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DYNAMICAL INSTABILITY OF COLLAPSING RADIATING FLUID

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We take the collapsing radiative fluid to investigate the dynamical instability with cylindrical symmetry. We match the interior and exterior cylindrical geometries. Dynamical instability is explored at radiative and nonradiative perturbations. We conclude that the dynamical instability of the collapsing cylinder depends on the critical value $\Upsilon < 1$ for both radiative and nonradiative perturbations.

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1. INTRODUCTION

The subject of dynamical instability of self-gravitating objects has attracted many astrophysicists due to explosions and evolution of these objects. The evolution of different self-gravitating objects during gravitational collapse for different ranges of instability is an important feature of dynamical instability. The static stellar model would be interesting if it remains stable under fluctuations. In this scenario, Chandrasekhar [1] found the instability range $\Upsilon < 4/3$ for the spherically symmetric spacetime with isotropic fluid. After that, many people investigated the effects of physical properties of the fluid in the onset of dynamical instability with spherical symmetry.

In [2], it was found that dissipation at Newtonian (N) limit reduces the stability of the sphere and boosts at post-Newtonian (pN) limit. The conclusion in [3] was that the effects of radiation appears similar to dissipation in the N and pN limits. The same authors [4] examined the instability of a spherically symmetric spacetime with shear viscosity and found that it decreases the instability of the fluid. The dynamical instability of a collapsing radiating star would be increased due to the presence of anisotropic pressure and shear viscosity [5].

Cylindrically symmetric spacetimes are idealized models in general relativity. The study of gravitational collapse of these astrophysical objects is an important problem. Some recent work [6–10] indicate great interest in cylindrical gravitational collapse with different fluids with and without an electromagnetic field. Recently, Sharif and his collaborators [11] investigated the dynamical instability for spherically and cylindrically symmetric spacetimes in general relativity and $f(R)$ gravity in the N and pN regimes, respectively. They have shown that the electromagnetic field, pressure anisotropy, dissipation, and $f(R)$ models have great relevance in the range of instability. The same authors [12] have also explored this problem for the thin-shell wormholes in nonlinear electrodynamics.

In this paper, we explore the dynamical instability of a collapsing radiating cylinder in the N and pN approximations. The paper is organized as follows. In Sec. 2, the field equations and matching conditions are developed. In Sec. 3, we formulate the dynamical instability at nonradiative and radiative perturbations. Finally, we discuss our results in Sec. 4.

2. FIELD EQUATIONS AND MATCHING CONDITIONS

The matter under consideration is assumed to be locally isotropic with pure radiation inside a cylindrical surface Σ. The energy–momentum tensor for such a fluid has the form

$$T_{\alpha\beta} = (\mu + p)w_\alpha w_\beta + pg_{\alpha\beta} + \varepsilon l_\alpha l_\beta, \qquad (1)$$

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and the following conditions are satisfied in comoving coordinates:

$$w^\alpha = A^{-1}\delta_0^\alpha, \quad l^\alpha = A^{-1}\delta_0^\alpha + B^{-1}\delta_1^\alpha,$$

$$w^\alpha w_\alpha = -1, \quad l_\alpha l^\alpha = 0.$$

Here μ, p, w_α, ε, and l_α are the energy density, the isotropic pressure, the four-velocity, the radiation density, and the null four-vector of the fluid. We consider the nonstatic cylindrically symmetric spacetime inside the hypersurface Σ with

$$ds^2 = -W^2(t,r)\,dt^2 + X^2(t,r)\,dr^2 + + Y^2(t,r)\,d\theta^2 + dz^2, \quad (2)$$

where

$$-\infty < t < \infty, \quad 0 \leqslant r < \infty,$$

$$-\infty < z < \infty, \quad 0 \leqslant \phi \leqslant 2\pi.$$

Using Eqs. (1) and (2), we can write the Einstein field equations

$$\kappa T_{\alpha\beta} = G_{\alpha\beta}$$

as

$$\kappa A^2(\mu + \varepsilon) = \left(\frac{W}{X}\right)^2 \left(\frac{X'Y'}{XY} - \frac{Y''}{Y}\right) + \frac{\dot{X}\dot{Y}}{XY}, \quad (3)$$

$$\kappa W X \varepsilon = \frac{\dot{Y}'}{Y} - \frac{\dot{Y}W'}{YW} - \frac{\dot{X}Y'}{XY}, \quad (4)$$

