ЖЭТФ, 2013, том 143, вып. 1, стр. 58–69

EXCITATION OF HELIUM ATOMS IN COLLISIONS WITH PLASMA ELECTRONS IN AN ELECTRIC FIELD

B. M. Smirnov*

Joint Institute for High Temperatures, Russian Academy of Sciences
125412, Moscow, Russia

Received August 28, 2012

The rate constants are evaluated for excitation of helium atoms in metastable states by electron impact if ionized helium is located in an external electric field and is supported by it, such that a typical electron energy is small compared to the atom excitation energy. Under these conditions, atom excitation is determined both by the electron traveling in the space of electron energies toward the excitation threshold and by the subsequent atom excitation, which is a self-consistent process because it leads to a sharp decrease in the energy distribution function of electrons, which in turn determines the excitation rate. The excitation rate constant is calculated for the regimes of low and high electron densities, and in the last case, it is small compared to the equilibrium rate constant where the Maxwell distribution function is realized including its tail. Quenching of metastable atom states by electron impact results in excitation of higher excited states, rather than transition to the ground electron state for the electric field strengths under consideration. Therefore, at restricted electron number densities, the rate of emission of resonant photons of the wavelength 58 nm, which results from the transition from the 2^1P state of the helium atom to the ground state, is close to the excitation rate of metastable atom states. The efficiency of atom excitation in ionized helium, i. e., the part of energy of an electric field injected in ionized helium that is spent on atom excitation, is evaluated. The results exhibit the importance of electron kinetics for an ionized gas located in an electric field.

DOI: 10.7868/S0044451013010058

1. INTRODUCTION

An ionized gas located in an external electric field, i. e., a gas discharge plasma, is a nonequilibrium system because, first, the energy is transferred from the electric field to electrons and, second, it is transferred to the gas as a result of collisions of electrons and atoms. Therefore, this system cannot be described by a universal thermodynamic method, and its properties depend on the properties of a certain gas and processes in this ionized gas (see, e. g., [1–4]). Therefore, types of gas-discharge plasmas are divided into many groups and conditions determined by certain processes [5]. But along with elementary processes, kinetics of electrons is of importance in an ionized gas under the action of an external electric field. The goal of this paper is to find the parameters of excitation of metastable atom states for ionized helium placed in an electric field at low electric field strengths where kinetics of electrons is of importance for atom excitation.

*E-mail: bmsmirnov@gmail.com

To demonstrate in what way electron kinetics may influence the atom excitation in a weakly ionized plasma located in an electric field, we analyze a general formula for the rate constant of atom excitation by electron impact in a plasma. We represent the rate of atom excitation by electron impact in an ionized gas as

$$\frac{dN_*}{dt} = N_a \int k_{ex}(\varepsilon) f_0(\varepsilon)\, d\mathbf{v},$$

where dN_*/dt is the number of excited atoms formed per unit volume and per unit time, ε is the electron energy, N_a is the atom number density, $\mathbf{v}$ is the electron velocity, $f_0(\varepsilon)$ is the distribution function of electrons, which is normalized to the electron number density N_e, i. e.,

$$\int f_0(\varepsilon)\, d\mathbf{v} = N_e,$$

and $k_{ex}(\varepsilon)$ is the rate constant of atom excitation by electron impact. It is convenient to express this rate constant through the rate constant k_q of quenching of an excited atom by electron impact on the basis of the principle of detailed balance because the quenching rate constant is independent of the electron energy for slow

electrons. The principle of detailed balance gives (see, e. g., [6])

$$k_{ex}(\varepsilon) = k_q \frac{g_*}{g_0}\sqrt{\frac{\varepsilon-\Delta\varepsilon}{\Delta\varepsilon}},$$

where g_0 and g_* are the respective statistical weights for the ground and excite states and $\Delta\varepsilon$ is the atom excitation energy. In the case of the Maxwell distribution function of electrons,

$$f_0 \propto \exp\left(-\frac{\varepsilon}{T_e}\right),$$

where T_e is the electron temperature, we obtain the excitation rate as

$$\left(\frac{dN_*}{dt}\right)_{eq} = N_a N_e k_q \frac{g_*}{g_0}\exp\left(-\frac{\Delta\varepsilon}{T_e}\right), \qquad (1.1)$$

which relates to the thermodynamic equilibrium of atoms in the ground and excited states, and the average excitation rate constant $\overline{k_{ex}}$ is given by

$$\overline{k_{ex}} = k_q \frac{g_*}{g_0}\exp\left(-\frac{\Delta\varepsilon}{T_e}\right). \qquad (1.2)$$

We analyze the assumptions used in deriving formula (1.2) for the excitation rate constant under equilibrium conditions in a plasma where the average electron energy is small compared with the excitation energy. The first is the assumption of a high rate of formation of fast electrons on the tail of the distribution function because of electron diffusion in the electron energy space due to electron collisions and interaction with an external electric field. Second, we ignore a decrease in the electron distribution function with an increasing electron energy above the excitation threshold owing to atom excitation. We below drop these assumptions and accurately take the indicated factors into account for the excitation of helium atoms in metastable states 2^3S and 2^1S by electron impact in weakly ionized uniform helium.

2. KINETICS OF ATOM EXCITATION BY ELECTRON IMPACT IN A GAS LOCATED IN AN EXTERNAL ELECTRIC FIELD

We first determine the rate of formation of fast electrons that are able to excite atoms. We assume in this consideration that each electron that reaches the excitation threshold excites an atom and becomes slow. In other words, these fast electrons are absorbed near the excitation threshold, i. e., the distribution function is zero,

$$f(\Delta\varepsilon) = 0,$$

at this boundary. To find the excitation rate under these conditions, it is necessary to analyze electron kinetics in the space of electron energies. The kinetic equation for the electron distribution function f below the excitation threshold has the form

$$\frac{e\mathbf{E}}{m_e}\cdot\frac{\partial f}{\partial \mathbf{v}} = I_{ee}(f) + I_{ea}(f), \qquad (2.1)$$

where $\mathbf{E}$ is the electric field strength, $\mathbf{v}$ is the electron velocity, m_e is the electron mass, and $I_{ee}(f)$ and $I_{ea}(f)$ are the collision integrals for electron–electron and electron–atom elastic collisions. Depending on the electron number density, there are two regimes of electron motion in the energy space of electrons. In the regime of a high electron number density, Eq. (2.1) has the form $I_{ee}(f) = 0$ and its solution is the Maxwell distribution function (as it was used above)

$$f_0 = N_e\left(\frac{m_e}{2\pi T_e}\right)^{3/2}\exp\left(-\frac{m_e v^2}{2T_e}\right), \qquad (2.2)$$

where v is the electron velocity. In another limit case, one electron moves in an atomic gas, and a small parameter m_e/M (M is the atom mass) allows expanding the distribution function over spherical harmonics and restricting by two harmonics, whence

$$f(\mathbf{v}) = f_0(v) + v_x f_1(v).$$

This problem was solved both for electrons of a semiconductor [7–10] and for a weakly ionized gas [11–16], and the results were summarized in [17]. The set of equations for spherical harmonics takes the form

$$a\frac{df_0}{dv} = -\nu_{ea} v f_1, \quad \frac{a}{3v^2}\frac{d}{dv}v^3 f_1 = I_{ea}(f_0), \qquad (2.3)$$

where $a = eE/m_e$, the rate of electron–atom collisions is $\nu_{ea} = N_a v\sigma^*_{ea}(v)$, and $\sigma^*_{ea}(v)$ is the diffusion cross section of electron–atom scattering. The collision integral from the isotropic part of the distribution function $I_{ea}(f_0)$ has the form of the right-hand side of the Fokker–Planck equation because of a small energy variation in a single collision with atoms. This collision integral is zero for the Maxwell distribution function [18, 19].

