ЖЭТФ, 2012, том 142, вып. 1 (7), стр. 77–84

MECHANISM OF “GSI OSCILLATIONS” IN ELECTRON CAPTURE BY HIGHLY CHARGED HYDROGEN-LIKE ATOMIC IONS

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Received January 11, 2012

We suggest a qualitative explanation of oscillations in electron capture decays of hydrogen-like ^{140}Pr and ^{142}Pm ions observed recently in an ion experimental storage ring (ESR) of Gesellschaft für Schwerionenforschung (GSI) mbH, Darmstadt, Germany. This explanation is based on the electron multiphoton Rabi oscillations between two Zeeman states of the hyperfine ground level with the total angular momentum $F = 1/2$. The Zeeman splitting is produced by a constant magnetic field in the ESR. Transitions between these states are produced by the second, sufficiently strong alternating magnetic field that approximates realistic fields in the GSI ESR. The Zeeman splitting amounts to only about 10^{-5} eV. This allows explaining the observed quantum beats with the period 7 s.

1. INTRODUCTION

The authors of Ref. [1] reported recently on time-modulated weak decays observed in the orbital electron capture of hydrogen-like $^{140}\mathrm{Pr}_{58+}$ and $^{142}\mathrm{Pm}_{60+}$ ions (these ions have odd–odd nuclei) coasting in the ion experimental storage ring (ESR) in Gesellschaft für Schwerionenforschung (GSI) mbH, Darmstadt, Germany. Using a nondestructive single-ion time-resolved Schottky mass spectrometry, they found that an exponential decay is modulated in time with a modulation period of about 7 seconds for both ions. The authors of Ref. [1] attributed this observation to a coherent superposition of finite-mass eigenstates of the electron neutrinos from the weak decay into a two-body final state. This idea was developed in Ref. [2], where time modulation was explained in terms of the interference of two massive-neutrino mass eigenstates. But it was concluded in Ref. [3] that the decay rate measured at GSI cannot oscillate using approach in Ref. [2] if only standard physics of the weak interaction is involved.

Further, another explanation was proposed in [4]. It is based on a mechanism related to Rabi multiphoton oscillations between atomic hyperfine levels that can offer such a period. The Rabi multiphoton frequency was derived using the multiphoton perturbation theory.

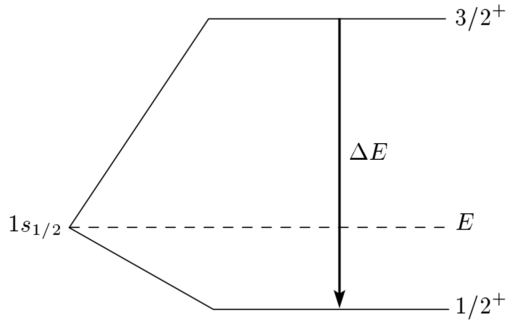

Fig. 1. Hyperfine splitting of the ground $1s$ state for odd–odd nuclei $^{140}\mathrm{Pr}_{59}$ and $^{142}\mathrm{Pm}_{61}$

Only a weak perturbing oscillating (magnetic) field was considered in [4]. Here, we extend this approach to a strong perturbing field based on our previous adiabatic approximations for a two-level system [5, 6]. The nuclear spin of odd–odd nuclei $^{140}\mathrm{Pr}_{59}$ and $^{142}\mathrm{Pm}_{61}$ is equal to one. The energy of the hydrogen-like $1s$ state with the nuclear charge $Z = 59$ is equal to

$$E = mc^2\sqrt{1-(Z\alpha)^2} - mc^2 \approx -52\ \mathrm{keV}.$$

This level is splits (Fig. 1) into two hyperfine levels with the total angular momentum $F = 1/2$ (lower) and $F = 3/2$ (higher). The electron capture decay from the $F = 3/2$ state is forbidden because the fully ionized daughter nucleus has the spin $I = 0$. 50 % of

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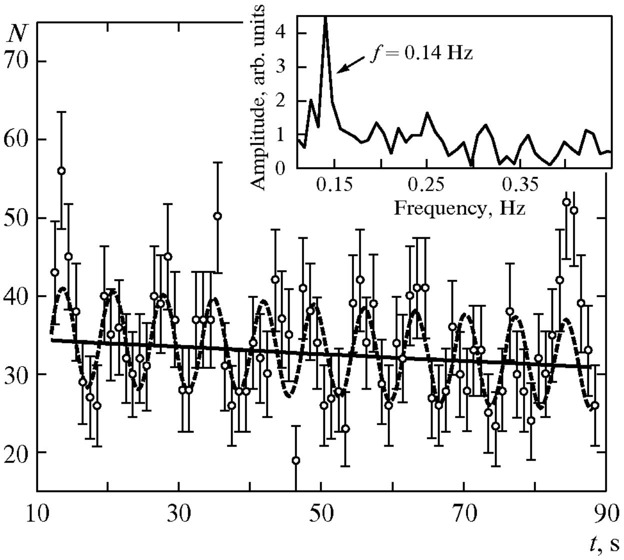

