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$\mu \to e\gamma$ DECAY VERSUS THE $\mu \to eee$ BOUND AND LEPTON FLAVOR VIOLATING PROCESSES IN SUPERNOVA

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Even tiny lepton flavor violation (LFV) due to some New Physics is able to alter the conditions inside a collapsing supernova core and probably to facilitate the explosion. LFV emerges naturally in a see-saw type-II model of neutrino mass generation. Experimentally, the LFV beyond the Standard Model is constrained by rare lepton decay searches. In particular, strong bounds are imposed on the $\mu \to eee$ branching ratio and on the μ–e conversion in muonic gold. Currently, the $\mu \to e\gamma$ is under investigation in the MEG experiment that aims at a dramatic increase in sensitivity in the next three years. We seek a see-saw type-II LFV pattern that fits all the experimental constraints, leads to $\mathrm{Br}(\mu \to e\gamma) \gtrsim \mathrm{Br}(\mu \to eee)$, and ensures a rate of LFV processes in supernova high enough to modify the supernova physics. These requirements are sufficient to eliminate almost all freedom in the model. In particular, they lead to the prediction $0.4 \cdot 10^{-12} \lesssim \mathrm{Br}(\mu \to e\gamma) \lesssim 6 \cdot 10^{-12}$, which will be testable by MEG in the nearest future. The considered scenario also constrains the neutrino mass-mixing pattern and provides lower and upper bounds on τ-lepton LFV decays. We also briefly discuss a model with a single bilepton in which the $\mu \to eee$ decay is absent at the tree level.

1. INTRODUCTION

Theoretical description of the collapse-driven supernova explosion is an important unsolved problem in astrophysics. Modern computer simulations of the explosion have already reached a high level of sophistication. Nevertheless, they cannot self-consistently explain the ejection of the supernova envelope in the whole range of the relevant presupernova masses and metallicities. The Standard Model (SM) is typically used as a microphysical input in the simulations. But lepton flavor violation (LFV) due to some New Physics at a ~ 1 TeV scale can substantially alter the conditions inside the collapsing core [1–5] .

In particular, LFV tends to an increase in the neutrino luminosity, thus facilitating the explosion and modifying the expected neutrino signal [5–7]. Therefore, if the true underlying theory beyond the Standard Model violates lepton flavor at a certain level, then LFV processes should be included in the supernova simulations in order to obtain reliable results[1)].

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1) In this paper, we consider LFV processes other than neutrino oscillations. These last do not occur below the neutrino sphere because of the high matter density of the supernova core. Therefore, they do not affect the neutrino transport below the neutrino sphere.

One of the appealing SM extensions is the see-saw type-II model of neutrino mass generation [6, 7]. In our previous papers in collaboration with Blinnikov [5, 7], we have shown that under certain conditions, this model predicts the rates of LFV processes in supernova high enough to alter the supernova physics. Here, we continue to explore the see-saw type-II model.

LFV is constrained by experiments searching for rare processes with charged leptons. Currently, only upper limits on the corresponding transition probabilities are reported. But a dramatic increase in statistics in such experiments is expected. In particular, the MEG collaboration [9] plans to reach the sensitivity of few $\times 10^{-13}$ for $\mathrm{Br}(\mu \to e\gamma)$ in the next few years. The preliminary result of the year 2009 run is $\mathrm{Br}(\mu \to e\gamma) < 1.5 \cdot 10^{-11}$ at 90 % CL [10], which is already close to the best previous result due to the MEGA experiment [11]:

$$\mathrm{Br}(\mu \to e\gamma) < 1.2 \cdot 10^{-11},\ 90\,\%\,\mathrm{CL}. \qquad (1)$$

In this paper, we consider a scenario in which the $\mu \to e\gamma$ decay probability is large enough to be measured by MEG in the nearest future, i. e.,

$$\mathrm{Br}(\mu \to e\gamma) = x \cdot 10^{-12}, \qquad (2)$$

where x is of the order of 1. In considering this scenario, the strong experimental bound on the $\mu \to eee$ decay put by SINDRUM collaboration [12]

$$\mathrm{Br}(\mu \to eee) < 1.0 \cdot 10^{-12}, \quad 90\,\%\mathrm{CL}, \tag{3}$$

must be taken into account.

Generically in the see-saw type-II model, the $\mu \to eee$ decay proceeds at the tree level, and the $\mu \to e\gamma$ decay — through one loop. Therefore, generically, $\mathrm{Br}(\mu \to e\gamma) \ll \mathrm{Br}(\mu \to eee)$ and the above scenario with $\mathrm{Br}(\mu \to e\gamma) \sim 10^{-12}$ is not feasible. But for certain values of the model parameters, the $\mu \to eee$ decay is suppressed at the tree level and the considered scenario can be realized [13, 14]. Is it possible to satisfy the additional requirement of a sufficiently large (i. e., relevant for neutrino transport) LFV rate in supernova? The goal of this paper is to explore this question. The result is as follows: we find a region in the parameter space of the model in which the answer is affirmative. We call this region the "Golden Domain" of the see-saw type-II model. Roughly speaking, this Golden Domain corresponds to the normal neutrino mass hierarchy and $\theta_{13} > 2°$; in this domain, the rates of LFV processes in supernova are high enough to alter the SN physics whenever $\mathrm{Br}(\mu \to e\gamma) \gtrsim 0.4 \cdot 10^{-12}$. The upper bound $\mathrm{Br}(\mu \to e\gamma) \lesssim 6 \cdot 10^{-12}$ is derived in our model from the experimental upper bound on the μ–e conversion in a muonic Au atom.

The rest of the paper is organized as follows. In Sec. 2, the see-saw type-II model is reviewed. In Sec. 3, the criterion is derived that ensures that the LFV processes in supernova alter the supernova physics significantly. In Sec. 4, the LFV charged lepton decays and μ–e conversion are discussed. In Sec. 5, interrelations between various bounds and restrictions are established and the Golden Domain of the parameter space of the see-saw type-II model is presented. In Sec. 6, we compare our results to what may be expected in other models, namely, in a model with a single charged bilepton and in the MSSM. In Sec. 7, we summarize our results.

2. SEE-SAW TYPE-II MODEL

In the see-saw type-II model [8], a heavy scalar triplet $\boldsymbol{\Delta}$ is introduced that is responsible for the generation of Majorana neutrino masses. The triplet is coupled to leptons and to the SM Higgs boson, the latter coupling producing a vacuum expectation value for the neutral component of the triplet. The neutrino masses are proportional to this vev.