$$\kappa X^2(p + \varepsilon) = \left(\frac{X}{W}\right)^2 \left(\frac{\dot{W}\dot{Y}}{WY} - \frac{\ddot{Y}}{Y}\right) + \frac{W'Y'}{WY}, \quad (5)$$

$$\kappa p = \frac{W''}{WX^2} + \frac{\dot{W}\dot{X}}{W^3X} - \frac{\ddot{X}}{W^2X} - \frac{W'X'}{X^3W}, \quad (6)$$

$$\kappa p = \frac{W''}{WX^2} - \frac{\ddot{X}}{W^2X} + \frac{\dot{W}\dot{X}}{W^3X} - \frac{W'X'}{WX^3} + \frac{\dot{W}\dot{Y}}{W^3Y} - \frac{\ddot{Y}}{W^2Y} - - \frac{X'Y'}{X^3Y} + \frac{Y''}{X^2Y} + \frac{W'Y'}{WX^2Y} - \frac{\dot{X}\dot{Y}}{W^2XY}. \quad (7)$$

The mass function proposed by Thorne [13] in the form of gravitational C-energy per unit specific length is defined as

$$m(t,r) = \frac{1}{8} - \frac{1}{8l^2}\nabla^\beta \tilde{r} \nabla_\beta \tilde{r}, \quad (8)$$

and satisfies the relations

$$\rho^2 = \eta_{(\theta)a}\eta_\theta^a, \quad l^2 = \eta_{(z)a}\eta_z^a, \quad \tilde{r} = \rho l,$$

where l, ρ, $\tilde{r}$, η_θ, and η_z are the specific length, circumference radius, areal radius, and two Killing vectors for the cylindrical geometry. Hence, the specific energy of the collapsing cylinder becomes [14]

$$m(r,t) = \frac{1}{8}\left[1 + \left(\frac{\dot{Y}}{W}\right)^2 - \left(\frac{Y'}{X}\right)^2\right]. \quad (9)$$

For the radiative fluid, the conservation of the stress–energy tensor,

$$(T^{\alpha\beta})_{;\beta} = 0,$$

yields

$$\dot{\mu} + \dot{\varepsilon} + \left(\frac{2W'}{W} + \frac{Y'}{Y} + \frac{\varepsilon'}{\varepsilon}\right)\varepsilon\frac{W}{Y} + (\mu + p + \varepsilon) \times \times \left(\frac{\dot{X}}{X} + \frac{\dot{Y}}{Y}\right) + \varepsilon\frac{\dot{X}}{X} = 0, \quad (10)$$

$$p' + \dot{\varepsilon}\frac{X}{W} + (\mu + p + 2\varepsilon)\frac{W'}{W} + \varepsilon\frac{Y'}{Y} + + \left(2\frac{\dot{X}}{X} + \frac{\dot{Y}}{Y}\right)\varepsilon\frac{X}{W} + \varepsilon' = 0. \quad (11)$$

We use the Darmois conditions [15] for the continuity of inner and outer manifolds. The cylindrical manifold in the exterior region is [16]

$$ds^2 = -\left(-\frac{2M}{R}\right)d\nu^2 - 2d\nu\,dR + + R^2(d\theta^2 + \gamma^2 dz^2), \quad (12)$$

where ν is known as the retarded time coordinate and γ has the dimension of $1/r$. From the Darmois conditions, as discussed in [11], it follows that

$$m - M \overset{\Sigma}{=} \frac{1}{8}, \quad p \overset{\Sigma}{=} 0. \quad (13)$$

This shows that the difference of interior and exterior masses is equal to 1/8 at the boundary surface and the isotropic pressure vanishes.