Fig. 2. The number N of electron capture decays of hydrogen-like $^{140}\mathrm{Pr}_{59}$ ions per second as a function of the time t after the injection into the experimental storage ring [1]. The inset shows the fast fourier transform of these data

the decay of the $^{140}_{81}\mathrm{Pr}_{59}$ nucleus is the Gamov–Teller $1^+ \to 0^+$ β^+-decay:

$$^{140}_{81}\mathrm{Pr}_{59} \to {}^{140}_{82}\mathrm{Ce}_{58} + e^+ + \nu_e.$$

The other 50 % of the decay of the $^{140}_{81}\mathrm{Pr}_{59}$ nucleus is an electron capture:

$$^{140}_{81}\mathrm{Pr}_{59} + e^- \to {}^{140}_{82}\mathrm{Ce}_{58} + \nu_e.$$

The relativistic hyperfine splitting of the $1s_{1/2}$ state with the charge Z for nuclear spin 1 is [7]

$$\Delta E = \frac{2Z^3}{\sqrt{1-(Z\alpha)^2}\left[2\sqrt{1-(Z\alpha)^2}-1\right]}\,\frac{\mu}{\mu_N}\,\frac{\mathrm{Ry}^2}{M_p c^2},$$

where μ_N is the nuclear magneton and $\mu = +0.88\mu_N$ is the magnetic moment of the odd–odd nucleus with nuclear spin 1. In the case of the $^{140}_{81}\mathrm{Pr}_{59}$ nucleus, this splitting is $\Delta E = 0.4$ eV. The M1 spontaneous gamma-decay $3/2^+ \to 1/2^+$ occurs during the short time

$$\tau = \frac{9}{4}\,\frac{\hbar^3}{me^4}\,\frac{e^2}{\hbar c}\left(\frac{mc^2}{\Delta E}\right)^3 = 0.03 \text{ s},$$

and we should therefore discuss electron capture from the lower hyperfine state $F = 1/2$.

The experimental data in Ref. [1] are shown in Fig. 2. The number of electron capture decays of hydrogen-like $^{140}\mathrm{Pr}$ ions per second is given as a function of time. The case of the $^{142}\mathrm{Pm}_{61}$ ion is similar.

In the GSI storage ring, ultrarelativistic atomic ions circulate in the plane perpendicular to the strong magnetic field about $H_0 = 0.1$ T. This is an average value of the field because the actual value varies. The radius of the circle in the ring is $r = 15$ m. The Zeeman splitting of the lower state $F = 1/2$ is $\hbar\omega_0 = 2\mu_e H_0 = 1.2\cdot 10^{-5}$ eV, where μ_e is an electron Bohr magneton. Hence, $\omega_0 \sim 10^{10}$ 1/s. The lower state is $(1/2, -1/2)$ and the upper state is $(1/2, +1/2)$. The second oscillating magnetic field in the ESR produces transitions between these states (the focusing of the ion beam occurs due to several quadrupole magnets). The oscillating magnetic field $h\sin\omega t$ is directed in the plane perpendicular to the constant magnetic field. The field has the frequency of the order of $\omega \sim Nc/r \sim 10^9$ 1/s. Here, $N \sim 100$ is the possible number of magnetic devices on the ring. Hence, $\omega_0/\omega \sim 10$.

2. ADIABATIC APPROXIMATION FOR A TWO-LEVEL SYSTEM IN A STRONG OSCILLATING FIELD

The goal of this section is to investigate the interaction of a strong classical magnetic field with a two-level system in the case of a multiphoton resonance [5]. The basic assumption is the smallness of the frequency of the field in comparison with the separation between levels (in atomic units), i. e., $\omega \ll \omega_0$. Of course, the well-known results of time-dependent perturbation theory are not applicable for strong alternating fields. We use the adiabatic approximation to calculate the rate for transition from the lower state $(+1/2)$ into the upper state $(-1/2)$. As is well known, it is mathematically equivalent to the WKB approximation for the problem of above-barrier reflection. We direct the constant magnetic field producing the Zeeman splitting along the z axis. We also first direct the oscillating magnetic field $h\sin(\omega t)$ along the x axis.

