$ -brackets are generators of  $(p+1)$ -dimensional diffeomorphisms.

Finally, for flat superspace, we found two sets of differential  $(p+1)$ -forms that couple to the super  $p$ -brane. This result originates from the reformulation and cannot be seen in the original form of the action.

#### REFERENCES

1. J. Bagger and N. Lambert, Phys. Rev. D **75**, 045020 (2007), ArXiv:hep-th/0611108; Phys. Rev. D **77**, 065008 (2008), ArXiv:hep-th/0711.0955; JHEP **0802**, 105 (2008), ArXiv:hep-th/0712.3738; A. Gustavsson, Nucl. Phys. B **811**, 66 (2009), ArXiv:hep-th/0709.1260.
2. A. Gustavsson, JHEP **0804**, 083 (2008), ArXiv:hep-th/0802.3456; M. A. Bandres, A. E. Lipstein, and J. H. Schwarz, JHEP **05**, 025 (2008), ArXiv:hep-th/0803.3242; A. Morozov, JHEP **05**, 076 (2008), ArXiv:hep-th/0804.0913; N. Lambert and D. Tong, Phys. Rev. Lett. **101**, 041602 (2008), ArXiv:hep-th/0804.1114; E. A. Bergshoeff, M. de Roo, and O. Hohm, Class. Quant. Grav. **25**, 142001 (2008), ArXiv:hep-th/0804.2201; G. Papadopoulos, JHEP **05**, 054 (2008), JHEP **0805**, 054 (2008), ArXiv:hep-th/0804.2662; Class. Quant. Grav. **25**, 142002 (2008), ArXiv:hep-th/0804.3567; S. Benvenuti, D. Rodriguez-Gomez, E. Tonni, and H. Verlinde, JHEP **0901**, 078 (2009), ArXiv:hep-th/0805.1087; S. Banerjee and A. Sen, ArXiv:hep-th/0805.3930; M. A. Bandres, A. E. Lipstein, and J. H. Schwarz, JHEP **0807**, 117 (2008), ArXiv:hep-th/0806.0054; S. Cecotti and A. Sen, ArXiv:hep-th/0806.1990; E. A. Bergshoeff, M. de Roo, O. Hohm, and D. Roest, JHEP **0808**, 091 (2008), ArXiv:hep-th/0806.2584; K. Lee and J.-H. Park, JHEP **0904**, 012 (2009), ArXiv:hep-th/0902.2417; I. A. Bandos and P. K. Townsend, Class. Quant. Grav. **25**, 245003 (2008), ArXiv:hep-th/0806.4777; JHEP **0902**, 013 (2009), ArXiv:hep-th/0808.1583.
3. Y. Nambu, Phys. Rev. D **7**, 2405 (1973).
4. V. T. Filippov, Sib. Mat. Zh. **26**, 126 (1985).
5. E. Bergshoeff, E. Sezgin, Y. Tanii, and P. K. Townsend, Ann. Phys. **199**, 340 (1990).
6. A. Schild, Phys. Rev. D **16**, 1722 (1977).
7. J. M. Evans, Class. Quant. Grav. **5**, L87 (1988).
8. M. Duff, Class. Quant. Grav. **6**, 1577 (1989).
9. T. Curtright and C. Zachos, New J. Phys. **4**, 83.1 (2002), ArXiv:hep-th/0205063; Phys. Rev. D **68**, 085001 (2003), ArXiv:hep-th/0212267; C. Zachos, Phys. Lett. B **570**, 82 (2003), ArXiv:hep-th/0306222.