Thus, because of a small energy change in electron–atom collisions, the velocity distribution of electrons moving in a gas in an external electric field is nearly symmetric with respect to directions of electron motion and can be represented in the form

$$I_{ea}(f_0) = \frac{1}{v}\frac{\partial}{\partial\varepsilon}\left[vB_{ea}(\varepsilon)\left(\frac{\partial f_0}{\partial\varepsilon}+\frac{f_0}{T}\right)\right], \qquad (2.4)$$

and the analog of the diffusion coefficient in the space of electron energies is given by

$$B_{ea}(\varepsilon) = T\frac{m_e^2 v^2}{M}\nu_{ea}, \tag{2.5}$$

where $\varepsilon = m_e v^2/2$ is the electron energy.

In considering elastic electron scattering on a helium atom, we use the fact that the diffusion electron–atom cross section [20] is almost constant at low collision energies, and it is convenient to approximate the measured cross section [20] with an accuracy of about 20 % as

$$\sigma_{ea}^*(\varepsilon) = \begin{cases} (6\pm 1)\text{Å}^2, & \varepsilon < 10 \text{ eV}, \\ \approx \dfrac{A}{\varepsilon}, & A \approx 60 \text{ eV}\cdot\text{Å}^2, \\ & 10 \text{ eV} < \varepsilon < 40 \text{ eV}. \end{cases} \tag{2.6}$$

The electron–electron collision integral, or the Landau collision integral [21], has a nonlinear form. But if we extract fast electrons from thermal electrons with the Maxwell distribution function, i. e., divide the electron subsystem into two, the collision integral for fast electrons has the diffusion form similar to collision integral (2.4) because of the relatively small variation of the electron energy in single electron–electron collisions. Correspondingly, in the regime of a high electron number density, the electron–electron collision integral is (see, e. g., [22])

$$I_{ee}(f_0) = \frac{1}{v}\frac{\partial}{\partial\varepsilon}\left[vB_{ee}(\varepsilon)\left(\frac{\partial f_0}{\partial\varepsilon} + \frac{f_0}{T_e}\right)\right], \qquad B_{ee}(\varepsilon) = \frac{4\pi}{3}e^4 v N_e \ln\Lambda, \tag{2.7}$$

where $\ln\Lambda$ is the Coulomb logarithm; below, we use its typical value for a glow gas discharge plasma $\ln\Lambda = 7$.

The regime of low electron number densities corresponds to the Druyvesteyn case [23, 24], where the electron–atom diffusion cross section is independent of the electron energy, and the distribution function, as a solution of the set of equations (2.3) in the regime of low electron number densities, has the form

$$f_0 = C\exp\left(-\frac{\varepsilon^2}{\varepsilon_0^2}\right), \quad \varepsilon_0 = \sqrt{\frac{M}{3m_e}}eE\lambda. \tag{2.8}$$

Here

$$\lambda = \frac{1}{N_a\sigma_{ea}^*},$$

C is the normalization coefficient, and in the helium case,

$$\varepsilon_0 = 0.82x, \tag{2.9}$$

where the characteristic energy ε_0 is measured in eV and the reduced electric field strength $x = E/N_a$ is always expressed in Td (1 Td $= 10^{-17}$ V $\cdot$ cm^2).

We now determine the rate of atom excitation under the assumption that each electron transferred through the excitation threshold loses its energy as a result of atom excitation and becomes slow. This corresponds to the boundary condition

$$f_0(\Delta\varepsilon) = 0.$$

In the regime of a high electron number density, this corresponds to the distribution function [25]

$$f_0 = C\left[\exp\left(-\frac{\varepsilon}{T_e}\right) - \exp\left(-\frac{\Delta\varepsilon}{T_e}\right)\right] \tag{2.10}$$

instead of formula (2.2). Reducing the kinetic equation to the isotropic part of the distribution function, we have

$$\frac{\partial f_0}{\partial t} + \frac{1}{v}\frac{\partial}{\partial\varepsilon}(vJ) = 0, \qquad J = \frac{m_e^2 v^2 a^2}{3\nu_{ea}}\frac{\partial f_0}{\partial\varepsilon} + B_{ee}\left(\frac{\partial f_0}{\partial\varepsilon} + \frac{f_0}{T}\right) \tag{2.11}$$

and hence the electron flux in the velocity space is the sum of two parts due to the electric field and electron–electron collisions. We note that because of a small electron concentration in ionized helium, we assume that variation of the electron momentum results from electron–atom collisions. This gives the excitation rate due to electron diffusion in the space of electron energies in the form

$$\frac{dN_*}{dt} = \int_{\varepsilon_0}^{\infty} 4\pi v^2 dv\frac{\partial f_0}{\partial t} = N_e\frac{2}{\sqrt{\pi}}\frac{\varepsilon_0^{1/2}}{T_e^{5/2}}\times \times\exp\left(-\frac{\varepsilon_0}{T_e}\right)\left(\frac{4\pi e^4}{3}v_0 N_e\ln\Lambda + \frac{2m_e\varepsilon_0 a^2}{3\nu_{ea}}\right), \tag{2.12}$$

where we use formula (2.10) for the distribution function. We can introduce the excitation rate constant $k_<$ as

$$\frac{dN_*}{dt} = k_< N_e N_a. \tag{2.13}$$

Taking $\sigma_{ea}^* = 6\,\text{Å}^2$, according to [20], in the basic part of the electron distribution, we have the electron temperature

$$T_e = 0.41x, \quad T_e \gg T. \tag{2.14}$$

Because $\sigma_{ea}^*(\Delta\varepsilon) = 2.7\,\text{Å}^2$, these results lead to the rate constant of excitation of the metastable state 2^3S

$$k_< = \frac{7.6 \cdot 10^{-3}}{x^{5/2}} \times \\ \times \exp\left(-\frac{48}{x}\right)(c_e + 2.0 \cdot 10^{-7} x^2), \quad (2.15)$$

where $c_e = N_e/N_a$ is the concentration of electrons.