We seek eigenstates of the adiabatic Schrödinger equation

$$\hat{H}(t)\Psi(t) = E(t)\Psi(t),$$

$$\hat{H}(t) = \mu_e\hat{\sigma}_z H_0 + \mu_e\hat{\sigma}_x h\sin(\omega t),$$

where $\hat{\sigma}_x$ and $\hat{\sigma}_z$ are Pauli matrices. The wave function is presented in the form of a superposition of the unperturbed lower and upper states

$$\Psi = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}.$$

We obtain the following equations for a_1 and a_2:

$$E(t)a_1 = -\frac{\hbar\omega_0}{2}\, a_1 + \mu_e h \sin(\omega t) a_2,$$
$$E(t)a_2 = +\frac{\hbar\omega_0}{2}\, a_2 + \mu_e h \sin(\omega t) a_1. \quad (1)$$

From system (1), we find that the energy eigenvalues are given by

$$E_{1,2}(t) = \mp\frac{\hbar\omega_0}{2}\sqrt{1+q^2\sin^2(\omega t)}\,, \quad (2)$$

where we introduce the dimensionless quantity

$$q = \frac{2\mu_e h}{\hbar\omega_0} = \frac{h}{H_0}\,. \quad (3)$$

Similarly to the problem of above-barrier reflection, we are only interested in the complex turning points t_k that lie in the upper half-plane. They are determined from the condition $E_1(t_k) = E_2(t_k)$, whence

$$t_K = \frac{k\pi}{\omega} + \frac{i}{\omega}\,\mathrm{arcsh}\,\frac{1}{q}, \quad k = 0, \pm 1, \pm 2, \ldots \quad (4)$$

These points are the fundamental branch points for $E_{1,2}(t)$.

We first consider the point t_0. According to [8, 9], we obtain the following result for the contribution to the transition probability introduced by the point t_0 ($k = 0$):

$$w_{12} = \exp\left\{-2\,\mathrm{Im}\int_{t_1}^{t_0}[E_2(t) - E_1(t)]\,dt\right\}. \quad (5)$$

Here, t_1 is an arbitrary point on the real time axis. Evaluating the integral, we find

$$w_{12} = R^2,$$
$$R = \exp\left\{-\frac{\omega_0}{\omega\sqrt{1+q^2}}\, D\left(\frac{1}{\sqrt{1+q^2}}\right)\right\}, \quad (6)$$

where $D(x)$ is the complete elliptic integral of the third kind. It follows that the quantity w_{12} is exponentially small in terms of the adiabatic parameter $\omega_0/\omega \gg 1$. Referring to this formula, we also emphasize that the unit pre-exponential factor is exact.

Taking the turning points with $k \neq 0$ into account (they all lie at the same distance from the real time axis) allows passing from absolute probabilities to probabilities per unit time (rates). According to the principle of superposition of quantum mechanics [10], the total amplitude A_{12} for a transition into the upper state is a sum of the amplitudes a_k associated with the individual turning points t_k. The resonance condition is that these amplitudes be added together coherently. The phase factor $\exp(iS)$ appears in the amplitude a_k in connection with the transition from a given turning point to the next. Here, the quantity

$$S = \frac{\omega_0}{\omega}\int_0^{\pi}\sqrt{1+q^2\sin^2\varphi}\,d\varphi \quad (7)$$

represents the accumulation of the classical action between neighboring turning points. The minus sign appears in front of the exponential in the phase factor because the WKB wave contains the factor $[E(t)]^{-1/2}$; it changes sign during the transition from the point t_k to the point t_{k+1}. It follows from Eq. (7) that

$$S = \frac{\omega_0}{\omega}\sqrt{1+q^2}\,E\left[\frac{q}{\sqrt{1+q^2}}\right], \quad (8)$$

where $E(x)$ is the complete elliptic integral of the second kind. The quantity S is a semiclassical phase (classical action) taken on the temporal interval $[0, \pi/\omega]$.