3. THE PERTURBATION SCHEME AND DYNAMICAL INSTABILITY

We perturb the field equations, conservation equations, and physical functions of the fluid (initially in hydrostatic equilibrium) up to the first order in $0 < \lambda \ll 1$. We use the perturbation scheme [11]

$$W(t,r) = W_0(r) + \lambda T(t) w(r), \quad (14)$$

$$X(t,r) = X_0(r) + \lambda T(t) x(r), \quad (15)$$

$$Y(t,r) = rX(t,r)[1 + \lambda T(t)\bar{y}(r)], \quad (16)$$

$$\mu(t,r) = \mu_0(r) + \lambda\bar{\mu}(t,r), \quad (17)$$

$$p(t,r) = p_0(r) + \lambda\bar{p}(t,r), \quad (18)$$

$$\varepsilon(t,r) = \lambda\bar{\varepsilon}(t,r), \quad (19)$$

$$m(t,r) = m_0(r) + \lambda\bar{m}(t,r). \quad (20)$$

The static (unperturbed) and perturbed forms of Eqs. (3)–(5) are given by

$$\kappa\mu_0 = \frac{1}{X_0^2}\left[\left(\frac{X_0'}{X_0}\right)^2 - \frac{1}{r}\frac{X_0'}{X_0} - \frac{X_0''}{X_0}\right], \qquad (21)$$

$$\kappa p_0 = \frac{1}{X_0^2}\frac{W_0'}{W_0}\left(\frac{X_0'}{X_0} + \frac{1}{r}\right), \qquad (22)$$

$$\kappa(\bar{\mu} + \bar{\varepsilon}) = -\frac{T}{X_0^2}\left[\left(\frac{x}{X_0} - \bar{y}\right)\left(\frac{X_0'}{X_0}\right)^2 + \frac{1}{X_0}\left((\bar{y}X_0')' + \frac{(rx')'}{r}\right) + \frac{2}{r}\left(\bar{y}' + \frac{r\bar{y}''}{2}\right) - \frac{X_0'}{X_0}\left(\frac{2x'}{X_0} - \frac{\bar{y}}{r}\right)\right] - \kappa\frac{T}{X_0^2}\left(\frac{3x}{X_0} + \bar{y}\right)\mu_0, \qquad (23)$$

$$\kappa W_0 X_0 \bar{\varepsilon} = \left[\left(\frac{x}{W_0 X_0}\right)' + \left(\frac{\bar{y}}{W_0}\right)' + \left(\frac{1}{r} + \frac{X_0'}{X_0}\right)\left(\frac{\bar{y}}{W_0}\right)\right]\dot{T}, \qquad (24)$$

$$\kappa(\bar{p} + \bar{\varepsilon}) = -\left(\frac{x}{X_0} + \bar{y}\right)\frac{\ddot{T}}{W_0^2} + \frac{T}{X_0^2}\left[\left(\frac{1}{r} + \frac{X_0'}{X_0}\right) \times \left(\frac{w}{W_0}\right)' + \frac{W_0'}{W_0}\left(\frac{x}{X_0} + \bar{y}\right)'\right] - 2\kappa\frac{Tx}{X_0}p_0. \qquad (25)$$

Similarly, Eqs. (10) and (11) turn out to be

$$\frac{W_0'}{W_0} = -\frac{p_0'}{(\mu_0 + p_0)}, \qquad (26)$$

$$\dot{\bar{\mu}} + \dot{\bar{\varepsilon}} + \left(\frac{2W_0'}{W_0} + \frac{X_0'}{X_0} + \frac{1}{r} + \frac{\bar{\varepsilon}'}{\bar{\varepsilon}}\right)\bar{\varepsilon}\frac{W_0}{X_0} + (\mu_0 + p_0)\left(2\frac{x}{X_0} + \bar{y}\right)\dot{T} = 0, \qquad (27)$$

$$\bar{p}' + \bar{\varepsilon}' + \dot{\bar{\varepsilon}}\frac{X_0}{W_0} + (\mu_0 + p_0)T\left(\frac{w}{W_0}\right)' + \bar{\varepsilon}\left(\frac{1}{r} + \frac{X_0'}{X_0}\right) + (\bar{\mu} + \bar{p} + 2\bar{\varepsilon})\frac{W_0'}{W_0} = 0. \qquad (28)$$

Inserting Eq. (24) in (27) and integrating, we obtain

$$\bar{\mu} = -\left(\frac{2x}{X_0} + \bar{y}\right)(\mu_0 + p_0)T - \frac{1}{\kappa X_0^2}\left(\Phi' + \left(\frac{X_0}{W_0} + \frac{1}{r} + \frac{W_0'}{W_0}\right)\Phi\right)T, \qquad (29)$$

where

$$\Phi(r) = \left[\left(\frac{x}{W_0 X_0}\right)' + \left(\frac{\bar{y}}{W_0}\right)' + \left(\frac{1}{r} + \frac{X_0'}{X_0}\right)\left(\frac{\bar{y}}{W_0}\right)\right]. \qquad (30)$$