We now analyze the character of excitation of the metastable state for the helium atom by electron impact above the excitation threshold, assuming that the excitation affects the distribution function and leads to its strong decrease in the course of removal from the excitation threshold. Then the electron distribution function f_0 of electrons satisfies the kinetic equation

$$\frac{a}{3v^2}\frac{d}{dv}\left(\frac{v^2}{\nu}\frac{df_0}{dv}\right) + \frac{m_e}{M}\frac{1}{v^2}\frac{d}{dv}(v^3\nu_{ea}f_0) - \\ - \nu_{ex}f_0 = 0. \quad (2.16)$$

We use the semiclassical solution of this equation that is based on a sharp decrease of the distribution function at removal from the excitation threshold; the distribution function is then taken in the form

$$f_0 = A\exp(S)$$

and in the semiclassical approach, we have $(S')^2 \gg S''$. Near the excitation threshold, Eq. (2.16) then yields [26]

$$f_0(v) = f(v_0)\exp(-S), \\ S = \kappa\left(\frac{\varepsilon - \Delta\varepsilon}{\Delta\varepsilon}\right)^{5/4}, \quad (2.17) \\ \kappa = \frac{2v_0}{5a}\sqrt{3\frac{g_*}{g_0}\nu_q\nu_{ea}},$$

where v_0 is the electron velocity at the excitation threshold, the quenching rate is $\nu_q = N_a k_q$, and g_* and g_0 are statistical weights of the helium atom in the ground and excited states. This regime is valid at $\kappa \gg 1$, and in the case of excitation of the metastable 2^3S state, this parameter is $\kappa \approx 270/x$, and this regime is realized in the region $x < 12$ Td under consideration. Introducing the excitation rate constant

$$k_> = \frac{1}{N_a N_e}\frac{dN_*}{dt},$$

we obtain hence this excitation rate as

$$k_> = \frac{1}{N_e}\int k_{ex}(v)f_0(v)\cdot 4\pi v^2 dv = \\ = \frac{4.6}{N_a N_e}\frac{g_*}{g_0}\nu_q v_0^3 f_0 \kappa^{-1.2}, \quad (2.18)$$

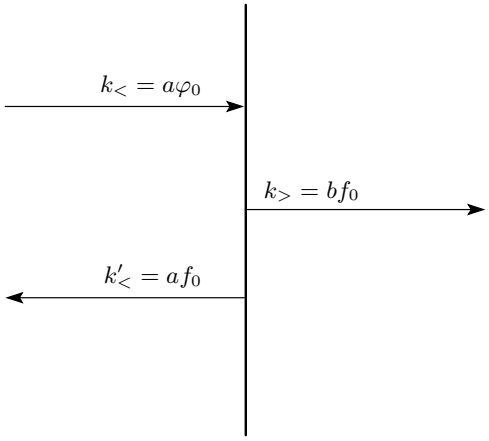

Fig. 1. The character of electron fluxes near the atom excitation threshold. The flux of electrons toward the excitation boundary $k_<$ is proportional to the derivative of the electron distribution function at the excitation threshold and is hence expressed through the distribution function φ_0 with the absorption process for fast electrons ignored. The reflected electron flux $k'_<$ is proportional to the electron distribution function at the absorption boundary

where $f_0 = f_0(v_0)$ is the electron distribution function at the excitation threshold. We note that formula (2.18) for the excitation rate constant is valid both for regimes of low and high electron number densities. In this formula, we substitute the Maxwell distribution function of electrons, which applies in the regime of a high electron number density and is conserved at the distribution function tail at $k_< \gg k_>$, to obtain the atom excitation rate by electron impact for electron energies above the excitation threshold,

$$k_> = 0.83\frac{g_*}{g_0}k_q\left(\frac{\Delta\varepsilon}{T_e}\right)^{3/2}\exp\left(-\frac{\Delta\varepsilon}{T_e}\right)\kappa^{-1.2}, \quad (2.19)$$

where the electron distribution function at the excitation threshold f_0 coincides with the Maxwell distribution function φ_0. Formula (2.19) gives the rate constant $k_>$ of excitation of the metastable 2^3S state by electron impact in the regime of a high electron number density:

$$k_> = \frac{3.1 \cdot 10^{-9}}{x^{0.3}}\exp\left(-\frac{48}{x}\right). \quad (2.20)$$

We thus obtain formula (2.15) for the rate constant $k_<$ of electrons traveling in the space of electron energies to the excitation threshold in the limit where the electron flux to the excitation threshold is relatively small and hence the electron distribution function at the excitation threshold is zero. This rate constant $k_<$

is determined by the distribution function φ_0 near the excitation threshold, where absorption on this boundary is not essential, and therefore the second term in formula (2.10) for the electron distribution function is relatively small. In addition, in the expression for the excitation rate constant, we neglect the reflection of electrons in the electron energy space from the absorption boundary. The excitation rate constant $k_>$ is proportional to the current distribution function f_0 of electrons at the excitation threshold. We can combine the rate constants $k_<$ and $k_>$ into the total excitation rate constant as is shown in Fig. 1; the parameters in this figures are

$$a = \frac{k_<}{\varphi_0}, \quad b = \frac{k_>}{f_0}.$$

Evidently, from the conservation of the excitation rate, we have

$$k_< = k'_< + k_>.$$

This gives the connection between the nondistorted distribution function φ_0 and the current distribution function f_0 at the excitation threshold, and the excitation rate constant

$$k_{ex} = \frac{k_< k_>}{k_< + k_>}, \tag{2.21}$$

where the excitation rate constant $k_<$ is taken under the boundary condition $f_0 = 0$, whereas the evaluation of the rate constant $k_>$ corresponds to the boundary condition $f_0 = \varphi_0$. This formula shows that the mechanisms of electron traveling to the excitation rate and the excitation rate above the excitation threshold do not compete, but they determine the distribution function at the excitation threshold, and the combination of these mechanisms gives the total excitation rate constant. We use formula (2.21) to find the excitation rate constant k_{ex} of the metastable state $\mathrm{He}(2^3S)$ by electron impact in an external electric field on the basis of formulas (2.15) and (2.20) in the regime of high electron number densities:

$$k_{ex} = \frac{3.1 \cdot 10^{-9}(c_e + 2.0 \cdot 10^{-7}x^2)}{x^{0.3}(c_e + 2.0 \cdot 10^{-7}x^2 + 4.2 \cdot 10^{-7}x^{2.2})} \times \\ \times \exp\left(-\frac{48}{x}\right). \tag{2.22}$$

The same can be done to find the excitation rate constant of atoms by electron impact in the regime of a low electron number density. Based on formulas (2.18) and (2.8), we obtain the rate constant of excitation of the metastable 2^3S helium atom by electron impact in an external field in the regime of a low electron number density:

$$k_> = \frac{1.6 \cdot 10^{-9}}{x^{0.3}} \exp\left(-\frac{580}{x^2}\right). \tag{2.23}$$

For simplicity, we continue the nonperturbed distribution function (2.8) up to the excitation threshold.