Summing the amplitudes from the N turning points and taking the absolute value squared of A_{12}, we find

$$|A_{12}|^2 = w_{12}\frac{\sin^2[N(S-\pi)/2]}{\sin^2[(S-\pi)/2]}. \quad (9)$$

The resonance condition has the form

$$(S-\pi)/2 = m\pi + \gamma, \quad \gamma \to 0, \quad (10)$$

where m is an integer. The time interval T is related to N by the formula $T = \pi N/\omega$. Finally, we express the transition rate as

$$W_{12} = \frac{|A_{12}|^2}{T} = \frac{2\omega^2}{\pi}\times$$
$$\times\,\delta\left[\frac{\omega_0}{\pi\omega}\int_0^{\pi}\sqrt{1+q^2\sin^2\varphi}\,d\varphi - K\omega\right], \quad (11)$$

where $K = 2m+1$. Therefore, the transition only occurs in odd harmonics. The δ-function in Eq. (11) expresses the energy conservation law, and its argument contains the magnitude of the energy shift due to the external field (the Stark effect).

We next proceed to an analysis of the transition rate W_{12}. If $Kq^2 \ll 1$, we use Eq. (11) to obtain the transition rate in the perturbation theory:

$$W_{12} = \frac{2\omega^2}{\pi}\left(\frac{eq}{4}\right)^{2K}\delta(\omega_0 - K\omega). \quad (12)$$

We see that the perturbation theory validity criterion with regard to the transition rate is more stringent than with regard to the Stark shift. On the other hand, we write the well-known expression for the transition rate obtained by direct application of time-dependent perturbation theory:

$$W_{12}(\text{exact}) = \frac{2\pi\omega^2}{[(K-1)!!]^4}\left(\frac{Kq}{4}\right)^{2K}\delta(\omega_0-K\omega). \quad (13)$$

Using Stirling's formula, it is easy to verify that Eqs. (12) and (13) agree exactly for $K \gg 1$. It turns out that the WKB approximation works reasonably well even for small values of K. For example, the ratio of these rates is equal to 0.75 for $K = 1$.

The adiabatic linearly independent basic functions are given by

$$\begin{aligned} \psi_1(t) &= \begin{pmatrix} \cos(\theta(t)/2) \\ -\sin(\theta(t)/2) \end{pmatrix} \exp\left[i\int_0^t \Omega(\tau)\,d\tau\right], \\ \psi_2(t) &= \begin{pmatrix} \sin(\theta(t)/2) \\ \cos(\theta(t)/2) \end{pmatrix} \exp\left[-i\int_0^t \Omega(\tau)\,d\tau\right], \end{aligned} \quad (14)$$

where we introduce the notation

$$\begin{aligned} \operatorname{tg}\theta(t) &= q\sin(\omega t), \\ \Omega(t) &= \frac{\omega_0}{2}\sqrt{1+q^2\sin(\omega t)}\,. \end{aligned} \quad (15)$$

The general solution on the temporal interval $[0, \pi/\omega]$ is a superposition of the linearly independent basic functions, Eq. (14):

$$\Psi = C_1\psi_1 + C_2\psi_2.$$

We continue this solution analytically along the real axis into the next temporal interval $[\pi/\omega, 2\pi/\omega]$, by passing the branch point t_1 (for the function $\psi_1(t)$) and t_1^* (for the function $\psi_2(t)$). The Hamiltonian $\hat{H}(t)$ is invariant under the transformation

$$\hat{T} = \hat{\sigma}_z \exp\left(\frac{\pi}{\omega}\frac{\partial}{\partial t}\right)$$

which is a translation by π/ω in the time t and a simultaneous rotation through the angle π around the z axis in the energy–spin space. Therefore, the solutions can be chosen such that they are eigenfunctions of $\hat{T}$:

$$\hat{T}\Psi(t) = \hat{\sigma}_z\Psi(t+\pi/\omega) = \exp(i\pi\varepsilon/\omega)\Psi(t). \quad (16)$$

Because applying the transformation $\hat{T}$ twice gives translation by a whole period, these solutions describe states with a definite quasi-energy ε.

Using conditions (16), we obtain the following equations for the coefficients C_1 and C_2:

$$\begin{aligned} &C_1\left[\exp(i\pi\varepsilon/\omega) - \sqrt{1-R^2}\exp(iS)\right] = \\ &\qquad = C_2R\exp(-iS), \\ &C_2\left[\exp(i\pi\varepsilon/\omega) + \sqrt{1-R^2}\exp(-iS)\right] = \\ &\qquad = C_1R\exp(iS). \end{aligned} \quad (17)$$

The quantities R and S are given by Eqs. (6) and (8). Equating the determinant of the system (17) to zero, we obtain the equation for the quasi-energy

$$\sin\left(\frac{\pi\varepsilon}{\omega}\right) = \sqrt{1-R^2}\,\sin S, \quad (18)$$

which has two solutions, ε and $\varepsilon' = \pi - \varepsilon$.

