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ARTIFICIAL NETS FROM SUPERCONDUCTING NANOGRANULES

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We show that a large transport current can flow through superconducting nets composed of nanoclusters. Although thermal and quantum fluctuations lead to a finite value of dissipation, this value can be very small in one- and two-dimensional systems for realistic parameters of the nanoclusters and distances between them. The value of the action for vortex tunneling at zero temperature can be made sufficiently large to make the dissipation negligibly small. We estimate the temperature T_0 of the transition from the thermal activation to quantum tunneling.

1. INTRODUCTION

It was found recently that small-size nanoclusters with a magic or close to magic number of electrons can have high values of the superconducting transition temperature [1–3]. In such systems, the resonance tunneling between nanoclusters can occur [4, 5] and the nonresonant part of the critical current can be significantly larger than the estimate of the critical current value made by Ambegaokar and Baratoff [6]. The resonant part of the critical current depends on geometry of the system. As a result, the self-organization can occur in the degenerate case. The high-T_c superconducting clusters are promising blocks to construct superconducting nets or chains.

An important question for such systems is the role of quantum and thermal fluctuations. The usual concept of the Coulomb blockade should be significantly modified. For example, the mass renormalization for nanoclusters can be much larger than the zero "mass" in the standard concept of the Coulomb blockade [7–9].

The quantum equation describing the Josephson effect is similar to that for motion of a particle in a washboard potential [10, 11]. The junction capacitance C_0 plays the role of mass (in the zeroth approximation). In the classical picture, the "particle" is localized in some potential well. In this picture, the usual concept of the Coulomb blockade (with $U_0 > e^2/2C_0$ [12, 13]) corresponds to the condition $U_0 > \omega_0/2$, where U_0 is the barrier height and ω_0 is the frequency of small oscillations near the bottom of the potential. This condition means that the energy of oscillations is smaller than the barrier height (otherwise, it is impossible to localize the "particle").

Quantum effects lead to a strong renormalization of C_0 ("zero mass") that corresponds to a single junction. According to [9], the dissipation in nets is related with transitions of quantum vortices between neighboring wells. It is essential that an artificial net is characterized by the intrinsic inhomogeneity (the positions of the vortex center inside the net are not equivalent), which leads to the appearance of pinning. There exists some crossover temperature T_0 such that at $T > T_0$, the "decay" rate is described by the Arrenius law $\exp(-U_0/T)$. At $T < T_0$, the quantum "tunneling" becomes the dominant factor. Then we are dealing with the factor $\exp(-A(T))$, where $A(T)$ is an effective action along the optimal "trajectory" (in imaginary time). The quantity $A(T)$ is finite as T tends to zero.

The construction of nets (in two or three dimensions) and chains from roadblocks allows obtaining macroscopic samples with superconducting properties. The characteristic current densities in such systems can be larger than 10^7 A/cm^2. In one- and two-dimensional systems, the presence of fluctuations leads to a finite value of dissipation [9]. The objective is therefore to

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obtain samples with a low level of dissipation created by classical (thermal) and quantum fluctuations.

First, we show that the value of the critical current in superconducting nanocluster chains decays very slowly as the number of links increases. Second, we show that under the realistic assumptions taking only the nonresonant part of the Josephson current and effective mass reorganization into account strongly suppresses quantum fluctuations (quantum tunneling of vortices). If the conditions for resonant tunneling are satisfied, the value of the critical current can increase by several orders of magnitude [5]. As a result, all the fluctuations are strongly suppressed and the effective action for vortex quantum tunneling is much larger than unity [9]. Finally, we estimate the characteristic size of a single vortex in a nanocluster net, which is important for investigation of the properties of nets in the presence of a magnetic field.

2. THE DEPENDENCE OF THE CRITICAL CURRENT OF A CHAIN OF NANOCLUSTERS ON THE NUMBER OF LINKS

We suppose that the critical current distribution function W of links in a chain has the Gaussian form

$$W = \frac{1}{\sqrt{\pi\delta}} \exp\left(-\frac{(J_c^0 - J)^2}{\delta}\right), \tag{1}$$

where J_c^0 is the mean value of the critical current through one link. The variation is

$$\delta/2 = \langle (J_c^0 - J)^2 \rangle. \tag{2}$$

The probability $W(I > J)$ that the critical current $I \geq J$ for one pair of clusters is given by

$$W(I \geq J) = \frac{1}{\sqrt{\pi\delta}} \int_J^\infty dI_1 \exp\left(-\frac{(J_c^0 - I_1)^2}{\delta}\right). \tag{3}$$