To find the rate constant $k_<$ in the limit of high electron number densities, we use the nonstationary kinetic equation in the regime of a low electron number density, which has the form

$$\frac{\partial f_0}{\partial t} = I_{ea}(f_0) + \frac{a^2}{3v^2}\frac{\partial}{\partial v}\left(\frac{v^2}{\nu}\frac{\partial f_0}{\partial v}\right) = \\ = \frac{\partial}{v^2 \partial v}\left(\frac{m_e}{M}v^3\nu\left[f_0 + \frac{\partial f_0}{m_e v \partial v}\left(T + \frac{Ma^2}{3\nu^2}\right)\right]\right).$$

From this, in the limit $\Delta\varepsilon \gg T$ and with the boundary condition $f_0 = 0$, we find the excitation rate constant owing to electron diffusion in the electron energy space from small electron energies

$$k_< = \frac{1}{N_a N_e}\frac{dN_*}{dt} = 4\pi\frac{m_e}{M}v_0^3\frac{\nu_{ea}(v_0)\varphi(v_0)}{N_a N_e}. \tag{2.24}$$

For the excitation of metastable helium atoms in the 2^3S state in the regime of a low electron number density, formula (2.24) gives

$$k_< = 3.8 \cdot 10^{-9}\frac{1}{x^{3/2}}\exp\left(-\frac{580}{x^2}\right). \tag{2.25}$$

From the above formulas, it follows that the total excitation rate constant for metastable 2^3S helium atoms in the regime of a low electron number density is given by

$$k_{ex} = \frac{1.6\exp(-580/x^2)}{x^{0.3}(1 + 0.43x^{1.2})}. \tag{2.26}$$

We note that the transition from the regime of low electron number densities to the regime of high electron number densities for excitation of the lowest metastable state $\mathrm{He}(2^3S)$ proceeds at the electron concentration [3]

$$c_e = N_e/N_a \sim 10^{-7}.$$

The above results correspond to the assumption that the diffusion cross section of the electron–atom cross section is independent of the electron velocity. We now drop this assumption and use approximation (2.6) for this cross section. We start from the regime of low electron number densities where a typical electron energy exceeds a typical atom energy significantly, and

hence the solution of Eq. (2.3) with the use of relations (2.4) and (2.5) has the form

$$f_0(\varepsilon) = C \exp\left(- \int\limits_0^{\varepsilon} \frac{6m_e}{M} \frac{\varepsilon d\varepsilon \sigma_{ea}^2}{e^2 x^2} \right).$$

Using expression (2.6) for the electron–atom cross section σ_{ea}, we reduce this formula to the form

$$f_0(\varepsilon) = \varphi(\varepsilon_c) \left[\exp\left(-\frac{6m_e A^2}{M x^2} \ln\left(\frac{\varepsilon}{\varepsilon_c}\right)\right)\right],$$

where we set $\varepsilon_c = 10$ eV and $A = 60$ eV $\cdot$ Å^2 in accordance with formula (2.6), and the electron distribution function $\varphi(\varepsilon_c)$ is determined by Druyvesteyn formula (2.8). This gives the distribution function of electrons in helium located in an electric field at the excitation threshold $\Delta\varepsilon$ in the form

$$f_0(\Delta\varepsilon) = \frac{N_e}{\pi\Gamma(3/4)} \left(\frac{m_e}{2\varepsilon_0}\right)^{3/2} \times \exp\left[-\frac{\varepsilon_c^2}{\varepsilon_0^2} \left(1 + 2\ln\frac{\Delta\varepsilon}{\varepsilon_c}\right)\right], \quad \varepsilon_0 = eE\lambda\sqrt{\frac{M}{3m_e}}, \quad (2.27)$$

and this distribution function differs from the Druyvesteyn one (2.8). Based on this distribution function, we obtain the excitation rate constants

$$k_< = 3.8 \cdot 10^{-9} \frac{1}{x^{3/2}} \exp\left(-\frac{350}{x^2}\right), \quad k_> = \frac{1.6 \cdot 10^{-9}}{x^{0.3}} \exp\left(-\frac{350}{x^2}\right), \quad k_{ex} = \frac{1.6\exp(-350/x^2)}{x^{0.3}(1+0.43x^{1.2})} \quad (2.28)$$

instead of those given by formulas (2.25), (2.23), and (2.26). Figure 2 contains the excitation rate constants in the regime of low number densities according to formulas (2.28).

In the regime of a high electron number density, the second term in expression (2.6) for the electron–atom cross section affects the electron temperature, in contrast to the regime of a low electron number density where it changes the tail of the distribution function. Therefore, formulas (2.20), (2.15), and (2.22) remain valid at small electric field strengths where the electrons predominantly have an energy below ε_c and the electron temperature T_e is related to the reduced electric field strength x by Eq. (2.14). In the general case,

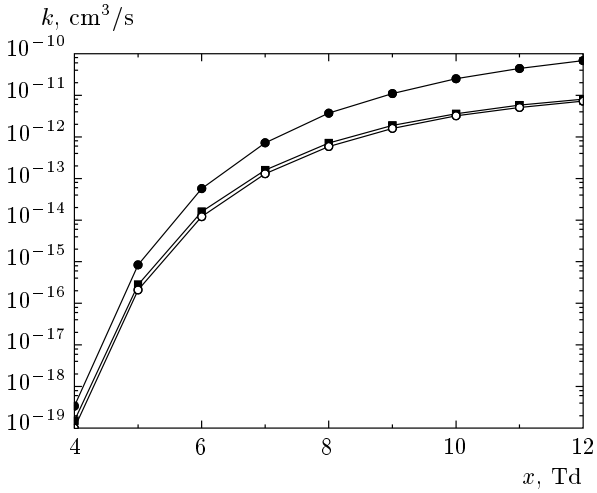

Fig. 2. Rate constants of the excitation of the metastable state He(2^3S) by electron impact in a constant electric field in the regime of low electron number densities in accordance with formula (2.28). Filled squares correspond to $k_<$, filled circles correspond to $k_>$, and open circles correspond to k_{ex}

the electron temperature T_e in a gas is related to the electric field strength x as [27]

$$T_e = \frac{Ma^2}{3} \frac{\langle v^2/\nu_{ea}\rangle}{\langle v^2\nu_{ea}\rangle}, \quad (2.29)$$

where an averaging in brackets is done with the Maxwell distribution function, ν_{ea} is the rate of electron–atom collisions, and $T_e \gg T$. Using cross section (2.6) in formula (2.29), we reduce it to the form

$$x^2 = \frac{300}{z^2} \frac{2 - e^{-z}(z+2)}{1 + e^{-z}(1+2/z)}, \quad (2.30)$$

where

$$z = \varepsilon_c/T_e = 10 \text{ eV}/T_e.$$

This formula is transformed into formula (2.14) in the limit $z \gg 1$. According to this relation, formulas (2.14) and (2.30) coincide for $x \le 6$ Td; $T_e = 6$ eV at $x = 10$ Td, and T_e depends on x sharply at larger x. As a result, in the general case, instead of formulas (2.20), (2.15), and (2.22), we obtain

$$k_< = \frac{8.1 \cdot 10^{-4}}{T_e^{5/2}} \exp\left(-\frac{19.8}{T_e}\right) \times (c_e + 2.0 \cdot 10^{-7} x^2), \quad k_> = 8.3 \cdot 10^{-10} \frac{x^{1.2}}{T_e^{1.5}} \exp\left(-\frac{19.8}{T_e}\right). \quad (2.31)$$