The see-saw type-II Lagrangian contains two major ingredients, a scalar–lepton interaction,

$$\mathcal{L}_{ll\Delta} = \sum_{l,l'} \lambda_{ll'} \overline{L_l^c} i\tau_2 \Delta L_{l'} + \mathrm{H.c.}, \tag{4}$$

and a scalar potential, which in its minimal form is given by

$$V = -M_H^2 H^\dagger H + f(H^\dagger H)^2 + M_\Delta^2 \mathrm{Tr}(\Delta^\dagger \Delta) + \\ + \frac{1}{\sqrt{2}}(\tilde{\mu} H^T i\tau_2 \Delta^\dagger H + \mathrm{H.c.}). \tag{5}$$

Here,

$$\Delta \equiv \boldsymbol{\Delta\tau}/\sqrt{2} = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}, \tag{6}$$

$$L_l \equiv \begin{pmatrix} (\nu_l)_L \\ l_L \end{pmatrix}$$

is a doublet of left-handed leptons of a flavor $l = e, \mu, \tau$, H is a Higgs doublet, and $\tilde{\mu}$ is a parameter with the dimension of mass.

We note that due to the anticommutation of the fermion fields, the 3×3 matrix $\Lambda \equiv ||\lambda_{ll'}||$ is symmetric,

$$\Lambda^T = \Lambda. \tag{7}$$

The vev of the neutral component of the triplet is given by

$$\langle \Delta^0 \rangle = \frac{\tilde{\mu} v^2}{2\sqrt{2} M_\Delta^2}, \tag{8}$$

where $v \equiv \sqrt{2}\langle H^0 \rangle = 246$ GeV. Due to the triplet vev, neutrinos acquire the Majorana mass according to

$$m = 2\langle \Delta^0 \rangle \Lambda, \tag{9}$$

where $m \equiv ||m_{ll'}||$ is the neutrino mass matrix in the flavor basis. It follows that in the see-saw type-II model, the neutrino mass matrix m is proportional to the coupling matrix Λ.

The neutrino mass matrix in the flavor basis is obtained from the diagonal mass matrix by the transformation [15]

$$m = U^* \mathrm{diag}(m_1, m_2, m_3) U^\dagger, \tag{10}$$

with $U \equiv ||U_{li}||$ ($l = e, \mu, \tau,\ i = 1, 2, 3$) being a PMNS neutrino mixing matrix,

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \times \\ \times \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathrm{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1). \tag{11}$$

The explicit expressions for the entries of m are [16]

$$
\begin{aligned}
m_{ee} &= ac_{13}^2 + s_{13}^2 m_3 e^{2i\delta}, \\
m_{\mu\mu} &= m_1 e^{-i\alpha_1}(s_{12}c_{23} + s_{13}e^{-i\delta}c_{12}s_{23})^2 + \\
&+ m_2 e^{-i\alpha_2}(c_{12}c_{23} - s_{13}e^{-i\delta}s_{12}s_{23})^2 + m_3 c_{13}^2 s_{23}^2, \\
m_{\tau\tau} &= m_1 e^{-i\alpha_1}(s_{12}s_{23} - s_{13}e^{-i\delta}c_{12}c_{23})^2 + \\
&+ m_2 e^{-i\alpha_2}(c_{12}s_{23} + s_{13}e^{-i\delta}s_{12}c_{23})^2 + m_3 c_{13}^2 c_{23}^2, \\
m_{e\mu} &= c_{13}[ds_{12}c_{12}c_{23} + s_{13}e^{i\delta}s_{23}(m_3 - ae^{-2i\delta})], \\
m_{e\tau} &= c_{13}[-ds_{12}c_{12}s_{23} + s_{13}e^{i\delta}c_{23}(m_3 - ae^{-2i\delta})], \\
m_{\mu\tau} &= s_{23}c_{23}(-b + c_{13}^2 m_3) - s_{13}de^{-i\delta} \times \\
&\times s_{12}c_{12}(c_{23}^2 - s_{23}^2) + s_{13}^2 ae^{-2i\delta}s_{23}c_{23}.
\end{aligned}
\tag{12}
$$

We here define the parameters with the dimension of mass:

$$
\begin{aligned}
a &\equiv m_1 e^{-i\alpha_1}c_{12}^2 + m_2 e^{-i\alpha_2}s_{12}^2, \\
b &\equiv m_1 e^{-i\alpha_1}s_{12}^2 + m_2 e^{-i\alpha_2}c_{12}^2, \\
d &\equiv m_2 e^{-i\alpha_2} - m_1 e^{-i\alpha_1}.
\end{aligned}
\tag{13}
$$

The best experimental bound on the mass of the doubly charged scalar Δ^{--} (which we are mainly interested in) is reported by the D0 collaboration [17]:

$$
M_{\Delta^{--}} > 150 \text{ GeV}, \quad 95\,\% \text{ CL}. \tag{14}
$$

A slightly weaker bound was earlier reported by the CDF collaboration [18]. Prospects for Δ^{--} searches on the LHC are discussed in a recent paper [19].

3. LFV PROCESSES IN SUPERNOVA

The see-saw type-II model gives rise to the following flavor-changing reactions in supernova [5][2)]:

$$
\begin{aligned}
&e^- e^- \to \mu^- \mu^-, \\
&e^- \nu_e \to \mu^- \nu_{e,\mu,\tau}, \\
&e^- \nu_e \to e^- \nu_{\mu,\tau}, \\
&\nu_e \nu_e \to \nu_l \nu_l, \quad l = \mu, \tau, \\
&\nu_e \nu_e \to \nu_l \nu_{l'}, \quad l, l' = e, \mu, \tau, \quad l \neq l'.
\end{aligned}
\tag{15}
$$

All the above processes are described by a tree diagram with Δ in the s-channel. For example, the first process is described by the diagram in Fig. 1.