Hence, the probability for a chain of $N+1$ clusters to have the critical current $I \geq J$ is

$$W_{N+1}(I \geq J) = \left[\frac{1}{\sqrt{\pi\delta}} \int_J^\infty dI_1 \exp\left(-\frac{(J_c^0 - I_1)^2}{\delta}\right)\right]^N. \tag{4}$$

From Eq. (4), we obtain a distribution function (the probability density) $\mathcal{W}_{N+1}$ for the critical current of a chain of $N+1$ clusters:

$$\mathcal{W}_{N+1} = -\frac{\partial}{\partial J} W_{N+1}(I \geq J) = \frac{N}{\sqrt{\pi\delta}} \times \exp\left\{-\frac{(J_c^0 - J)^2}{\delta} + (N-1) \times \ln\left[\frac{1}{\pi\delta}\int_J^\infty dI \exp\left(-\frac{(J_c^0 - I)^2}{\delta}\right)\right]\right\}. \tag{5}$$

The mean of the critical current of a chain with N links is given by

$$\langle J \rangle_{N+1} = \int_0^\infty dJ\, J \mathcal{W}_{N+1}(J). \tag{6}$$

For $N \gg 1$, the extremal point J_{extr} of $\mathcal{W}_{N+1}(J)$ is a solution of the equation

$$2\kappa = \frac{N-1}{\sqrt{\pi}} \frac{\exp(-\kappa^2)}{\dfrac{1}{2} + \dfrac{1}{\sqrt{\pi}} \displaystyle\int_0^\kappa dy \exp(-y^2)}, \tag{7}$$

where

$$\kappa = \left(J_c^0 - J_{extr}\right)/\sqrt{\delta}. \tag{8}$$

Equation (7) is quite accurate even for $N \sim 1$. From Eq. (6), we can obtain the exact value of $\langle J \rangle_3$:

$$\langle J \rangle_3 = J_c^0 - \sqrt{\frac{\delta}{2\pi}}. \tag{9}$$

Substituting $N = 3$ in Eq. (7), we obtain $\kappa(3) = 0.541$.

For $N \gg 1$, we obtain

$$\kappa = \ln^{1/2}\left(\frac{N-1}{2\kappa\sqrt{\pi}}\right) \tag{10}$$

from Eq. (7). It follows from Eq. (10) that the critical current decreases very slowly with the length of the chain. The value of the length at which the chain is still superconducting can be estimated as

$$N < 2\sqrt{\pi}\frac{J_c^0}{\sqrt{\delta}} \exp\left(\frac{(J_c^0)^2}{\delta}\right). \tag{11}$$

Even for a relatively broad distribution $\sqrt{\delta}/J_c^0 \sim 0.1$, we obtain

$$N < 10^{45}. \tag{12}$$

Hence, the main problem is to suppress the thermal and quantum fluctuations.

3. THE AMPLITUDE OF THE NONRESONANT PART OF THE JOSEPHSON CURRENT

The nonresonant part $J_{n.r.}$ of the Josephson current J_c^0 is quite simple for magic or near-magic clusters because the form is nearly spherical for such clusters (see [4, 5]):

$$J_{n.r.} = \frac{e\hbar^3}{m^2}\sum_{L,L_1} T\sum_{\omega}\frac{|\Delta|^2}{[\omega^2+E^2(L)][\omega^2+E^2(L_1)]}\times$$
$$\times\frac{(2L+1)(2L_1+1)Z_0(L)Z_0(L_1)}{a^6}\,\frac{E_0}{\delta U_0}\left(\frac{\hbar}{\sqrt{2m\delta U_0}}\right)^2\times$$
$$\times K^2_{L+1/2}\left(\frac{\sqrt{2m\delta U_0}}{\hbar}\frac{D}{2}\right)\left[K_{L-1/2}\left(\frac{\sqrt{2m\delta U_0}}{\hbar}a\right)+\right.$$
$$\left.+\frac{(L+1)\hbar}{a\sqrt{2m\delta U_0}}K_{L+1/2}\left(\frac{\sqrt{2m\delta U_0}}{\hbar}a\right)\right]^{-2}\times$$
$$\times K^2_{L_1+1/2}\left(\frac{\sqrt{2m\delta U_0}}{\hbar}\frac{D}{2}\right)\left[K_{L_1-1/2}\left(\frac{\sqrt{2m\delta U_0}}{\hbar}a\right)+\right.$$
$$\left.+\frac{(L_1+1)\hbar}{a\sqrt{2m\delta U_0}}K_{L_1+1/2}\left(\frac{\sqrt{2m\delta U_0}}{\hbar}a\right)\right]^{-2}. \qquad (13)$$