The unperturbed form of the mass function becomes

$$m_0 = \frac{1}{8}\left[1 - \left(\frac{1}{r} + \frac{X_0'}{X_0}\right)^2\right], \qquad (31)$$

which can be rewritten as

$$\frac{X_0'}{X_0} = -\frac{1}{r} + \sqrt{1 - 8m_0}. \qquad (32)$$

Now we can write the differential equation by considering Eqs. (24) and (25) and using the condition

$$p_0 \overset{\Sigma}{=} 0$$

as

$$\psi T - 2\phi\dot{T} \overset{\Sigma}{=} \ddot{T}, \qquad (33)$$

where

$$\psi(r) \overset{\Sigma}{=} \left(\frac{W_0}{X_0}\right)^2\left(\frac{x}{X_0} + \bar{y}\right)^{-1}\left[\left(\frac{1}{r} + \frac{X_0'}{X_0}\right) \times \left(\frac{w}{W_0}\right)' + \frac{W_0'}{W_0}\left(\frac{x}{X_0} + \bar{y}\right)'\right], \qquad (34)$$

$$\phi(r) \overset{\Sigma}{=} \frac{1}{2}\frac{W_0}{X_0}\left(\frac{x}{X_0} + \bar{y}\right)^{-1}\left[\left(\frac{x}{W_0 X_0}\right)' + \left(\frac{\bar{y}}{W_0}\right)' + \left(\frac{1}{r} + \frac{X_0'}{X_0}\right)\left(\frac{\bar{y}}{W_0}\right)\right]. \qquad (35)$$

The solution of the above equation yields

$$T(t) = -\exp\left[\left(-\phi_\Sigma + \sqrt{\psi_\Sigma + \phi_\Sigma^2}\right)t\right], \qquad (36)$$

where we choose

$$\psi_\Sigma > 0, \quad \phi_\Sigma < 0$$

for the solution to be real. According to the above equation, the cylinder proceeds to collapse at $t = -\infty$ and continues with the increase in t.

To find dynamical instability, we introduce the adiabatic index Υ defined in [2]:

$$\bar{p} = \Upsilon\left(\frac{p_0}{\mu_0 + p_0}\right)\bar{\mu}, \qquad (37)$$

where quantities with a bar represent the perturbed energy density and isotropic pressure. Substituting Eq. (29) in the above equation, we obtain

$$\bar{p} = -\Upsilon p_0 \left[\left(\frac{2x}{X_0} + \bar{y}\right) T + \right.$$
$$\left. + \frac{1}{\kappa X_0^2}\left(\Phi' + \left(\frac{W_0'}{W_0} + \frac{X_0}{W_0} + \frac{1}{r}\right)\Phi\right)\frac{T}{\mu_0 + p_0}\right]. \quad (38)$$

Equation (25) with the matching condition leads to

$$\left(\frac{w}{W_0}\right)' = \left(\frac{1}{r} + \frac{X_0'}{X_0}\right)^{-1} \times$$
$$\times \left\{\frac{X_0^2}{W_0^2}\left(\frac{x}{X_0} + \bar{y}\right)\frac{\ddot{T}}{T} - \frac{W_0'}{W_0}\left(\frac{x}{X_0} + \bar{y}\right)' - \right.$$
$$- \left[\left(\frac{2x}{X_0} + \bar{y}\right)\kappa \Upsilon p_0 X_0^2 - \left(\Phi' + \left(\frac{W_0'}{W_0} + \frac{X_0}{W_0} + \frac{1}{r}\right)\Phi\right) \times \right.$$
$$\left.\left. \times \Upsilon\left(\frac{p_0}{\mu_0 + p_0}\right)\right] + \frac{X_0}{W_0}\Phi\frac{\dot{T}}{T} + \kappa p_0 x X_0\right\}. \quad (39)$$