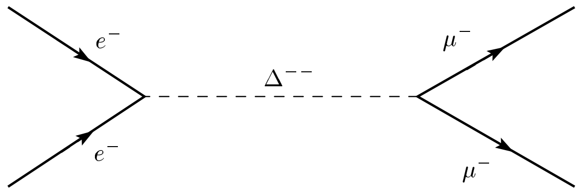

Fig. 1. $ee \to \mu\mu$ LFV transition mediated by the doubly charged scalar Δ^{--}

Neglecting the electron mass, we obtain the following cross sections:

$$
\begin{aligned}
\sigma(ee \to \mu\mu) &= \frac{|\lambda_{ee}|^2 |\lambda_{\mu\mu}|^2}{M_\Delta^4} \times \\
&\times \left(1 - \frac{m_\mu^2}{2E^2}\right) \sqrt{1 - \frac{m_\mu^2}{E^2}}\, \frac{E^2}{2\pi}, \\
\sigma(e\nu_e \to \mu\nu_l) &= \frac{|\lambda_{ee}|^2 |\lambda_{\mu l}|^2}{M_\Delta^4} \times \\
&\times \left(1 - \frac{m_\mu^2}{4E^2}\right)^2 \frac{E^2}{2\pi}, \quad l = e, \mu, \tau, \\
\sigma(e\nu_e \to e\nu_l) &= \frac{|\lambda_{ee}|^2 |\lambda_{el}|^2}{M_\Delta^4} \frac{E^2}{2\pi}, \quad l = \mu, \tau, \\
\sigma(\nu_e\nu_e \to \nu_l\nu_l) &= 2\frac{|\lambda_{ee}|^2 |\lambda_{ll}|^2}{M_\Delta^4} \frac{E^2}{\pi}, \quad l = \mu, \tau, \\
\sigma(\nu_e\nu_e \to \nu_l\nu_{l'}) &= 4\frac{|\lambda_{ee}|^2 |\lambda_{ll'}|^2}{M_\Delta^4} \frac{E^2}{\pi}, \\
l, l' = e, \mu, \tau, &\quad l \neq l',
\end{aligned}
\tag{16}
$$

where E is the energy of the initial electron or neutrino in the center-of-momentum frame[3)].

The rate of conversion of electron flavor to μ- and τ-flavors inside the proto-neutron star can be estimated as

$$
\begin{aligned}
R_{LFV} \approx \frac{n_e^2}{2}\sigma(ee \to \mu\mu) + n_e n_{\nu_e} \sum_{f,f'} \sigma(e\nu_e \to ff') + \\
+ \frac{n_{\nu_e}^2}{2} \sum_{f,f'} \sigma(\nu_e\nu_e \to ff'),
\end{aligned}
\tag{17}
$$

where f and f' denote various final neutrinos and charged leptons (see Eq. (15)). If this rate is comparable with the rate of decrease of the total lepton number due to neutrino diffusion out of the proto-neutron star,

---

2) It was argued in [5] that only reactions with $|\Delta L_e|$, $|\Delta L_\mu|$, $|\Delta L_\tau| = 0, 2$ are relevant because non-diagonal matrix elements of Λ should be small in order to suppress the yet unobserved LFV decays of charged leptons. This conclusion is valid generically; but in the present paper, we consider a special domain in the model parameter space in which the $\mu \to e\gamma$ decay probability is close to its experimental bound. Therefore, we should consider all LFV reactions.

3) These cross sections were calculated in [5]; however, unfortunately, some numerical factors in [5] are incorrect. Namely, $\sigma(e\nu_e \to \mu\nu_l)$ and $\sigma(\nu_e\nu_e \to \nu_l\nu_l)$ in [5] respectively have erroneous extra factors $1/2$ and $1/4$.

Table 1. Lepton flavor-violating processes: experimental constraints [22, 23] and predicted values. The latter correspond to the three selected points from the Golden Domain of the see-saw type-II model (see Table 2) and are normalized to $x \equiv \mathrm{Br}(\mu \to e\gamma)/10^{-12}$. The "$\lesssim$" symbol is used whenever the probability of the process vanishes at the tree level and is given by higher-order loop diagrams. The "branching ratio" for the μ–e conversion on Au is defined as $\Gamma_{Au}(\mu \to e)/\Gamma_{capt}$, where $\Gamma_{capt} = 13.07 \cdot 10^6$ s^{-1} [24] is the muon capture rate in muonic gold

Process	Experimental upper bound on Br	Br(process)/x		
		I	II	III
$\mu \to e\gamma$	$1.2 \cdot 10^{-11}$	10^{-12}		
$\mu^- \to e^+e^-e^-$	$1.0 \cdot 10^{-12}$	$\lesssim 10^{-13}$	$4.1 \cdot 10^{-13}$	$3.3 \cdot 10^{-13}$
μ Au $\to e$ Au ($M_\Delta = 150$ GeV)	$7 \cdot 10^{-13}$	$1.2 \cdot 10^{-13}$	$1.9 \cdot 10^{-13}$	$1.7 \cdot 10^{-13}$
μ Au $\to e$ Au ($M_\Delta = 1$ TeV)		$3.1 \cdot 10^{-13}$	$4.2 \cdot 10^{-13}$	$3.8 \cdot 10^{-13}$
$\tau^- \to \mu^+\mu^-\mu^-$	$3.2 \cdot 10^{-8}$	$1.1 \cdot 10^{-9}$	$3.5 \cdot 10^{-10}$	$9.1 \cdot 10^{-10}$
$\tau^- \to e^+\mu^-\mu^-$	$2.3 \cdot 10^{-8}$	$7.4 \cdot 10^{-11}$	$3.9 \cdot 10^{-11}$	$6.1 \cdot 10^{-11}$
$\tau^- \to e^+e^-e^-$	$3.6 \cdot 10^{-8}$	$9.1 \cdot 10^{-13}$	$9.3 \cdot 10^{-13}$	$6.7 \cdot 10^{-13}$
$\tau^- \to \mu^+e^-e^-$	$2.0 \cdot 10^{-8}$	$1.3 \cdot 10^{-11}$	$8.4 \cdot 10^{-12}$	$1.0 \cdot 10^{-11}$
$\tau^- \to e^+e^-\mu^-$	$2.7 \cdot 10^{-8}$	$\lesssim 10^{-11}$	$3.8 \cdot 10^{-13}$	$4.2 \cdot 10^{-13}$
$\tau^- \to \mu^+e^-\mu^-$	$3.7 \cdot 10^{-8}$	$\lesssim 10^{-13}$	$3.4 \cdot 10^{-12}$	$6.3 \cdot 10^{-12}$
$\tau \to \mu\gamma$	$3.3 \cdot 10^{-8}$	$1.6 \cdot 10^{-11}$	$5.4 \cdot 10^{-12}$	$1.4 \cdot 10^{-11}$
$\tau \to e\gamma$	$4.4 \cdot 10^{-8}$	$3.4 \cdot 10^{-13}$	$2.7 \cdot 10^{-13}$	$3.0 \cdot 10^{-13}$

R_{diff}, then the physics of the collapse is substantially altered compared to the SM case. In particular, the neutrino signal is modified and the explosion is probably facilitated [5, 7]. To be specific, we demand that

$$R_{LFV} > R_{diff} \approx 4 \cdot 10^{36}\ \mathrm{cm}^{-3} \cdot \mathrm{s}^{-1}. \tag{18}$$

This numerical value is based on the supernova simulations in Ref. [20]. Matter in the center of supernova after the core bounce is characterized by $n_B \approx \approx 2 \cdot 10^{38}$ cm^{-3}, $Y_e \equiv n_e/n_B \approx 0.28$, $Y_{\nu_e} \equiv \equiv n_{\nu_e}/n_B \approx 0.07$, $\mu_e \approx (240$–$280)$ MeV, and $\mu_{\nu_e} \approx \approx (160$–$220)$ MeV (these values can be obtained, e. g., from paper [20] or using the open-code programm BOOM described in [21]). For the numerical estimates, we conservatively take $E = 160$ MeV. We use the above numerical values to establish the relation between the $\mu \to e\gamma$ decay probability, R_{LFV}, and R_{diff} in Sec. 5.