Here, $K_\nu(x)$ is the Bessel function, a is the radius of a cluster, D is the distance between the centers of the clusters, δU_0 is the height of the potential barrier, E_0 is the energy of the last occupied shell, and $Z_0(L)$ are zeros of the Bessel function (the states near the Fermi level),

$$E^2(L) \equiv \Delta^2(\omega) + (\tilde{\mu} - E_1^0)^2, \qquad (14)$$

where $\tilde{\mu}$ is the chemical potential and E_1^0 is the electron energy in the normal state with the angular momentum L.

If the resonance conditions are satisfied, the resonant part of the critical current exceeds the nonresonant part (by many orders of magnitude). Our goal is to show that although we take only the nonresonant part of the current into account, the value of the action for vortex tunneling can be made much larger than unity. This guarantees that the resonant tunneling completely suppresses quantum fluctuations.

It is convenient to represent Eq. (13) for the value of the critical current as

$$J_{n.r.} = \frac{e\hbar^3}{2\pi m^2 a^4}\,\frac{E_0}{\delta U_0}\left(\frac{\hbar}{\sqrt{2m\delta U_0}a}\right)^2\times$$
$$\times\sum_{L,L_1}(2L+1)(2L_1+1)Z_0(L)Z_0(L_1)C_{LL_1}B_{LL_1}, \qquad (15)$$

where

$$C_{LL_1} = 2\pi T\sum_{\omega}|\Delta(\omega)|^2\times$$
$$\times\left[\omega^2+|\Delta(\omega)|^2+(\tilde{\mu}-E_L^0)^2\right]^{-1}\times$$
$$\times\left[\omega^2+|\Delta(\omega)|^2+(\tilde{\mu}-E_{L_1}^0)^2\right]^{-1},$$

$$B_{LL_1} = K^2_{L+1/2}\left(\frac{\sqrt{2m\delta U_0}}{\hbar}\frac{D}{2}\right)\times$$
$$\times\left[K_{L-1/2}\left(\frac{\sqrt{2m\delta U_0}}{\hbar}a\right)+\right.$$
$$\left.+\frac{(L+1)\hbar}{a\sqrt{2m\delta U_0}}K_{L+1/2}\left(\frac{\sqrt{2m\delta U_0}}{\hbar}a\right)\right]^{-2}\times$$
$$\times K^2_{L_1+1/2}\left(\frac{\sqrt{2m\delta U_0}}{\hbar}\frac{D}{2}\right)\left[K_{L_1-1/2}\left(\frac{\sqrt{2m\delta U_0}}{\hbar}a\right)+\right.$$
$$\left.+\frac{(L_1+1)\hbar}{a\sqrt{2m\delta U_0}}K_{L_1+1/2}\left(\frac{\sqrt{2m\delta U_0}}{\hbar}a\right)\right]^{-2}. \qquad (16)$$

For small junctions, the geometric capacity C_0 between two granules can be much smaller (see [9]) than the effective value of the renormalized capacity C [7, 8]. The value of the renormalized capacity C determines the value of the action for vortex tunneling [9]. In the low-temperature regime, the value of C is given by (see [7, 8])

$$C = C_0 + \frac{3e\hbar J_c}{16\Delta^2}. \qquad (17)$$

4. DETAILED INVESTIGATION OF THE PARTICULAR CASE OF A MAGIC CLUSTER WITH $N_l = 168$

We consider the magic cluster Al_{56} ($N_e = 168$) with the parameters (HOS is the highest occupied state, LUS is the lowest unoccupied state)

$$\begin{gathered} E_0 = 1.467\cdot10^{-11}\ \text{erg},\\ \text{HOS}(L=7,\ Z_0(7)=11.657),\\ \text{LUS}(L=4,\ Z_0(4)=11.7049),\\ a = 6.07\cdot10^{-8}\ \text{cm}. \end{gathered} \qquad (18)$$