In the neighborhood of a multiphoton resonance, the semiclassical phase is

$$S = \frac{(2K+1)\pi}{2} + \Delta S, \quad \Delta S \to 0$$

and the quasi-energy is of the form

$$\varepsilon = \frac{(2K+1)\omega}{2} \pm \frac{\omega}{\pi}\left[(\Delta S)^2 + R^2\right]^{1/2}. \quad (19)$$

Corresponding to the two values of the quasi-energy, ε and $\varepsilon' = \pi - \varepsilon$, there are two sets of coefficients C_1 and C_2:

$$\begin{aligned} C_1^{\pm} &= \left\{\frac{1}{2}\left[1 \pm \sqrt{\frac{1-R^2}{1+R^2\operatorname{tg}^2 S}}\right]\right\}^{1/2}, \\ C_2^{\pm} &= \pm\left\{\frac{1}{2}\left[1 \mp \sqrt{\frac{1-R^2}{1+R^2\operatorname{tg}^2 S}}\right]\right\}^{1/2}. \end{aligned} \quad (20)$$

When the interaction is turned off adiabatically, the solution Ψ^+ goes over into the wave function

$$\Psi^+ \to \begin{pmatrix} 1 \\ 0 \end{pmatrix} \exp\left(\frac{1}{2}i\omega_0 t\right)$$

which describes an electron in the lower hyperfine state. It suffices to define the wave function of a quasi-energy state on the temporal interval $[0, \pi/\omega]$ only, because it can be constructed easily on other temporal intervals by means of condition (16).

The probability of a transition from the lower hyperfine state $(1/2, +1/2)$ to the upper hyperfine state $(1/2, -1/2)$ during the adiabatic turning on of the perturbation is determined by the square of the lower component of the spinor Ψ^+:

$$W(t) = \frac{1}{2}\left\{1 - \cos\theta(t)\sqrt{\frac{1-R^2}{1+R^2\,\mathrm{tg}^2\,S}} - \sin\theta(t)\times\right.$$
$$\left.\times\frac{R\cos\left[S - 2\int\limits_0^t \Omega(\tau)\,d\tau\right]}{\sqrt{\cos^2 S + R^2\sin^2 S}}\right\}. \qquad (21)$$

The quantity $\Omega(\tau)$ is defined in Eq. (15). The last term in this expression describes rapid oscillations with a small amplitude except in the neighborhood of a multiphoton resonance, where these oscillation are modulated by the function $\sin\theta(t)$.

Averaging (21) over time leads to the transition probability from the lower hyperfine state $(1/2, +1/2)$ to the upper hyperfine state $(1/2, -1/2)$, which is independent of time:

$$W = \frac{1}{2}(1-P),$$
$$P = \frac{2}{\pi}\sqrt{\frac{1-R^2}{(1+q^2)(1+R^2\,\mathrm{tg}^2\,S)}}\,K\left(\frac{q}{\sqrt{1+q^2}}\right). \qquad (22)$$

Here, $K(x)$ is the complete elliptic integral of the first kind. At the resonance point, $P = 0$ and $W = 1/2$. That the populations are equal at the exact resonance is a fact not related to the adiabatic approximation, and is also true for the exact solution of the problem.

We now turn to the case of instantaneous turning on the perturbation. In this case, the solution is a superposition of the quasi-energy states Ψ^+ and Ψ^-. The coefficients of this superposition are found from the condition that at the initial instant $t = 0$, the system is in the lower state. Omitting the intermediate steps, we find that the average probability of population of the upper level is

$$W = \frac{1}{2}(1-P^2), \qquad (23)$$

where the value of P is given by Eq. (22).

At points where the condition

$$\frac{\omega_0}{\omega}\sqrt{1+q^2}\,E\left(\frac{q}{\sqrt{1+q^2}}\right) = \frac{2K+1}{2}\pi \qquad (24)$$

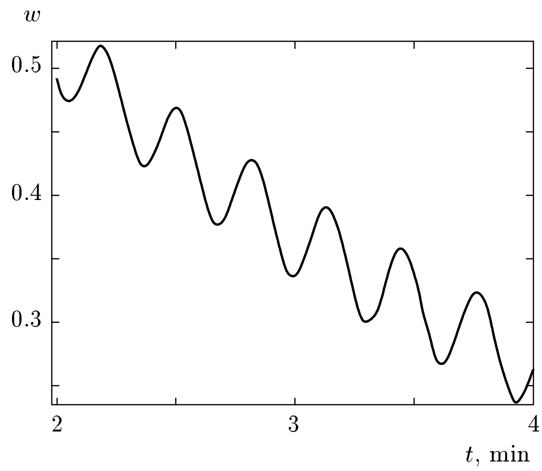

Fig. 3. Derived dependence of the capture decay on time t taking Rabi oscillations into account

holds (with K an integer), the occupation probability for each level is $1/2$. Condition (24) gives the positions of multiphoton resonances in the case of a strong field and is independent of the way the external field is turned on, adiabatically or instantaneously.