4. RARE LEPTON DECAYS

The present experimental constraints on so-called "rare" (in fact, still unobserved) LFV lepton processes are summarized in the second column of Table 1. A detailed analysis of LFV charged lepton decays mediated by a scalar triplet is given in [25]. Three-lepton rare decays normally proceed at the tree level and their widths are given by

$$\Gamma(\mu^- \to e^+e^-e^-) = \frac{m_\mu^5}{768\pi^3 M_\Delta^4}|\lambda_{e\mu}\lambda_{ee}|^2, \tag{19}$$

$$\Gamma(\tau^- \to l^+l'^-l'^-) = \frac{m_\tau^5}{768\pi^3 M_\Delta^4}|\lambda_{l\tau}\lambda_{l'l'}|^2, \tag{20}$$

$$\Gamma(\tau^- \to l^+l'^-l''^-) = \frac{m_\tau^5}{384\pi^3 M_\Delta^4}|\lambda_{l\tau}\lambda_{l'l''}|^2, \quad l' \neq l''. \tag{21}$$

We note that the decays with two identical leptons of equal sign in the final state (see Eqs. (19) and (20)) have an additional factor 1/2 compared to decay (21) with different leptons of equal sign in the final state.

Radiative $l \to l'\gamma$ decays are described by penguin diagrams, and therefore their widths contain an additional factor $\sim \alpha$ [13]:

$$\Gamma(l \to l'\gamma) = \frac{27}{16}\frac{\alpha}{4\pi}\frac{m_l^5}{192\pi^3 M_\Delta^4} \times \\ \times |\lambda_{le}\lambda^*_{el'} + \lambda_{l\mu}\lambda^*_{\mu l'} + \lambda_{l\tau}\lambda^*_{\tau l'}|^2. \quad (22)$$

One could therefore expect that generically

$$\mathrm{Br}(l_1 \to l_2\gamma) \ll \mathrm{Br}(l_1 \to l_2 l_3 l_4).$$

This relation implies that $\mathrm{Br}(\mu \to e\gamma) \ll 10^{-12}$ due to the strong $\mu \to eee$ experimental bound, which makes the $\mu \to e\gamma$ decay unobservable in the MEG experiment. But because the matrix Λ is related to the neutrino masses and mixing, we can expect a hierarchy of couplings and therefore of decay rates. Indeed, we show in the next section that the above-mentioned contradiction may be avoided for a certain choice of Λ (allowed by the experimental data). It is clear that this choice should lead to the suppression of the tree $\mu \to eee$ decay amplitude.

Another strong bound on LFV is imposed by the results of the SINDRUM II collaboration on the μ–e conversion on gold [26]. This experiment investigated the fate of muonic atoms with heavy nuclei. The most probable event is the capture of a muon by the nucleus with a muon neutrino emission. An LFV mode is the μ–e transition that results in a monoenergetic electron emission. This process was first theoretically explored in Ref. [27]. An approximate expression for the width of the μ–e conversion can be written in a model-independent way as [13]

$$\Gamma_{(A,Z)}(\mu \to e) = 4\alpha^5 m_\mu^5 Z_{eff}^4 Z |F_p(q^2)|^2 \times \\ \times (|A_1^L + A_2^R|^2 + |A_1^R + A_2^L|^2), \quad (23)$$

where Z_{eff} is an effective charge felt by a muon bound in the atom, $F_p(q^2)$ is a form-factor related to the proton density in the nucleus, and $q^2 \approx m_\mu^2$ and $A_{1,2}^{L,R}$ are the model-dependent form-factors that enter the effective low-energy LFV violating electromagnetic current

$$j^\alpha = \bar{e}[q^2\gamma_\alpha(A_1^L P_L + A_1^R P_R) + \\ + m_\mu i\sigma_{\alpha\beta}q^\beta(A_2^L P_L + A_2^R P_R)]\mu. \quad (24)$$

Formula (23) demonstrates how the μ–e conversion rate depends on the quantities involved; however, it strongly depends on the quantities Z_{eff} and $F_p(q^2)$, which cannot be expressed analytically. A thorough analysis of the μ–e transition rate is presented in Refs. [28, 29]. We use their results, which are reproduced if we take $Z_{eff} = 33.5$, $F_p(q^2) = 0.16$ for gold [29].

The form-factors A_1^L and A_2^R for the see-saw type-II model are given by [13]

$$A_1^L = \sum_l f_l \frac{\lambda^*_{el}\lambda_{l\mu}}{12\pi^2 M_\Delta^2}, \quad A_2^R = \sum_l \frac{3\lambda^*_{el}\lambda_{l\mu}}{32\pi^2 M_\Delta^2}, \quad (25)$$

while A_1^R and A_2^L vanish due to the electron chirality conservation. Here,

$$f_l = \ln\frac{m_l^2}{M_\Delta^2} + 4\frac{m_l^2}{|q^2|} + \left(1 - 2\frac{m_l^2}{|q^2|}\right) \times \\ \times \sqrt{1 + 4\frac{m_l^2}{|q^2|}}\ln\frac{\sqrt{|q^2| + 4m_l^2} + \sqrt{|q^2|}}{\sqrt{|q^2| + 4m_l^2} - \sqrt{|q^2|}}, \quad (26)$$

where $|q^2| \approx m_\mu^2$. This general expression is simplified for specific flavors:

$$f_e \approx \ln\frac{|q^2|}{M_\Delta^2} = -18.3, \\ f_\mu \approx \ln\frac{m_\mu^2}{M_\Delta^2} + \left(4 - \sqrt{5}\ln\frac{3+\sqrt{5}}{2}\right) = -16.5, \quad (27) \\ f_\tau \approx \ln\frac{m_\tau^2}{M_\Delta^2} + \frac{5}{3} = -11.0,$$

where the numerical values are given for $M_\Delta = 1$ TeV. Large logarithmic factors in f_l appear due to the diagram in which the photon couples to a charged fermion in the loop. Contracting the propagator of the Δ-boson in this diagram yields the photon polarization operator, which contains this famous logarithm responsible for the running of the electromagnetic coupling α. Due to the large logarithmic factor, A_1^L dominates over A_2^R in the probability of the μ–e conversion.