In the low-temperature regime, we obtain the order parameter Δ and the chemical potential $\tilde{\mu}$:

$$\begin{gathered} \Delta \equiv \Delta(\omega) = \frac{B\langle\Omega^2\rangle^{1/2}}{1+C\omega^2/\langle\Omega^2\rangle},\\ \langle\Omega^2\rangle^{1/2} = 350\ \text{K},\quad B = 0.455,\quad C = 0.065, \end{gathered} \qquad (19)$$

$$\tilde{\mu} - E^0_{L=7} = E^0_{L=7}\left\{0.72\left[\left(\frac{Z_0(4)}{Z_0(7)}\right)^2 - 1\right]\right\},$$

$$\tilde{\mu} - E^0_{L=4} = -E^0_{L=7}\left\{0.28\left[\left(\frac{Z_0(4)}{Z_0(7)}\right)^2 - 1\right]\right\}.$$

Substituting the above value of the order parameter Δ in Eq. (16), we obtain

$$E^0_{L=4} - E^0_{L=7} = 1.467\cdot 10^{-11}\frac{10^{12}}{1.6}\times \\ \times\left[\left(\frac{11.7049}{11.657}\right)^2 - 1\right] = 0.0755 \text{ eV},$$

$$C_{44} = \frac{0.530046}{\langle\Omega^2\rangle^{1/2}},\quad C_{77} = \frac{4.2186\cdot 10^{-2}}{\langle\Omega^2\rangle^{1/2}}, \\ C_{47} = \frac{0.141228}{\langle\Omega^2\rangle^{1/2}} \tag{20}$$

as $T \to 0$.

We consider the following special cases

$$\delta U_0 = \{5 \text{ eV}, 2.5 \text{ eV}, 1 \text{ eV}\}, \\ D = \{15 \text{ Å}, 14 \text{ Å}, 13.5 \text{ Å}\}. \tag{21}$$

1. For $\delta U_0 = 5$ eV, it then follows from Eq. (16) that

$$\begin{aligned}
&\{B_{44} = 3.9397\cdot 10^{-5}, && B_{77} = 6.3545\cdot 10^{-6}, && B_{47} = 1.5822\cdot 10^{-5}\}, && D = 15 \text{ Å},\\
&\{B_{44} = 9.770609\cdot 10^{-4}, && B_{77} = 2.33012\cdot 10^{-4}, && B_{47} = 4.77144\cdot 10^{-4}\}, && D = 14 \text{ Å},\\
&\{B_{44} = 4.94382\cdot 10^{-3}, && B_{77} = 1.460418\cdot 10^{-3}, && B_{47} = 2.68701\cdot 10^{-3}\}, && D = 13.5 \text{ Å}.
\end{aligned} \tag{22}$$

2. For $\delta U_0 = 2.5$ eV, it follows from Eq. (16) that

$$\begin{aligned}
&\{B_{44} = 2.093629\cdot 10^{-4}, && B_{77} = 1.8725\cdot 10^{-5}, && B_{47} = 6.261247\cdot 10^{-5}\}, && D = 15 \text{ Å},\\
&\{B_{44} = 2.448279\cdot 10^{-3}, && B_{77} = 3.581055\cdot 10^{-4}, && B_{47} = 9.363453\cdot 10^{-4}\}, && D = 14 \text{ Å},\\
&\{B_{44} = 8.535846\cdot 10^{-3}, && B_{77} = 1.631306\cdot 10^{-3}, && B_{47} = 3.731565\cdot 10^{-3}\}, && D = 13.5 \text{ Å}.
\end{aligned} \tag{23}$$

3. For $\delta U_0 = 1$ eV, it follows from Eq. (16) that

$$\begin{aligned}
&\{B_{44} = 4.714718\cdot 10^{-4}, && B_{77} = 1.956326\cdot 10^{-5}, && B_{47} = 9.603918\cdot 10^{-5}\}, && D = 15 \text{ Å},\\
&\{B_{44} = 2.995027\cdot 10^{-3}, && B_{77} = 2.30809\cdot 10^{-4}, && B_{47} = 8.314328\cdot 10^{-4}\}, && D = 14 \text{ Å},\\
&\{B_{44} = 7.732681\cdot 10^{-3}, && B_{77} = 8.315719\cdot 10^{-4}, && B_{47} = 2.5358\cdot 10^{-3}\}, && D = 13.5 \text{ Å}.
\end{aligned} \tag{24}$$

Using Eqs. (15), (20), (22)–(24), we can obtain the values of the critical current and the renormalized capacity for different values of the distance D between centers of the clusters and the height of the potential barrier δU_0.