Next, we develop the main equation used for the instability range at nonradiative and radiative perturbations. It follows by inserting Eqs. (24), (29), (38), and (39) in (28) that

$$\left[-p_0\Upsilon\left(\frac{2x}{X_0} + \bar{y}\right) - \frac{\Upsilon}{\kappa X_0^2}\left(\frac{p_0}{\mu_0 + p_0}\right) \times \right.$$
$$\left. \times \left(\Phi' + \left(\frac{W_0'}{W_0} + \frac{X_0}{W_0} + \frac{1}{r}\right)\Phi\right)\right]' +$$
$$+ \frac{1}{\kappa}\left[\frac{1}{W_0 X_0}\Phi\right]'\frac{\dot{T}}{T} + \frac{X_0}{W_0}\Phi\frac{\ddot{T}}{T} + \frac{1}{\kappa} \times$$
$$\times \left(\frac{1}{r} + \frac{X_0'}{X_0}\right)\frac{1}{W_0 X_0}\Phi\frac{\dot{T}}{T} + (\mu_0 + p_0)\left(\frac{1}{r} + \frac{X_0'}{X_0}\right)^{-1} \times$$
$$\times \left\{\frac{X_0^2}{W_0^2}\left(\frac{x}{X_0} + \bar{y}\right)\frac{\ddot{T}}{T} - \frac{W_0'}{W_0}\left(\frac{x}{X_0} + \bar{y}\right)' - \right.$$
$$- \left[\left(\frac{2x}{X_0} + \bar{y}\right)\kappa\Upsilon p_0 X_0^2 - \right.$$
$$\left. - \left(\Phi' + \left(\frac{W_0'}{W_0} + \frac{X_0}{W_0} + \frac{1}{r}\right)\Phi\right)\Upsilon\left(\frac{p_0}{\mu_0 + p_0}\right)\right] +$$
$$\left. + \frac{X_0}{W_0}\Phi\frac{\dot{T}}{T} + \kappa p_0 x X_0\right\} - \left(\frac{2x}{X_0} + \bar{y}\right)(\mu_0 + p_0 + \Upsilon p_0) \times$$
$$\times \frac{W_0'}{W_0} - \frac{1}{\kappa X_0}\left(\Phi' + \left(\frac{W_0'}{W_0} + \frac{X_0}{W_0} + \frac{1}{r}\right)\Phi\right) \times$$
$$\times \frac{\Upsilon W_0'}{W_0}\left(\frac{p_0}{\mu_0 + p_0} + 1\right) = 0. \quad (40)$$

3.1. Nonradiative perturbation

For a nonradiative perturbation, we assume that $\varepsilon = 0$. Then the integration of Eq. (24) leads to

$$x = W_0 X_0, \quad \bar{y}' = -\bar{y}\left(\frac{p_0'}{\mu_0 + p_0} + \sqrt{1 - 8m_0}\right). \quad (41)$$

Using this fact, we see from Eqs. (30) and (35) that

$$\Phi(r) = 0, \quad \phi(r) = 0.$$

Substituting these results in Eq. (40), we obtain the instability equation as

$$\left[-\left(2\frac{x}{X_0} + \bar{y}\right)\Upsilon p_0\right]' + (\mu_0 + p_0)\left(\frac{1}{r} + \frac{X_0'}{X_0}\right)^{-1} \times$$
$$\times \left[\kappa p_0 x X_0 + \frac{X_0^2}{W_0^2}\left(\frac{x}{X_0} + \bar{y}\right)\frac{\ddot{T}}{T} - \right.$$
$$\left. - \kappa\left(2\frac{x}{X_0} + \bar{y}\right)\Upsilon p_0 X_0^2 - \frac{W_0'}{W_0}\left(\frac{x}{X_0} + \bar{y}\right)'\right] -$$
$$- \left(2\frac{x}{X_0} + \bar{y}\right)(\mu_0 + p_0 + \Upsilon p_0)\frac{W_0'}{W_0} = 0. \quad (42)$$

3.1.1. Newtonian approximation

To find the instability range in the N approximation, we use that

$$W_0 = 1, \quad X_0 = 1,$$

and ignore terms like p_0/μ_0 that are of the order of m_0/r in Eq. (42), which gives

$$-2p_0'\Upsilon + 2p_0' + \psi_\Sigma\mu_0 = 0, \quad (43)$$

assuming $p_0' < 0$ for the collapsing fluid. Consequently, the instability condition turns out to be

$$\Upsilon < 1. \quad (44)$$

This equation shows that the instability of the collapsing fluid depends on the critical value 1.