We note that all rare decay probabilities have the same $\sim M_\Delta^{-4}$ dependence on the scalar mass. If we fix the coupling matrix Λ up to a common factor λ and introduce an effective four-fermion constant $G_{LFV} = \lambda^2/M_\Delta^2$, then all rare decay probabilities depend only on G_{LFV} but not separately on M_Δ and λ. Therefore, the values of the rare decay widths in the third column of Table 1 do not explicitly depend on M_Δ. By contrast, the μ–e conversion probability has an additional logarithmic dependence on M_Δ, and we therefore quote two different values for it in Table 1, which correspond to two different values of M_Δ (our reference value $M_\Delta = 1$ TeV and the experimental lower bound $M_\Delta = 150$ GeV).

5. GOLDEN DOMAIN OF THE SEE-SAW TYPE-II MODEL

We now consider the "Golden Domain" of the see-saw type-II model in which:

1) all the experimental constraints from neutrino oscillations, μ–e conversion, and rare lepton decays are satisfied,

2) $\mathrm{Br}(\mu \to e\gamma) \sim 10^{-12}$ (as explained above, this implies the suppression of the tree-level amplitude for the $\mu \to eee$ decay),

3) the rate of LFV in supernova is high enough to affect the neutrino transport (see Eq. (18)).

A natural and convenient way to parameterize the coupling matrix Λ (up to an overall factor) of the see-saw type-II model is to use the neutrino masses, mixing angles, and phases as parameters. This natural parameterization involves five continuous parameters and one discrete ambiguity that are not fixed (but possibly restricted) by neutrino oscillation experiments. They are: the absolute scale of neutrino masses, the angle θ_{13}, the phases δ, α_1, and α_2 (continuous) and the mass hierarchy (discrete). In what follows, we use this natural parameterization to explore the experimentally allowed part of the parameter space of the see-saw type-II model. The ranges of mixing angles and phases are chosen according to the generally accepted convention [30]: $\theta_{12}, \theta_{23}, \theta_{13} \in [0, \pi/2]$, $\delta \in (-\pi, \pi]$, and $\alpha_{1,2} \in [0, \pi]$[4].

To suppress the $\mu \to eee$ decay, we should choose [13, 14]

$$\lambda_{e\mu} \approx 0. \tag{28}$$

Another possible way to suppress the $\mu \to eee$ decay would be to set $\lambda_{ee} \approx 0$; but this would also suppress the LFV processes in supernova (15), and is therefore not acceptable.

Condition (28) implies that $m_{e\mu}$ should vanish. To see how this can occur, we consider the case where $m_1 \ll m_2 \ll m_3$. From Eq. (12), we then obtain

$$m_{e\mu} \approx e^{-i\alpha_2} \cos\theta_{23} \times \\ \times \left(\frac{1}{2} \sin 2\theta_{12} m_2 + \mathrm{tg}\,\theta_{23} \sin\theta_{13} e^{i(\delta+\alpha_2)} m_3 \right). \tag{29}$$

If $|\delta + \alpha_2| \approx \pi$, then the cancelation of two terms in Eq. (29) occurs for $\theta_{13} \approx 5°$ [13]. Thus we are able to fit condition (28) by choosing an experimentally allowed mass-mixing pattern.

To obtain the general picture, we numerically scan the parameter space of the see-saw type-II model. As a result, we find a single Golden Domain of the parameter space that satisfies all the imposed requirements. Some of the 2D projections of this domain are presented in Fig. 2. The main features of this domain are as follows.

1. The normal mass hierarchy with $m_1 < m_2 \ll \ll m_3$. Neutrino masses can take the following values:

$$0 < m_1 \lesssim 0.021 \text{ eV}, \\ 0.009 \text{ eV} \lesssim m_2 \lesssim 0.023 \text{ eV}, \quad m_3 \approx 0.05 \text{ eV}. \tag{30}$$

Moreover, as follows from Fig. 2, the case of quasidegenerate m_1 and m_2 (with $m_1 \gtrsim 0.005$) is only marginally allowed; on the contrary, substantially hierarchical values $m_1 \ll m_2$ occupy the major part of the Golden Domain.

2. The value of θ_{13} can vary in a broad range, but cannot be too small:

$$2° \lesssim \theta_{13} \lesssim 12°. \tag{31}$$

3. The combination of phases $|\delta + \alpha_2|$ does not deviate too much from $180°$:

$$||\delta + \alpha_2| - 180°| \lesssim 40°. \tag{32}$$

4. The value of α_1 can vary in a broad range, especially if $m_1 \ll m_2$. This is easy to understand because α_1 enters the mixing matrix only in the expression $m_1 e^{i\alpha_1/2}$, which may be disregarded when m_1 vanishes.

In Table 1, we show the predictions for the probabilities of LFV processes for three selected points in the parameter space. These points are defined in Table 2, and the corresponding coupling matrices are given in Table 3. Finally, the rates of LFV processes in supernova are $2.6xR_{diff}$, $1.8xR_{diff}$, and $2xR_{diff}$ for the respective points I, II, and III.

It can be seen that as soon as the $\mu \to eee$ experimental constraint is made harmless, the most pressing current experimental bound stems from the SINDRUM II experiment on μ–e conversion, which bounds $\mathrm{Br}(\mu \to e\gamma)$ from above. On the other hand, condition (18) leads to the lower bound on $\mathrm{Br}(\mu \to e\gamma)$. As a results, we obtain

$$0.4 \lesssim x \lesssim 6. \tag{33}$$

We note that for vanishing $\lambda_{e\mu}$, the $\mu \to e\gamma$ decay and the μ–e conversion on nuclei proceed only through the virtual τ or ν_τ in the loop. We also note that the tree contributions to the $\mu \to eee$, $\tau^- \to e^+e^-\mu^-$, and $\tau^- \to \mu^+e^-\mu^-$ decays then vanish, and the decays proceed through the exchange of a virtual photon. The width of the $\mu^- \to e^-\gamma^* \to e^-e^+e^-$ process, compared

[4] It is argued in [30] that in the presence of nonstandard neutrino interactions, one should in general extend these ranges, e. g., take $\theta_{12}, \theta_{23} \in [-\pi/2, \pi/2]$. But it is straightforward to verify that in the see-saw type-II model, the transformation $\theta_{12} \to -\theta_{12}$ is equivalent to the transformation $\delta \to \pi - \delta$, $L_\mu \to -L_\mu$, $\mu_R \to -\mu_R$, $L_\tau \to -L_\tau$, $\tau_R \to -\tau_R$ (in the sense that the coupling matrix Λ is changed in the same way under these two transformations), and $\theta_{23} \to -\theta_{23}$ — to $\delta \to \pi - \delta$, $L_e \to -L_e$, $e_R \to -e_R$, $L_\mu \to -L_\mu$, $\mu_R \to -\mu_R$. Therefore, there is no need to extend the ranges of θ_{12} and θ_{23} in the case under consideration.