For $\delta U_0 = 5$ eV,

$$J_{n.r.}\,[\text{CGSE}] = \begin{bmatrix} 1.22696\cdot 10^2 & (D = 15 \text{ Å})\\ 3.247735\cdot 10^3 & (D = 14 \text{ Å})\\ 1.7111\cdot 10^4 & (D = 13.5 \text{ Å})\end{bmatrix},\quad Y\,[\text{cm}] = \begin{bmatrix} 1.09748\cdot 10^{-7} & (D = 15 \text{ Å})\\ 2.905\cdot 10^{-6} & (D = 14 \text{ Å})\\ 1.53061\cdot 10^{-5} & (D = 13.5 \text{ Å})\end{bmatrix}, \tag{25}$$

for $\delta U_0 = 2.5$ eV,

$$J_{n.r.}\,[\text{CGSE}] = \begin{bmatrix} 2.40842\cdot 10^3 & (D = 15 \text{ Å})\\ 3.0055\cdot 10^4 & (D = 14 \text{ Å})\\ 1.09238\cdot 10^5 & (D = 13.5 \text{ Å})\end{bmatrix},\quad Y\,[\text{cm}] = \begin{bmatrix} 2.15427\cdot 10^{-6} & (D = 15 \text{ Å})\\ 2.68833\cdot 10^{-5} & (D = 14 \text{ Å})\\ 9.77106\cdot 10^{-5} & (D = 13.5 \text{ Å})\end{bmatrix}, \tag{26}$$

and for $\delta U_0 = 1$ eV,

$$J_{n.r.}\,[\text{CGSE}] = \begin{bmatrix} 3.13935\cdot 10^4 & (D = 15 \text{ Å})\\ 2.11697\cdot 10^5 & (D = 14 \text{ Å})\\ 5.68733\cdot 10^5 & (D = 13.5 \text{ Å})\end{bmatrix},\quad Y\,[\text{cm}] = \begin{bmatrix} 2.808065\cdot 10^{-5} & (D = 15 \text{ Å})\\ 1.89357\cdot 10^{-4} & (D = 14 \text{ Å})\\ 5.08717\cdot 10^{-4} & (D = 13.5 \text{ Å})\end{bmatrix}, \tag{27}$$

where

$$Y = \frac{3e\hbar J_c}{16\Delta^2} \tag{28}$$

(see Eq. (20)). The effective action value A for the vortex underbarrier tunneling at $T = 0$ and zero value of the transport current I_{tr} is given by (see [9])

$$A = 4.522\sqrt{\frac{\hbar}{8e^3} J_{n.r.} Y}. \tag{29}$$

For several values of $\{\delta U_0, D\}$, the following values of the action A follow from Eq. (26)–(28):

$$\begin{gathered} \{\delta U_0 = 2.5 \text{ eV},\ D = 13.5 \text{ Å},\ A = 15.706\}, \\ \{\delta U_0 = 1 \text{ eV},\ D = 14 \text{ Å},\ A = 30.438\}, \\ \{\delta U_0 = 1 \text{ eV},\ D = 13.5 \text{ Å},\ A = 81.774\}. \end{gathered} \tag{30}$$

In those three cases, the action value is much larger than unity. Hence, the nets with the parameters $\{\delta U_0,\ D,\ \Delta\}$ have large values of the critical current. For example, when $\{\delta U_0 = 1 \text{ eV},\ D = 14 \text{ Å}\}$, the expected value of the critical current density is $j \sim 10^9$ A/cm^2.

If the resonant conditions for tunneling processes are satisfied, the critical current value exceeds the nonresonant part $J_{n.r.}$ by several orders of magnitude [5]. As a result, the admissible range of values of the parameters $\{\delta U_0,\ D,\ \Delta\}$ for which the thermal and quantum fluctuations do not completely destroy the superconducting current is significantly expanded.