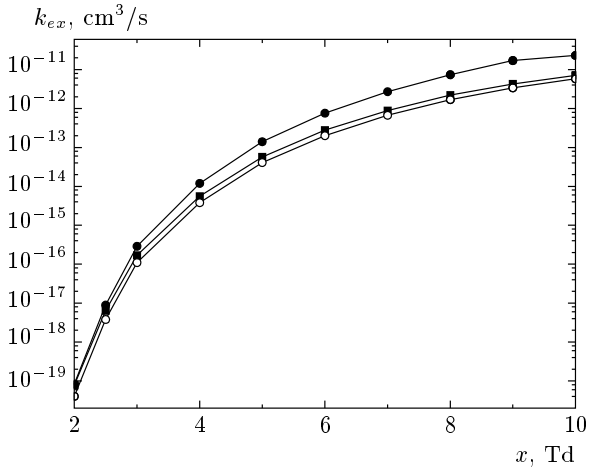

Fig. 3. Rate constants of the helium atom excitation to the metastable state $\mathrm{He}(2^3S)$ by electron impact in a constant electric field in the regime of a high electron number density at the electron concentration $c_e = 10^{-6}$ evaluated based on formulas (2.31), where the relation between the electron temperature and the electric field strength is given by (2.30). Closed squares correspond to $k_<$, closed circles correspond to $k_>$, and open circles correspond to k_{ex}

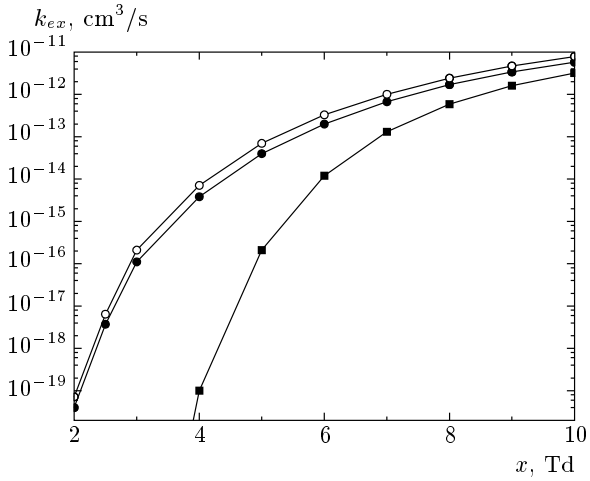

Fig. 4. Rate constants k_{ex} of the helium atom excitation to the metastable state $\mathrm{He}(2^3S)$ by electron impact in a constant electric field. Filled squares correspond to the regime of low electron number densities in accordance with formula (2.28), the rate constants for the regime of high electron number densities are determined by formula (2.31). Filled circles correspond to the electron concentration $c_e = 10^{-6}$ and open circles correspond to $c_e = 10^{-5}$

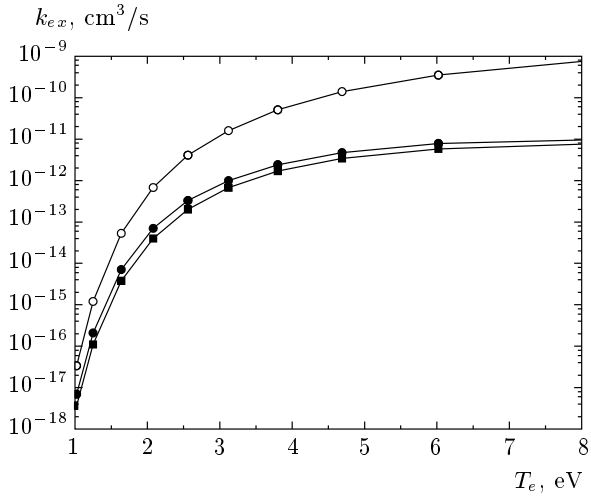

Fig. 5. Rate constants k_{ex} of the helium atom excitation to the metastable state $\mathrm{He}(2^3S)$ by electron impact in a constant electric field. Open circles correspond to thermodynamic equilibrium according to formula (1.2). The excitation rates for the regime of high electron number densities in accordance with formula (2.28) are represented by filled squares for the electron concentration $c_e = 10^{-6}$ and by filled circles for $c_e = 10^{-5}$

For electron temperatures below (2–3) eV, where formula (2.14) for the electron temperature is valid, formulas (2.31) are converted into formulas (2.20), (2.15), and (2.22). Figure 3 contains the excitation rate constants of helium atoms by electron impact under the action of an external electric field on weakly ionized helium in the regime of a high electron number density in accordance with formulas (2.31). In addition, the excitation rate constants k_{ex} are compared in Fig. 4 in the regimes of low and high electron number densities. These rate constants in the regime of high electron number densities are compared in Fig. 5 with thermodynamic rate constant (1.2), where along with the equilibrium inside the electron system, electrons establish thermodynamic equilibrium between atoms in the ground and excited states.

3. ATOM QUENCHING BY ELECTRON IMPACT IN A GAS LOCATED IN AN EXTERNAL ELECTRIC FIELD

To find the concentrations of metastable helium atoms in a given field, it is necessary to include the quenching of excited atoms by electron impact into consideration. This parameter is known sufficiently well for atom transition $2^3S \to 1^1S$ and its averaging over

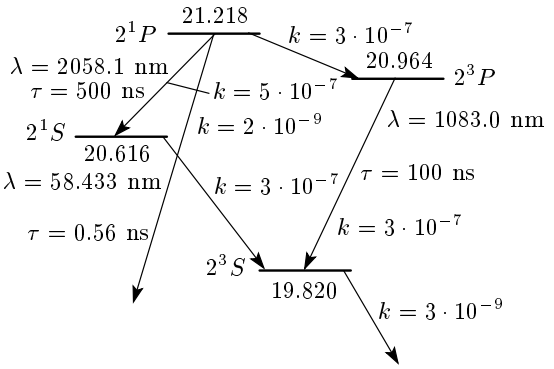

Fig. 6. The lowest excited states of a helium atom and the rates of transitions involving these states [28], where the rate constants k of atom quenching by electron impact are given in cm^3/s