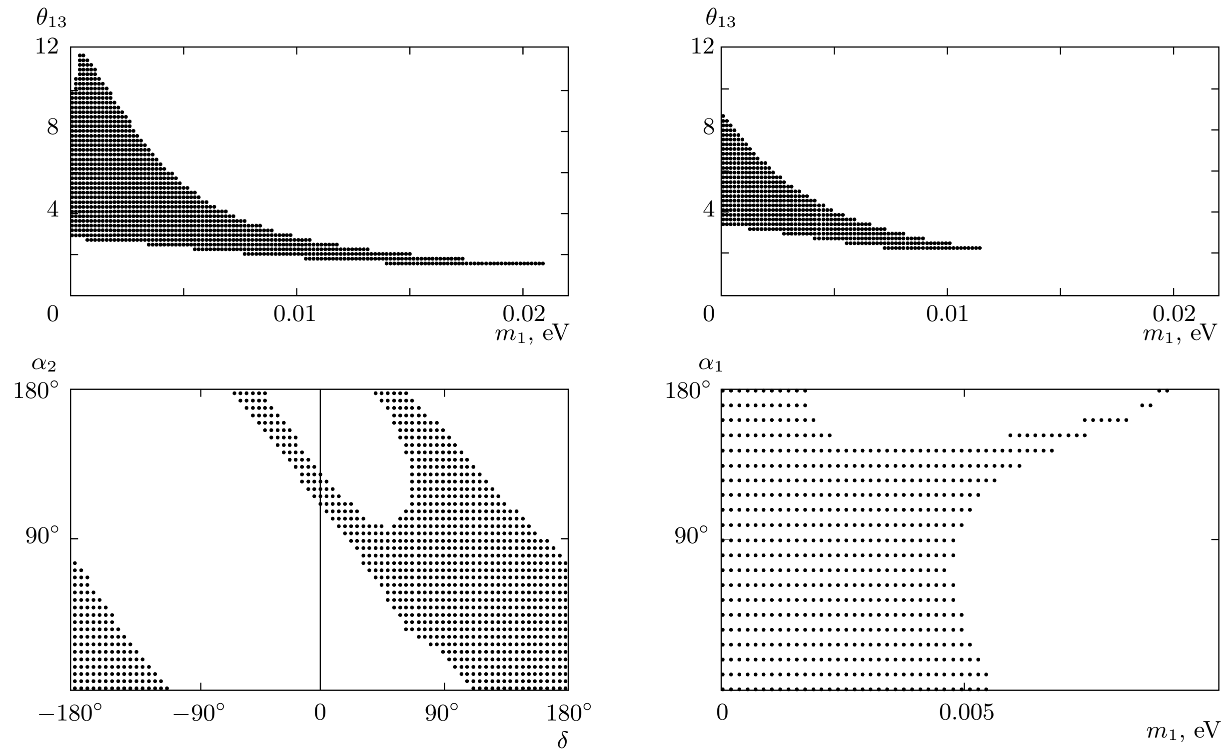

Fig. 2. Projections of the Golden Domain of the see-saw type-II model. The Golden Domain consists of the points in the neutrino mass-mixing parameter space that provide $\mathrm{Br}(\mu \to e\gamma) = x \cdot 10^{-12}$ and fit all the experimental constraints in the framework of the see-saw type-II model. $x = 3$ for the upper right plot and $x = 1$ for the other three plots. For all plots, the mass hierarchy is normal, with the masses m_2 and m_3 related to m_1 through the well-known mass-squared differences, $\Delta m^2_{21} = 0.76 \cdot 10^{-4}$ eV^2 and $\Delta m^2_{31} = 24.3 \cdot 10^{-4}$ eV^2 [31]. The remaining experimentally undetermined parameters are fixed as follows: two upper plots correspond to $\delta = \alpha_1 = \alpha_2 = 0$, the lower left plot to $m_1 = 0$, $\theta_{13} = 5°$, $\alpha_1 = 0$, and the lower right plot to $\theta_{13} = 5°$, $\delta = \alpha_2 = 0$

Table 2. Neutrino mass-mixing parameters for three selected points in the Golden Domain. These reference points are used in Table 1

	m_1	m_2	m_3	θ_{12}	θ_{23}	θ_{13}	δ	α_1, α_2
I	0	$0.9 \cdot 10^{-2}$ eV	$5 \cdot 10^{-2}$ eV	34°	45°	5°	180°	0
II	$0.1 \cdot 10^{-2}$ eV	$0.9 \cdot 10^{-2}$ eV	$5 \cdot 10^{-2}$ eV	34°	45°	8°	180°	0
III	0	$0.9 \cdot 10^{-2}$ eV	$5 \cdot 10^{-2}$ eV	34°	45°	5°	150°	0

to the $\mu \to e\gamma$ decay, is suppressed by α and by the ratio $\Phi_3/\Phi_2 \sim 1/4\pi$ of the three-particle to two-particle phase volumes, but is enhanced by a square of the large logarithm $(\ln m^2_\tau/M^2_\Delta)^2$:

$$\mathrm{Br}(\mu \to eee) \sim \frac{\alpha}{4\pi}\left(\ln\frac{m^2_\tau}{M^2_\Delta}\right)^2 \mathrm{Br}(\mu \to e\gamma) \lesssim \lesssim 10^{-1}\mathrm{Br}(\mu \to e\gamma). \quad (34)$$

Analogous estimates are valid for the $\tau^- \to e^+e^-\mu^-$ and $\tau^- \to \mu^+e^-\mu^-$ decay probabilities. We use these estimates in Table 1.

It follows from Table 3 that $\lambda_{ll'}/M_\Delta[\mathrm{TeV}] < 0.05$. This allows estimating the contribution of the new scalar field to the anomalous muon magnetic moment: $\delta a \sim m^2_\mu A^R_2 < 2 \cdot 10^{-13}$. This is well beyond the present experimental sensitivity.