According to Eq. (19) the quasi-energies in the exact K-photon resonance are of the form

$$\varepsilon = \frac{(2K+1)\omega}{2} \pm \frac{\omega}{\pi}R.$$

Hence, the Rabi frequency of oscillations between two levels (in the case of an exact multiphoton resonance) is

$$\Omega_{Rabi} = \frac{\omega}{\pi}R =$$
$$= \frac{\omega}{\pi}\exp\left\{-\frac{\omega_0}{\omega}\,\frac{D\left[(1+q^2)^{-1/2}\right]}{\sqrt{1+q^2}}\right\}. \qquad (25)$$

Substituting $\omega_0/\omega \sim 10$ (see above) and using an asymptotic expression for the elliptic integral, we obtain

$$\Omega_{Rabi} = 10^9\exp\left\{-10\ln\frac{4H_0}{h}\right\}\ \mathrm{s}^{-1}. \qquad (26)$$

The experimental value of the multiphoton Rabi frequency is of the order of $0.1\ \mathrm{s}^{-1}$. It follows from Eq. (26) that $h = 0.4H_0 = 0.04$ T, which is in agreement with experimental data.

We assume that the capture rate from the upper sublevel $(m = +1/2)$ is equal to $\lambda + \delta$ and the capture rate from the lower sublevel $(m = -1/2)$ is $\lambda - \delta$. The electron capture decay is modulated by Rabi oscillations:

$$w(t) = [\mathrm{ch}(\delta t) + \mathrm{sh}(\delta t)\cos(2\Omega_{Rabi}t)]\exp(-\lambda t),$$

$$\delta \ll \lambda \ll \Omega_{Rabi}.$$

In particular, if we set $\lambda = 1/3\ \text{min}^{-1}$, $\delta = 1/30\ \text{min}^{-1}$, and $\Omega_{Rabi} = 10\ \text{min}^{-1}$, then the time dependence of the electron capture decay is as depicted in Fig. 3.

3. TWO-LEVEL SYSTEM IN A CIRCULARLY POLARIZED FIELD

In the preceding sections, we assumed that the oscillating magnetic field is directed along the x axis. We now consider the evolution of a two-level system in a circularly polarized magnetic field in the plane (x, y) that is perpendicular to the constant magnetic field, which is directed along the z axis.

We seek a solution of the Schrödinger equation

$$i\hbar \frac{\partial \Psi(t)}{\partial t} = \hat{H}(t)\Psi(t),$$

$$\hat{H}(t) = \mu_e \hat{\sigma}_z H_0 + \mu_e h \left(\hat{\sigma}_x \cos(\omega t) + \hat{\sigma}_y \sin(\omega t)\right),$$

where $\hat{\sigma}_x$, $\hat{\sigma}_y$, $\hat{\sigma}_z$ are the Pauli matrices. The wave function is presented in the form of a superposition of the unperturbed lower and upper states

$$\Psi = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}.$$

We obtain the following equations for the quantities a_1 and a_2:

$$\begin{aligned} i\hbar \frac{da_1}{dt} &= \mu_e H_0 a_1 + \mu_e h \exp(-i\omega t)\, a_2, \\ i\hbar \frac{da_2}{dt} &= -\mu_e H_0 a_2 + \mu_e h \exp(i\omega t)\, a_1. \end{aligned} \tag{27}$$

The exact solution of these equations satisfying the initial condition $a_1(0) = 1$, $a_2(0) = 0$ is of the form

$$\Psi(t) = \frac{1}{\hbar\omega_0} \begin{pmatrix} [\hbar\omega_0 \cos(\omega_0 t) - i(\hbar\omega/2 - \mu_e H_0)\sin(\omega_0 t)]\exp(-i\omega t/2) \\ -i\mu_e h \sin(\omega_0 t)\exp(i\omega t/2) \end{pmatrix}. \tag{28}$$