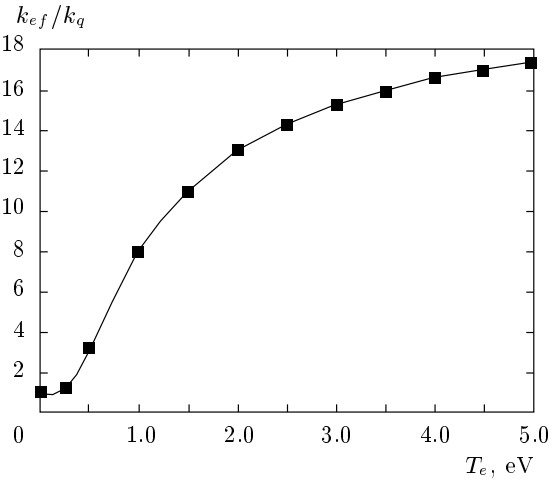

Fig. 7. The ratio of the total rate constant of quenching of the metastable state $\mathrm{He}(2^3S)$ of the helium atom by electron impact in a helium gas discharge plasma to the rate constant in the absence of transitions between excited states as a function of the electron temperature T_e

some measurement gives $k_q = 3.1 \cdot 10^{-9}$ cm^3/s [29]. But quenching of metastable states may also proceed through excited states, and Fig. 6 gives the scheme of processes involving lower excited states of the helium atoms with the rates of these processes. We consider the number densities of electrons if the decay of 2^1P state results from its radiation rather than quenching by electron impact, and then the number density of atoms in this state is small compared with that under thermodynamic equilibrium.

Using the parameters in Fig. 6, we obtain the effective rate of quenching of the metastable state 2^3S by electron impact in cm^3/s as

$$k_{ef} = 3 \cdot 10^{-9} + \frac{5 \cdot 10^{-7} \exp(-1.398/T_e)}{3 + 5 \exp(-0.602/T_e)}. \qquad (3.1)$$

Figure 7 gives the ratio of the effective quenching rate constant k_{ef} of the metastable atom $\mathrm{He}(2^3S)$ by electron impact to the quenching rate constant k_q with transition into the ground atom state. As can be seen, excitation of the metastable atom $\mathrm{He}(2^3S)$ by electron impact in higher excited states with subsequent radiation of the $\mathrm{He}(2^1P)$ gives the leading contribution to the quenching of an $\mathrm{He}(2^3S)$ atom at temperatures $T_e \sim 1$ eV that are of interest for gas discharge plasma.

From this, we can determine the concentration of metastable atoms $\mathrm{He}(2^3S)$ in the regime of high electron number densities if excitation and quenching of metastable atoms in weakly ionized helium result from collisions with free electrons and excitation of this gas proceeds due to the action of a constant electric field. The concentration of metastable 2^3S helium atoms is given by

$$c(2^3S) = \frac{k_{ex}}{k_{ef}}. \qquad (3.2)$$

In this case, formation of metastable atoms $\mathrm{He}(2^3S)$ is determined by collision with fast electrons whose energy exceeds the excitation threshold, and the excitation rate constants k_{ex} are determined by formula (2.31). Because the energy distribution function of electrons decreases sharply as the electron energy increases above the excitation threshold of the metastable state 2^3S, excitation of other excited states proceeds in a stepwise way through the metastable state 2^3S. In particular, according to the scheme of transitions between excited states given in Fig. 6, the concentration of helium atoms $c(2^1S)$ in the metastable state 2^3S is given by

$$c(2^1S) = c(2^3S) \frac{\exp(-0.796/T_e)}{3 + 5 \exp(-0.602/T_e)}. \qquad (3.3)$$

Figure 8 shows the concentrations of metastable atoms in an ionized helium according to formulas (3.2) and (3.3).

4. RESONANT RADIATION OF IONIZED GAS

We consider the regime of excitation of metastable helium states for ionized helium in an external electric field where quenching of metastable states results from excitation of the resonantly excited $\mathrm{He}(2^1P)$ state and a subsequent atom transition to the ground $\mathrm{He}(1^1S)$ state owing to radiation of a resonant photon of the

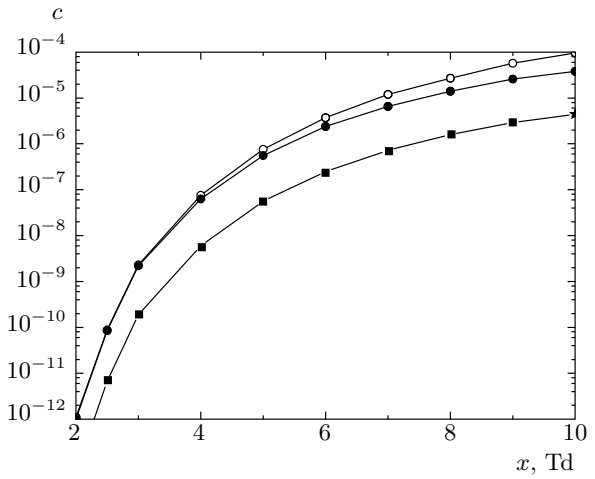

Fig. 8. Concentrations of metastable helium atoms in an ionized helium placed in an external electric field as a function of the reduced electric field at the electron concentration $c_e = 10^{-6}$. Open circles correspond to $c(2^3S)$, Eq. (3.2), filled squares correspond to $c(2^1S)$, Eq. (3.3). For comparison, the concentrations of metastable atoms $\mathrm{He}(2^3S)$ (filled circles) where the contribution to the decay of metastable atoms as a result of their ionization in collisions with electrons taken into account

wavelength about 58 nm. Then the number of resonant photons created per unit time and unit volume is

$$\frac{dN(2^1P)}{dt} = N_e N_a k_{ex} \frac{k_{ef} - k_q}{k_{ef}} \approx N_e N_a k_{ex}, \qquad (4.1)$$

where the total rate constant k_{ef} of quenching of the metastable 2^3S state is given by formula (3.1) and greatly exceeds the rate constant of metastable atom quenching k_q with a transition to the ground state under appropriate electron temperatures, as is shown in Fig. 7. Formula (4.1) means that each formed metastable atom is subsequently transformed into a resonantly excited atom and produces a resonant photon. Below, we estimate the conditions for this case, being guided by a glow gas discharge plasma with the parameters [2] $N_e \sim 10^{10}$–10^{12} cm^{-3}, $N_a \sim 10^{16}$–10^{17} cm^{-3} and the plasma size $L \sim 1$ cm.