Table 3. Coupling matrices corresponding to three selected points in the Golden Domain (see Table 2). Each matrix should be multiplied by $x^{1/4}M_\Delta/(1\ \mathrm{TeV})$

I	II
$\begin{pmatrix} 0.0053 & 0 & -0.099 \\ 0 & 0.048 & 0.037 \\ -0.099 & 0.037 & 0.047 \end{pmatrix}$	$\begin{pmatrix} 0.0057 & -0.0026 & -0.0093 \\ -0.0026 & 0.037 & 0.028 \\ -0.0093 & 0.028 & 0.036 \end{pmatrix}$
III	
$\begin{pmatrix} 0.0048-0.0005i & 0.0006+0.0027i & -0.0089+0.0027i \\ 0.0006+0.0027i & 0.046+0.0003i & 0.036+0.00001i \\ -0.0089+0.0027i & 0.036+0.00001i & 0.045-0.00028i \end{pmatrix}$	

6. COMPARISON WITH OTHER MODELS

6.1. Singlet bilepton model

As is clear from the above discussion, the strong experimental bounds on the $\mu \to eee$ decay and μ–e conversion on Au create a certain pressure on the allowed range of the probability of the $\mu \to e\gamma$ decay in the see-saw type-II model. It is interesting to note that there exists a "close relative" of the see-saw type-II model in which this pressure is completely absent. This is a simple model that extends the Standard Model by one charged heavy bilepton (i. e., scalar with the lepton number 2) coupled to leptons as follows [32]:

$$\mathcal{L}_{ll\tilde{\Delta}} = \sum_{l,l'} \tilde{\lambda}_{ll'} \overline{L_l^c} i\tau_2 L_{l'} \tilde{\Delta} + \mathrm{H.c.} \tag{35}$$

Such a scalar also appears in more sophisticated extensions of the Standard Model, e. g., in the Zee–Babu model of loop neutrino mass generation [33]. The difference from Eq. (4) is that $\tilde{\Delta}$ is a singlet and Δ is a triplet (see [34] for a systematical classification of bileptons). An important feature of the above coupling is that the coupling matrix $||\tilde{\lambda}_{ll'}||$ is antisymmetric, in contrast to a symmetric coupling matrix in the see-saw type-II model. As a consequence, the $\mu \to eee$ decay is forbidden at the tree level (as are the τ-lepton decays with two identical leptons in the final state). As regards the μ–e conversion, its probability does not obtain a large $\ln^2$ enhancement because only neutrinos (not charged leptons) enter the loop of the corresponding penguin diagram. At the same time, the $\mu \to e\gamma$ decay probability is of the same order as in the see-saw type-II model. The LFV processes in supernova cannot proceed at the tree level because the corresponding tree amplitudes would be proportional to λ_{ee}. However, e or ν_e may change flavor while scattering on a charged particle through the exchange of a virtual photon in the t-channel. An example of such a process is

$$\nu_e p \to \nu_\mu p. \tag{36}$$

We note that in the case of neutrino scattering, a charged lepton enters the loop of the penguin diagram, and the cross section acquires the $\ln^2$ enhancement. A detailed study of LFV in this singlet bilepton model will be carried out elsewhere.

6.2. MSSM

In the MSSM, all LFV processes proceed through loop diagrams. The radiative decays proceed through penguin diagrams, while the three-lepton decays of μ and τ, through the box diagrams and (for some decays) through the penguin diagrams with a virtual photon decaying into the lepton–antilepton pair. Therefore, generically $\mathrm{Br}(\mu \to eee) \sim g^2 \mathrm{Br}(\mu \to e\gamma)$. Moreover, heavy sleptons (not light charged leptons) enter loop diagrams, and therefore there is no $\ln^2$ enhancement of the μ–e conversion probability. Hence, the above-mentioned pressure of strong experimental bounds on the $\mu \to eee$ and μ–e conversion probabilities is absent in the MSSM. But the absence of the tree-level LFV processes and of a logarithmic enhancement of the γ^* emission amplitude generically severely suppresses the LFV rate in supernova.

We note that in the MSSM, the vertices in the above-mentioned penguin and box diagrams contain the elements of the unitary PMNS matrix U. Therefore, if all sleptons are degenerate, then all the LFV probabilities are zero due to the GIM mechanism. By contrast, in the above discussed models with bileptons, $||\lambda_{ll'}||$ and $||\tilde{\lambda}_{ll'}||$ are not unitary matrices and therefore the GIM mechanism does not work.

The degeneracy of sleptons is removed by the heavy τ-lepton. Therefore, the amplitudes of LFV processes in the MSSM are proportional to $\sin\theta_{13}$ (see, e.g., a recent paper [35] and the references therein). In the Golden Domain of the see-saw type-II model, θ_{13} also cannot be too small (see Eq. (31)), but this similarity between the see-saw type-II and MSSM is accidental.

7. SUMMARY AND CONCLUSIONS

We have discussed a number of requirements on the lepton flavor violation in the see-saw type-II model. Apart from the mandatory requirement of satisfying all the experimental bounds, we impose two supplementary requirements that severely constraint the parameter space of the model. The first one was previously discussed in the literature [13, 14]: the branching ratio of the $\mu \to e\gamma$ decay is of the order of 10^{-12} (which ensures its soon discovery at MEG), i.e., close to the experimental upper bound on the branching ratio of the $\mu \to eee$ decay. This is possible only if the tree amplitude of the $\mu \to eee$ decay is suppressed by a vanishing (or very small) coupling constant, either λ_{ee} or $\lambda_{e\mu}$. The second requirement is that the rates of LFV processes in supernova are high enough to alter the supernova physics (such alteration may facilitate the explosion). This is possible in some region of the parameter space [5, 7], in particular, where $\lambda_{e\mu} \approx 0$, but not where $\lambda_{ee} \approx 0$. As a consequence of the imposed requirements, we obtain a "Golden Domain" in the neutrino mass-mixing parameter space.

In the Golden Domain, the experimental results on the μ–e conversion on gold [26] impose the most restrictive upper bound on $\mathrm{Br}(\mu \to e\gamma)$, as is clear from Table 1. On the other hand, condition (18) on the LFV rates in supernova provides a lower bound. In total, the imposed constraints appear to be strong enough to force $\mathrm{Br}(\mu \to e\gamma)$ to lie in a narrow window,

$$0.4 \cdot 10^{-12} \lesssim \mathrm{Br}(\mu \to e\gamma) \lesssim 6 \cdot 10^{-12} \qquad (37)$$

(see Eq. (33)). We should take into account that the upper bound corresponds to the minimal experimentally allowed scalar mass 150 GeV; this bound becomes tighter if the mass is increased.