5. CURRENT DISTRIBUTION IN A SINGLE VORTEX

At large distances ($\rho \gg D$) from a vortex center, the distribution of currents in a granular artificial net is similar to that in thin films (a Pearl vortex [14]). The Maxwell equation for the vector potential $\mathbf{A}$ can be written as

$$\text{rot rot}\mathbf{A} = \frac{4\pi}{c}\frac{J_c}{\hbar}\delta(z)\left(\frac{1}{\rho} - \frac{2eA}{c}\right)\mathbf{e}_\theta, \tag{31}$$

where $\mathbf{e}_\theta = (-\sin\theta, \cos\theta)$ and $\mathbf{A} = A(z,\rho)\mathbf{e}_\theta$.

In the considered gauge, $\operatorname{div}\mathbf{A} = 0$ and the Fourier transformation of Eq. (31) gives

$$\begin{gathered} (k^2 + g^2)\mathbf{A}_{kg} = -\frac{2}{\lambda_{eff}} \times \\ \times \mathbf{A}_g - \frac{i8\pi^2 J_c}{\hbar c g}(-\sin\theta_g, \cos\theta_g), \\ \lambda_{eff}^{-1} = \frac{4\pi e J_c}{\hbar c^2}, \end{gathered} \tag{32}$$

where c is the speed of light.

From Eq. (32), we readily obtain

$$\mathbf{A}_g = \frac{\lambda_{eff}}{1 + \lambda_{eff} g}\left(-\frac{4i\pi^2 J_c}{cg}\right)(-\sin\theta_g, \cos\theta_g). \tag{33}$$

Substituting (33) in the right-hand side of Eq. (31), we obtain the current density $\tilde{j}$ (per unit length) in the form

$$\begin{gathered} \tilde{j} = \int j\, dz = \frac{\lambda_{eff} J_c}{\rho} \times \\ \times \int_0^\infty dq \frac{J_0(g\rho)}{(1 + \lambda_{eff} g)^2}(-\sin\theta, \cos\theta), \end{gathered} \tag{34}$$

where $J_0(x)$ is the Bessel function. Because $J_0(0) = 1$ and $\int_0^\infty dx J_0(x) = 1$, we obtain [10]

$$\begin{aligned} \tilde{j}_{(\rho \ll \lambda_{eff})} &= \frac{J_c}{\rho}(-\sin\theta, \cos\theta), \\ \tilde{j}_{(\rho \gg \lambda_{eff})} &= \frac{\lambda_{eff} J_c}{\rho^2}(-\sin\theta, \cos\theta). \end{aligned} \tag{35}$$

In the particular case where $D = 14$ Å and $\delta U_0 = 1$ eV, we use (35) to obtain the value of the "penetration depth" or the vortex characteristic size

$$\lambda_{eff} \approx 7 \cdot 10^{-4} \text{ cm}. \tag{36}$$

The characteristic magnetic field H_1 at which vortices start overlapping is determined by the condition

$$H_1 \lambda_{eff}^2 = \Phi_0, \tag{37}$$

where $\Phi_0 = \pi\hbar c/e$ is the quantum flux. If λ_{eff} is given by Eq. (36), then the value of H_1 is very small:

$$H_1 \text{ [Oe]} \approx 0.4. \tag{38}$$

However, the distances essential for the pinning phenomenon are of the order of D. The characteristic magnetic field H_2 that corresponds to such distances is extremely high:

$$H_2 \text{ [T]} \approx \frac{\Phi_0}{D^2} \approx 500. \tag{39}$$

Therefore, in a wide range of the magnetic fields, we can expect a weak dependence of the critical current on the magnetic field.

6. CONCLUSIONS

We showed that a large-density transport current can flow through superconducting nanoclusters nets. For realistic values of the parameters, the energy dissipation due to thermal and quantum fluctuations can be made negligibly small. For such systems, the value of the temperature T_0 defined as the temperature of the transition from thermal activations to quantum tunneling can be estimated as [9]

$$T_0 = 7.136 \cdot 10^{-2} \frac{1}{k_B} \sqrt{\frac{2e\hbar J_c}{Y}} = 0.233 \frac{\Delta \hbar}{k_B}. \tag{40}$$

For the parameter value given by (19), we obtain

$$T_0 = 37 \text{ K}. \tag{41}$$

Equation (40) is valid under the condition

$$C_0 \ll \frac{3e\hbar J_c}{16\Delta^2}. \tag{42}$$

We note that the effective mass M^* near the top of the barrier of the vortex tunneling in the net is approximately 30 times larger than the renormalized mass of a single junction [9].

The results obtained here indicate that superconducting nets are promising objects for obtaining high-temperature superconducting lines with a high transport current density and a weak dependence on the external magnetic field.

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