We first estimate the lifetime of a resonant photon inside an ionized gas. The broadening of the spectral line for the transition

$$\mathrm{He}(2^1P) \to \mathrm{He}(1^1S) + \hbar\omega \qquad (4.2)$$

has the Lorenz character for large frequency shifts and is determined by the dipole–dipole interaction of atoms. The spectral line width ν is given by [30]

$$\nu = \frac{1}{2}\langle N_a v \sigma_t \rangle, \qquad (4.3)$$

where σ_t is the total cross section of atom collision, and the average is taken over the atom distribution function. The total cross section of collision for atoms in states ^1S and ^1P due to the dipole–dipole atom interaction is given by [31]

$$\sigma_t = \frac{4.8\pi d^2}{\hbar v}, \qquad (4.4)$$

where d is the matrix element of the atom dipole moment that is taken between the states of collided atoms. It is connected with the oscillator strength f of the radiative transition between these states as

$$f = \frac{2m_e d^2 \Delta\varepsilon}{e^2\hbar^2}, \qquad (4.5)$$

and $\Delta\varepsilon$ is the transition energy between ^1S and ^1P states. As a result, we find the absorption coefficient for resonant photons in the line center of the transition (4.2):

$$k_0 = 1.8 \cdot 10^6 \text{ cm}^{-1}. \qquad (4.6)$$

It is important that the absorption coefficient at the line center is independent of both the number density of atoms and the electron temperature. We assume here that the criterion of the Lorenz broadening of spectral lines holds.

The probability of propagation of resonant radiation at a distance L for the Lorenz shape of the spectral line is given by [32]

$$P(L) = \frac{1}{\sqrt{\pi k_0 L}} \qquad (4.7)$$

and a typical lifetime τ_r of resonant photons inside a gas is

$$\tau_r \propto \frac{\tau}{P(L)}.$$

Here, $\tau = 0.56$ ns is the radiative lifetime of an isolate helium atoms in the state 2^1P. From this, we have that for helium with the indicated parameters, the radiation lifetime of resonant photons inside a uniform gas increases by three orders of magnitude compared to an isolated atom and is estimated as $\tau_r \sim 1$ μs.

The criterion of the above regime of atom excitation by electron impact in weakly ionized helium located in the electric field is given by [26]

$$N_e k_Q \tau_r \ll 1, \qquad (4.8)$$

where $k_Q = 5 \cdot 10^{-7}$ cm^3/s is the rate constant of quenching of $\mathrm{He}(2^1P)$ state with the transition into

the 2^1S state. Under the above conditions, this criterion has the form $N_e \ll 2 \cdot 10^{12}$ cm^{-3}, and therefore the above description is suitable for a glow discharge plasma.

We note that this regime resembles the capillary discharge (see, e. g., [33]), which is the source of resonant radiation when an ionized gas is located inside a filament of a diameter of hundreds microns. But the capillary discharge plasma is characterized by higher number densities of atoms and electrons compared with those parameters, and hence corresponds to another plasma regime.

5. PROPERTIES OF IONIZED GAS LOCATED IN ELECTRIC FIELD

The above analysis allows discussing the character of excitation of an ionized gas under the action of an electric field. It follows that a helium gas discharge plasma exists in the regime under consideration in a restricted range of electric field strengths that corresponds to typical electron energies approximately between 1 and 5 eV. At lower electric field strengths, processes of atom excitation and ionization are weak and are not able to support this system, while these processes proceed fast at larger electric field strengths and lead to the ionization instability of an ionized gas.

We estimate the conditions under which an ionized gas is a thermodynamic system and the excitation rate constant of atoms is given by formula (1.2). This holds if

$$k_< \gg k_>, \tag{5.1}$$

and hence the excitation does not disturb the electron distribution function at the excitation threshold. In addition, for the conservation the electron distribution function above the excitation threshold, the criterion

$$\kappa \ll 1 \tag{5.2}$$

is required, where the parameter κ is given by formula (2.17). As can be seen, criteria (5.1) and (5.2) are not fulfilled in the helium case under consideration.

We now evaluate one more parameter of the excitation process, the efficiency of the excitation process ξ, i. e., the part of energy that is introduced into the ionized gas and is consumed for the excitation process. This quantity is given by

$$\xi = \frac{\Delta\varepsilon k_{ex} N_a}{p_{el} + \Delta\varepsilon k_{ex} N_a}, \tag{5.3}$$

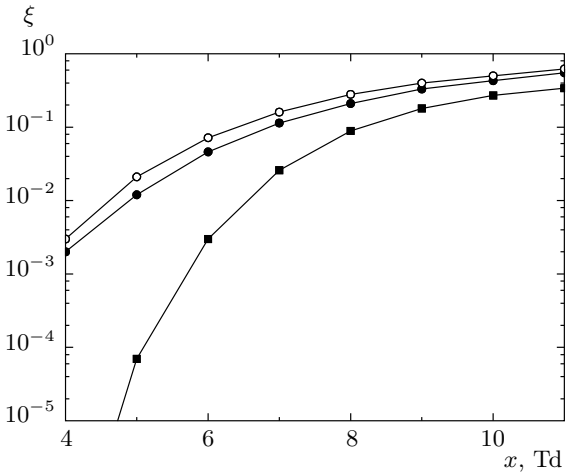

Fig. 9. Efficiency of the excitation of the metastable state 2^3S in ionized helium located in an external electric field. Filled squares correspond to the regime of low electron number densities; open and filled circles correspond to the regime of high electron number densities with the respective electron concentration $c_e = 10^{-5}$ and $c_e = 10^{-6}$

where $\Delta\varepsilon$ is the atom excitation energy, $p_{el} = eEw_e$ is the power per electron that is consumed on elastic electron scattering on atoms, and w_e is the electron drift velocity in the gas under the action of the electric field. Using the expressions for the electron drift velocity (see, e. g., [27]), in helium in the regime of low electron number density, we have

$$\frac{p_{el}}{N_a} = 5.3 \cdot 10^{-12} x^{3/2}, \tag{5.4}$$

and in the regime of high electron number density,

$$\frac{p_{el}}{N_a} = 3.7 \cdot 10^{-12} \frac{x^2}{\sqrt{T_e}}. \tag{5.5}$$

Here the reduced power p_{el}/N_a for the electron elastic scattering on helium atoms is given in eV $\cdot$ cm^3/s. Formulas (5.4) and (5.5) are based on the assumption that the diffusion cross section of the electron–atom elastic scattering is independent of the electron energy. Figure 9 contains the values of the efficiency ξ of the excitation of helium atoms in ionized gas under the action of an electric field.

6. CONCLUSION

We are based on the position that an ionized gas located in an electric field, i. e., a gas discharge plasma,

is a nonequilibrium system. Therefore, universal methods of the description of a gas discharge plasma [5] have a qualitative character, and the analysis of its properties requires the rate constants for processes involving atoms in the ground and excited states. In reality, we face the absence of such information, but in the case of helium, these rate constants are known, we encounter some accuracy and the analysis of the properties of helium excited by an external electric field gives us a useful experience to understand the physics of this object.