3.1.2. Post-Newtonian approximation

In the pN approximation, we assume that

$$W_0 = 1 - \frac{m_0}{r}, \quad X_0 = 1 + \frac{m_0}{r},$$

and take the terms of the order of m_0/r. Consequently, Eq. (42) becomes

$$-(2 + \bar{y})p_0'\Upsilon + 2p_0' + \Upsilon p_0 + \psi_\Sigma(1 + \bar{y})\frac{\mu_0}{r} -$$
$$- \kappa(2 + \bar{y})\Upsilon\mu_0 p_0 + \kappa\mu_0 p_0 = 0, \quad (45)$$

and the instability range in this limit turns out to be

$$\Upsilon < 1 + \frac{1}{|p_0'|}\left[\frac{1}{2}\kappa\mu_0 p_0 - \frac{\mu_0}{2r}(1+\bar{y})\psi_\Sigma\right]. \tag{46}$$

In the above inequality, the third term, which comes from the static background of energy density, enhances the instability and is decreased by the last term.

3.2. Radiative perturbation

In the radiative case, we take $\varepsilon \neq 0$, and hence we can write the perturbed solution of Eq. (41) as

$$x(r) = W_0 X_0 \left[1 + \xi f(r)\right], \tag{47}$$

where $\xi > 0$ is an arbitrary function. Substituting this result in Eqs. (30) and (35), we find that $\Phi(r)$ and $\phi(r)$ are of the relativistic order m_0/r.

3.2.1. Newtonian approximation

Considering the restrictions similar to those used in the above case, we have the instability condition for the radiative perturbation in the N approximation in the form

$$-(2+2\xi f)p_0'\Upsilon + (2+2\xi f)p_0' + (1+\xi f)\mu_0\psi_\Sigma = 0. \tag{48}$$

It is known that for a collapsing fluid, $\dot{T} < 0$ and $\bar{\varepsilon} > 0$, leading to $\bar{y} < 0$ and $f' < 0$ in accordance with Eq. (24). In this limit, the corresponding range is

$$\Upsilon < 1 + \left[\frac{\mu_0\psi_\Sigma\xi|f|}{2|p_0'|}\right], \tag{49}$$

which shows that the radiation density increases the instability range in the Newtonian limit. This result is analogous for the heat conduction [6].

3.2.2. Post-Newtonian approximation

The instability equation in the pN approximation turns out to be

$$\begin{aligned} &-(2+2\xi f+\bar{y})p_0'\Upsilon + (2+2\xi f+\bar{y})p_0' - \\ &\quad -(2+2\xi f+\bar{y})\kappa\Upsilon\mu_0 p_0 + (1+\xi f+\bar{y})\mu_0\psi_\Sigma + \\ &\quad + \kappa\mu_0 p_0 + \mu_0\xi f' p_0' + 2\xi f' p_0\Upsilon = 0, \end{aligned} \tag{50}$$

where we use

$$W_0 = 1 - \frac{m_0}{r}, \quad X_0 = 1 + \frac{m_0}{r},$$

and the relativistic correction terms. We require $p_0' < 0$ for the instability condition. Therefore, we can write

$$\begin{aligned} \Upsilon < 1 + \bigg[&\frac{\kappa\mu_0 p_0}{2|p_0'|} + \frac{\mu_0\psi_\Sigma\xi|f|}{2|p_0'|} - \\ &- \frac{\mu_0\xi|f'|}{2|p_0'|} - \frac{(1+\bar{y})\mu_0\psi_\Sigma}{2r|p_0'|}\bigg]. \end{aligned} \tag{51}$$

Here, we see that relativistic correction terms due to radiation increase and decrease the instability range. These results show the effect that different matter terms have on the instability of the system.

4. CONCLUSION

We have explored the dynamical instability of a collapsing fluid producing pure radiation with cylindrical symmetry. We have used the N and pN approximations for nonradiative and radiative perturbations. The critical value (instability range) is found to be 1 for the isotropic perfect fluid in the N regime. Thus, the stability or instability of the system corresponds to the respective value of adiabatic index $\Upsilon > 1$ or $\Upsilon < 1$.

We have seen from Eqs. (44) and (46) that the instability range is 1 for the isotropic perfect fluid and is increased by the relativistic correction terms of the static background configuration. Also, the free streaming radiation increases the instability of the system as shown in Eqs. (49) and (51). We note that the effects of radiation look qualitatively similar to the one obtained for the radial heat flux.

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