The branching ratios of the LFV τ decays in the Golden Domain of the see-saw type-II model are presented in Table 1. Evidently, the most promising decay is $\tau \to \mu\mu\mu$. In the Golden Domain, we have

$$1.4 \cdot 10^{-10} \lesssim \mathrm{Br}(\tau \to \mu\mu\mu) \lesssim 7 \cdot 10^{-9}, \qquad (38)$$

which is not too far from the current experimental bound.

A nice feature of the see-saw type-II model is that the coupling matrix determines the mass-mixing pattern of neutrinos. In the Golden Domain, the neutrino mass hierarchy is normal, the angle θ_{13} is moderately large ($2°$–$12°$), the phases δ and α_2 satisfy $||\delta + \alpha_2| - 180°| \lesssim 40°$, and α_1 is loosely bounded.

To conclude, we have considered a scenario of lepton flavor violation in the see-saw type-II model leading to alteration of supernova dynamics and manifesting itself in a variety of phenomenological consequences observable in the current and forthcoming experiments, including $\mu \to e\gamma$ searches at MEG, $\Delta^{\pm\pm}$ searches at the LHC, $\tau \to \mu\mu\mu$ searches at super-B factories, μ–e conversion searches at Mu2e (Fermilab) and COMET (J-PARC), and θ_{13} searches in short-base reactor disappearance and accelerator ν_e-appearance experiments. On the other hand, in the considered scenario, a direct neutrino mass measurement (the KATRIN experiment) and $2\beta0\nu$ detection will be unaccessible in the near future due to low neutrino masses.

We have also briefly outlined a scenario of lepton flavor violation in the singlet-bilepton model, which demonstrates drastically different signatures, which can nevertheless be probed in the future experiments.

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REFERENCES

1. E. W. Kolb, D. L. Tubbs, and D. A. Dicus, Astrophys. J. **255**, L57 (1982).

2. G. A. Fuller, R. W. Mayle, J. R. Wilson, and D. N. Schramm, Astrophys. J. **322**, 795 (1987).

3. P. S. Amanik, G. M. Fuller, and B. Grinstein, Astropart. Phys. **24**, 160 (2005).

4. P. S. Amanik and G. M. Fuller, Phys. Rev. D **75**, 083008 (2007).

5. O. Lychkovskiy, S. Blinnikov, and M. Vysotsky, Europ. Phys. J. C **67**, 213 (2010).

6. O. Lychkovskiy and S. Blinnikov, Phys. Atom. Nucl. **73**, 614 (2010).

7. O. Lychkovskiy, S. Blinnikov, and M. Vysotsky, arXiv:1010.0883 [hep-ph].

8. M. Magg and C. Wetterich, Phys. Lett. B **94**, 61 (1980); G. Lazarides, Q. Shafi, and C. Wetterich, Nucl. Phys. B **181**, 287 (1981); R. N. Mohapatra and G. Senjanovic, Phys. Rev. D **23**, 165 (1981).

9. S. Mihara [MEG Collaboration], Nucl. Phys. A **844**, 150C (2010).

10. G. Cavoto, arXiv:1012.2110 [hep-ex].

11. M. L. Brooks et al. [MEGA Collaboration], Phys. Rev. Lett. **83**, 1521 (1999); arXiv:hep-ex/9905013.

12. U. Bellgardt et al. [SINDRUM Collaboration], Nucl. Phys. B **299**, 1 (1988).

13. M. Kakizaki, Y. Ogura, and F. Shima, Phys. Lett. B **566**, 210 (2003); arXiv:hep-ph/0304254.

14. E. J. Chun, K. Y. Lee, and S. C. Park, Phys. Lett. B **566**, 142 (2003); arXiv:hep-ph/0304069.

15. J. Schechter and J. W. F. Valle, Phys. Rev. D **22**, 2227 (1980).

16. A. G. Akeroyd, M. Aoki, and H. Sugiyama, Phys. Rev. D **77**, 075010 (2008); arXiv:0712.4019 [hep-ph].

17. V. M. Abazov et al. [D0 Collaboration], Phys. Rev. Lett. **101**, 071803 (2008); arXiv:0803.1534 [hep-ex].

18. D. E. Acosta et al. [CDF Collaboration], Phys. Rev. Lett. **93**, 221802 (2004); arXiv:hep-ex/0406073.

19. A. G. Akeroyd and H. Sugiyama, arXiv:1105.2209 [hep-ph].

20. A. Burrows and J. M. Lattimer, Astrophys. J. **307**, 178 (1986).

21. Joseph Chen-Yu Wang, PhD dissertation at University of Texas at Austin (1998); http://en.wikiversity.org/wiki/BoomCode.

22. K. Nakamura et al. [Particle Data Group], J. Phys. G **37**, 075021 (2010), p. 95; http://pdglive.lbl.gov/FFconlaw.brl?lawin=LF&group=CNXX&inscript==N&fsizein=1.

23. W. H. Bertl et al. [SINDRUM II Collaboration], Europ. Phys. J. C **47**, 337 (2006).

24. T. Suzuki, D. F. Measday, and J. P. Roalsvig, Phys. Rev. C **35**, 2212 (1987).

25. A. G. Akeroyd, M. Aoki, and H. Sugiyama, Phys. Rev. D **79**, 113010 (2009); arXiv:0904.3640 [hep-ph].

26. W. H. Bertl et al. [SINDRUM II Collaboration], Europ. Phys. J. C **47**, 337 (2006).

27. S. Weinberg and G. Feinberg, Phys. Rev. Lett. **3**, 111 (1959).

28. A. Czarnecki, W. J. Marciano, and K. Melnikov, AIP Conf. Proc. **435**, 409 (1998); arXiv:hep-ph/9801218.

29. R. Kitano, M. Koike, and Y. Okada, Phys. Rev. D **66**, 096002 (2002); Erratum, ibid **76**, 059902 (2007); arXiv:hep-ph/0203110.

30. A. de Gouvea and J. Jenkins, Phys. Rev. D **78**, 053003 (2008); arXiv:0804.3627 [hep-ph].

31. K. Nakamura et al. [Particle Data Group], J. Phys. G **37**, 075021 (2010), p. 571; http://pdglive.lbl.gov//Rsummary.brl?nodein=S067.

32. W. Konetschny and W. Kummer, Phys. Lett. B **70**, 433 (1977).

33. A. Zee, Nucl. Phys. B **264**, 99 (1986); K. S. Babu, Phys. Lett. B **203**, 132 (1988).

34. F. Cuypers and S. Davidson, Europ. Phys. J. C **2**, 503 (1998).

35. M. Davidkov and D. I. Kazakov, arXiv:1102.1582 [hep-ph].