We note that an ionized gas supported by an electric field exists in a restricted range of electric field strengths. Low electric fields cannot support this system, whereas high electric fields lead to the ionization instability. In the case of helium, we are restricted by a range of electric field strengths with typical electron energies 1–5 eV. We assume the rate constants of quenching of excited atom states by electron impact to be independent of the electron energy. This latent assumption is important for the calculations fulfilled. We note that the approximation (2.6) for the elastic cross section of electron–atom scattering is not of principle in the above evaluations and may be replaced by numerical computer calculations, but this approximation allowed us to obtain the results in a vivid and transparent form.

We discuss the above results from the standpoint of the general approaches of a gas discharge plasma. We analyze excitation of helium in an external electric field for both regimes of low and high electron number densities, with a typical electron energy being small compared to the atom excitation energy. Then an electron energy variation in a single collisions is relatively small and formation of fast electrons has a stepwise character. In this case, the universal thermodynamic description of the excitation process is not valid for two reasons. First, the electron energy distribution function above the excitation threshold drops sharply because the excitation process leads to a decrease in the distribution function and this in turn decreases the excitation rate, making the atom excitation above the excitation threshold a self-consistent process. Second, after the atom excitation, fast electrons become slow and must be replaced by electrons accelerated in the electric field, but this process is not fast because the decrease in the electron energy proceeds in small steps. As a result, the excitation rate constant is lower by one to two orders of magnitude than that in (1.2) for an equilibrium system (see Fig. 5).

One more experience of this analysis is that the quenching of metastable atoms does not make them transfer to the ground state, but follows from their subsequent excitation (see Fig. 7). This may lead to different regimes of quenching depending on the electron and radiative processes involving excited atoms. The number of plasma regimes increases with additional processes involving excited atoms, for example, the Penning process that occurs if other atoms are present in helium with a small concentration or the ionization process results from collisions of two metastable atoms. Thus, the complexity of the gas discharge plasma consists in a large number of regimes for this nonequilibrium system, and our analysis of the excitation rate constants for helium atoms by electron impact corroborates this statement.

It should be noted that although we do not consider here ionization processes that support a weakly ionized gas, the excitation processes under consideration are important for this. Indeed, if the decay of metastable atoms proceeds in collisions with electrons, whose concentration is shown in Fig. 8, the subsequent formation of electrons results in the ionization of metastable atoms. This process occurs at the reduced electric field strengths 2 Td $< x <$ 10 Td, and we used above just this range of electric field strengths. In this regime, the electron distribution function decreases sharply as the electron energies increase above the excitation threshold, and direct ionization of atoms by electron impact is impossible. On the contrary, the classical direct mechanism of atom ionization by electron impact for self-maintaining of gas ionization [34] occurs in the range of electric fields strengths 10 Td $< x <$ 1000 Td, and the lifetime of metastable atoms is small in this regime. This stresses once more a variety of regimes for a gas discharge plasma and the complexity of their description.

REFERENCES

1. B. M. Smirnov, UFN **179**, 591 (2009) [Phys. Usp. **52**, 519 (2009)].
2. A. A. Kudryavtsev, A. A. Smirnov, and L. D. Tsendin, *Physics of Glow Discharge*, Lan'Petersburg (2009) (in Russian).
3. B. M. Smirnov, *Properties of Gas Discharge Plasma*, Politech. Univ. Press, Petersburg (2010) (in Russian).
4. L. D. Tsendin, UFN **180**, 113 (2010) [Phys. Usp. **53**, 133 (2010)].
5. Y. P. Raizer, *Gas Discharge Physics*, Springer, Berlin (1991).
6. B. M. Smirnov, *Plasma Processes and Plasma Kinetics*, Wiley, Weinheim (2007).

7. L. D. Landau and A. Kompaneetz, Phys. Zs. Sowjetunion **6**, 163 (1934); Zh. Eksp. Theor. Fiz. **5**, 276 (1935).

8. B. I. Davydov, Phys. Zs. Sowjetunion **9**, 433 (1936).

9. L. D. Landau and E. M. Lifshitz, Phys. Zs. Sowjetunion **9**, 477 (1936).

10. J. Yamashita and M. Watanabe, Prog. Theor. Phys. **12**, 433 (1954).

11. B. I. Davydov, Phys. Zs. Sowjetunion **8**, 59 (1935).

12. P. M. Morse, W. P. Allis, and E. S. Lamar, Phys. Rev. **48**, 412 (1935).

13. W. P. Allis and E. S. Lamar, Phys. Rev. **48**, 412 (1935).

14. B. I. Davydov, Phys. Zs. Sowjetunion **12**, 269 (1937).

15. W. P. Allis and H. W. Allen, Phys. Rev. **52**, 703 (1937).

16. S. Chapman and T. G. Cowling, *The Mathematical Theory of Non-Uniform Gases*, Cambr. Univ. Press, Cambridge (1939).

17. W. P. Allis, *Handbuch der Physik*, ed. by S. Flugge, Springer, Berlin (1956), Vol. 21, p. 383.

18. L. E. Gurevich, *Introduction to Physical Kinetics*, GIITL, Leningrad (1940) (in Russian).

19. E. M. Lifshits and L. P. Pitaevskii, *Physical Kinetics*, Pergamon Press, Oxford (1981).

20. J. L. Pack et al., J. Appl. Phys. **71**, 5363 (1992).

21. L. D. Landau, Zh. Eksp. Theor. Fiz. **7**, 203 (1937).

22. B. M. Smirnov, *Principles of Statistical Physics*, Wiley, Berlin (2006).

23. M. J. Druyvesteyn, Physica **10**, 61 (1930).

24. M. J. Druyvesteyn, Physica **1**, 1003 (1934).

25. A. V. Eletskii and V. D. Kulagin, Sov. J. Plasma Phys. **5**, 98 (1979) (in Russian).

26. B. M. Smirnov, *Physics of Ionized Gases*, Wiley, New York (2001).

27. B. M. Smirnov, *Physics of a Weakly Ionized Gases*, Fizmatgiz, Moscow (1972) (in Russian).

28. B. M. Smirnov, *Fundamental of Ionized Gases*, Wiley, Berlin (2012).

29. B. M. Smirnov, *Excited Atoms*, Energoatomizdat, Moscow (1982) (in Russian).

30. I. I. Sobelman, *Radiative Spectra and Radiative Transitions*, Springer-Verlag, Berlin (1979).

31. Yu. A. Vdovin and V. M. Galitskii, Sov. Phys. JETP **52**, 1345 (1967).

32. B. A. Veklenko, Zh. Eksp. Theor. Fiz. **33**, 629; 817 (1957).

33. G. Schönhense and U. Heinzmann, J. Phys. E **16**, 74 (1983).

34. J. S. Townsend, *Electricity in Gases*, Clarendon Press, Oxford (1915).

35. J. Dutton, J. Phys. Chem. Ref. Data **4**, 577 (1975).