Here, we introduce the notation

$$\hbar\omega_0 = \sqrt{\mu_e^2 h^2 + (\hbar\omega/2 - \mu_e H_0)^2}\,. \tag{29}$$

In the resonance case $\hbar\omega/2 = \mu_e H_0$, we obtain the well-known result

$$\Psi(t) = \begin{pmatrix} \cos(\omega_0 t)\exp(-i\omega t/2) \\ -i\sin(\omega_0 t)\exp(i\omega t/2) \end{pmatrix} \tag{30}$$

and the Rabi frequency is

$$\Omega_{Rabi} = \omega_0 = \mu_e h/\hbar. \tag{31}$$

However, this Rabi frequency is too high at any realistic values of the magnetic field h. Therefore, circularly polarized magnetic field cannot explain the observed oscillations of the electron capture decay.

4. DISCUSSION

Kienle [11, 12] theoretically investigated the time dependence of the orbital electron capture decays of H-like ^{140}Pr, ^{142}Pm, and ^{122}I ions in a heavy-ion storage ring at GSI Darmstadt, Germany, and found that the electron capture rate is not purely exponential but is in addition time-modulated with the respective periods $T = 7.06$ s, 7.10 s, and 6.11 s for ^{140}Pr, ^{142}Pm, and ^{122}I, measured in the laboratory system of the ions moving with 71 % of the speed of light (Lorentz factor is equal to 1.43). The modulation amplitude is $a = 0.20$ on average for all three ions. Such modulation periods correspond to the small energy difference $8.6\cdot 10^{-16}$ eV, and $7.5 \cdot 10^{-16}$ eV, for the phenomenon of a quantum beat type. Kienle attributed it to the mixing of massive electron neutrinos emitted in the decays with the squared mass difference $(\Delta m)^2 = 2.20 \cdot 10^{-4}$ eV^2. It is about 2.9 times larger than latest value reported by the antineutrino oscillation experiment.

In recent review papers [13, 14], the authors conclude that it is probable that the H-like ions are produced in a coherent superposition of the two hyperfine states with the total angular momenta $F = 1/2$ and $F = 3/2$. This could lead to well-known quantum beats with the beat period $T = \hbar/\Delta E$, where ΔE is the hyperfine splitting. However, the ΔE values in ^{140}Pr and ^{142}Pm ions are about 1 eV, which leads to beat periods of more than twelve orders of magnitude shorter than the observed ones. In contrast to that approach, we consider two Zeeman states of the lower hyperfine state with the total angular momentum $F = 1/2$. The Zeeman splitting amounts to only about 10^{-5} eV, and therefore quantum beats with the period of 7 s are possible.

A new explanation of the "GSI oscillations" is suggested in Refs. [15, 16]. The two-particle wave function of neutrino and recoil nucleus is found as a solution of an initial-value problem in the far zone for a time longer than the electron capture decay lifetime of a hydrogen-like ion. The neutrino-recoil entanglement arising in such a process is a consequence of the momentum conservation and is closely related to the wave packet structure of the state. Because of the neutrino mixing, the joint wave packet involves a coherent superposition of neutrino mass eigenstate packets. This is the new physical realization of the Einstein–Podolsky–Rosen thought experiment, which has no analogue in quantum optics and quantum informatics. A class of possible experiments for the registration of a neutrino and a recoil nucleus is proposed. It is shown that due to spatial correlations, neutrino and recoil oscillations can be observed in the coincidence experiment. However, the recoil nucleus does not oscillate. Observing the spatial correlation and neutrino oscillations requires a correlation experiment in which an experimental event involves an independent registration of the neutrino and the recoil nucleus in two detectors.

In Ref. [17], corrections to the ratio of electron capture rates in hydrogen- and helium-like ions are calculated. It follows that the most significant contribution is the electron screening effect. The correction has the simple form $(1 - 5/16Z)^3$, which ranges from almost 50 % in helium to 1 % in heavier nuclei.

The authors of Ref. [18] also studied a model for the “GSI anomaly” in which they obtained the time evolution of the population of parent and daughter particles directly in real time, by explicitly considering the quantum entanglement between the daughter particle and neutrino mass eigenstates in the two-body decay. They confirmed that the decay rate of the parent particle and the growth rate of the daughter particle do not feature a time modulation from interference of neutrino mass eigenstates. The lack of interference is a consequence of the orthogonality of the mass eigenstates. This result also follows from the density matrix obtained by taking the trace over the unobserved neutrino states.

In Ref. [19], the authors investigated the influence of the magnetic field of the GSI ESR on the periodic time dependence of the orbital K-shell electron capture decay rates of H-like heavy ions. They approximated the magnetic field of the ESR by a homogeneous magnetic field. In contrast to the assertion in other works, they showed that the motion of an H-like heavy ion in a homogeneous magnetic field cannot be the origin of a periodic time dependence of the electron capture decay rates of the H-like heavy ions. They concluded that the time modulation of the electron capture decay rates of H-like heavy ions with periods of the order of a few seconds, observed at GSI, cannot be explained by means of the interactions of spins of nuclei and bound electrons of H-like heavy ions with the magnetic field of the GSI ESR. However, the real magnetic field in the ESR is not homogeneous. The ESR contains six segments with magnetic dipoles with a homogeneous magnetic field $H_0 = 1.2$ T. Each dipole magnet bends the ion beam by 60°. There is a transition region of about 30 cm with a gradient of the magnetic field, where the magnetic field increases from zero to $H_0 = 1.2$ T. The focusing of the ion beam occurs due to quadrupole magnets. Of course, the magnetic field in the ESR is not homogeneous overall. But due to the stability of the orbits of ion beams in the ESR, the approximation of the real magnetic field by a homogeneous magnetic field seems to be rather good. We can conclude from this consideration that the value of the effective constant magnetic field in the GSI experiment should be diminished. Approximating several quadrupole magnets by an oscillating magnetic field is, of course, a qualitative approach.

The authors of Ref. [20] discuss a model in which a recently reported modulation in the decay of the hydrogen-like $^{140}Pr_{58+}$ and $^{142}Pm_{60+}$ ions arises from the coupling of rotation to the spin of electron and nuclei (Thomas precession). A similar model describes the electron modulation in muon experiments correctly. An agreement with the GSI experimental results is obtained for the current values of the bound electron g-factors, $g(^{140}Pr_{58+}) = 1.872$ and $g(^{142}Pm_{60+}) = 1.864$, if the Lorentz factor of the bound electron is equal to 1.88. This value is fixed by either of the two sets of experimental data. The model predicts that the modulation is not observable if the motion of the ions is linear, or if the ions are stopped in a target.

However, the goal of the comment in [21] is to show that the explanations in Refs. [2, 12, 20] are not satisfactory. The motion of an ion (along a circular orbit in the ring) does not change the value of the momentum F, it only changes the evolution of the direction of F, which does not affect the electron capture rate however. The author of Ref. [21] notes that any link between the GSI oscillations and atomic phenomena would not look natural. Indeed, the energy intervals and interactions in the ion including the interactions with external fields in the storage ring in the GSI experiment exceed 0.1 Hz by many orders of magnitude. He postulates that phenomena such as Rabi oscillations with a noticeable modulation amplitude would require a special experimental arrangement and do not happen accidentally.

In recent paper [22] (October 2011), the authors investigated the deviations from the usual exponential decay law for quantum mechanical systems. They have studied a typical behavior of unstable particles in the general framework: an exponential decay law with superimposed oscillations. They have shown that this behavior is quite common as soon as the Breit–Wigner distribution of the unstable state is left and (even very simple) form factors are taken into account, which suppress the Breit–Wigner distribution far away from the peak. Using a simple cut-off model, they have shown that the qualitative behavior of the oscillations seen in the GSI experiment can be reproduced. If their interpretation is correct, they predict that if the GSI experimentalists manage to measure the system at times smaller than roughly 5 s, then they would find that the number of decays per seconds rapidly drops to zero due to a fundamental property of quantum systems: the quadratic behavior of the survival probability at short times after the preparation of the system. Finally, the authors note that possible indications of the presence of oscillations superimposed on the exponential decay were also found in other unstable, but different, quantum systems, for example, the tunneling of cold atoms out of a trap [23]. They speculate that these superimposed oscillations are a manifestation of the same fundamental phenomenon of quantum mechanics.

In conclusion, we here suggested a qualitative explanation of oscillations in the electron capture decays of hydrogen-like $^{140}\mathrm{Pr}_{58+}$ and $^{142}\mathrm{Pm}_{60+}$ ions observed in the GSI experiment. This explanation is based on the multiphoton Rabi oscillations between two Zeeman states of the hyperfine ground level with the total angular momentum $F = 1/2$. Oscillations are produced by a sufficiently strong alternating magnetic field in the ESR.

This work was supported by the RFBR (grant № 10-02-00054).

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