

SHOOTING QUASIPARTICLES FROM ANDREEV BOUND STATES IN A SUPERCONDUCTING CONSTRICTION

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A few-channel superconducting constriction provides a set of discrete Andreev bound states that may be populated with quasiparticles. Motivated by recent experimental research, we study the processes in an a.c. driven constriction whereby a quasiparticle is promoted to the delocalized states outside the superconducting gap and flies away. We distinguish two processes of this kind. In the process of ionization, a quasiparticle present in the Andreev bound state is transferred to the delocalized states leaving the constriction. The refill process involves two quasiparticles: one flies away while another one appears in the Andreev bound state. We notice an interesting asymmetry of these processes. The electron-like quasiparticles are predominantly emitted to one side of the constriction while the hole-like ones are emitted to the other side. This produces a charge imbalance of accumulated quasiparticles, that is opposite on opposite sides of the junction. The imbalance may be detected with a tunnel contact to a normal metal lead.

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1. INTRODUCTION

Superconducting mesoscopic structures are among the most promising candidates to realize quantum computation devices in the solid state [1]. Apart from extrinsic sources of decoherence that might get in the way, quasiparticle poisoning constitutes one of the major obstacles inherent to superconductors [2]. In a Cooper pair box, the presence of quasiparticles leads to a coupling of even and odd charge modes, providing a channel of decoherence for the charge qubit [3, 4]. In addition, quasiparticle excitations can break the fermion parity required for the protection of a Majorana state [5–7]. Naively, the superconducting gap Δ should ensure an exponentially suppressed quasiparticle population at sufficiently low temperature. However, various experiments indicate that a long-lived, non-equilibrium quasiparticle population persists in the superconductor, harming the desired operation of superconducting devices [8–13].

This makes it important to develop the means of an active control of the quasiparticle population in bound states associated with a nano-device. Thus motivated, we theoretically investigate the control of the population of quasiparticles in the Andreev bound states at a superconducting constriction by means of pulses of microwave irradiation. We concentrate on the generic case of a few-channel superconducting constriction with highly transparent channels. Such constrictions are made on the basis of atomic break junctions [14]. The simplicity of their theoretical description enabled detailed theoretical research [15–17]. In the presence of a phase difference at the constriction, an Andreev bound state is formed in each channel [18, 19]. In a recent experiment, the population of such a single bound state has been detected by its effect on the supercurrent in the constriction. The spectroscopy of Andreev states has also been successfully performed [20, 21] in this setup.

In this work, we investigate the processes that switch the Andreev bound state population. We assume low temperatures that permit to neglect the population of delocalized quasiparticle states. Let us con-

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sider a quasiparticle with energy $E_A < \Delta$, Δ being the superconducting gap edge in the leads. If we modulate the superconducting phase with the frequency $\hbar\Omega > \Delta - E_A$, we can transfer this quasiparticle to the states of the delocalized spectrum. This is an ionization process. Suppose we start with no quasiparticle in the constriction and wish to fill the bound state. This can be achieved by the absorption of a quantum of the high-frequency phase modulation, provided the quantum energy exceeds $E_A + \Delta$. In the course of such a refill process, one quasiparticle emerges in the Andreev level while another one is promoted to the delocalized states and leaves the constriction.

We will utilize a master equation approach to describe the corresponding transitions. As usual, this works if the transition rates in energy units are much smaller than the energies involved. In our situation, the energy scale is Δ while the transition rates due to the modulation with amplitude $\delta\phi$ can be estimated as $\Delta(\delta\phi)^2$. Therefore, the master equation approach is justified if $|\delta\phi| \ll 1$, that is, in the limit of small modulations.

We compute the rates of the ionization and refill processes in the lowest order in the phase modulation amplitude and shortly explain how to control the population by applying the a.c. pulses that initiate the process.

We find an interesting asymmetry of the quasiparticles emitted in the course of these processes. The quasiparticles fly with equal probability to both leads. However, more electron-like quasiparticles leave to one of the leads while more hole-like ones leave to the opposite one. This results in a net charge transfer per process and in principle can be regarded as a non-equilibrium addition to the supercurrent in the constriction. Similar to the supercurrent, the effect changes sign upon changing the sign of the superconducting phase.

The effect leads to charge imbalance [22, 23] of the non-equilibrium quasiparticles that are accumulated in the leads on the spatial scale set by the inelastic relaxation of the quasiparticles [24]. This charge imbalance can be measured with a normal-metal voltage probe attached to the superconductor: the method proposed in [24] and widely applied in recent years [25, 26].

This paper is organized as follows. We formulate the model in Sec. 2 and we give results for the rates in Sec. 3. Section 4 is dedicated to the estimations of the charge imbalance effect in the voltage-probe setup.

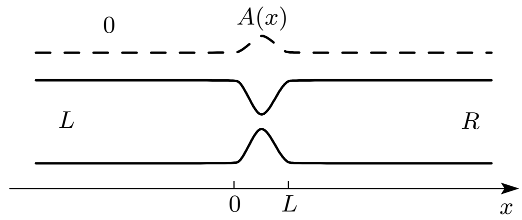

Fig. 1. $1d$ model of the superconducting constriction

2. MODEL

We model the superconducting weak link with a $1d$ quantum Hamiltonian corresponding to a single transport channel, x being the coordinate (see also Fig. 1). The constriction of the length L is modelled by a scattering potential $V(x)$. In addition, a finite vector potential $A(x)$ on a local support provides a phase bias between the left and right contact, $\phi = 2e\int dx\, A(x)$, e being the elementary charge. We focus on the regime where the excitation energy is much smaller than the Fermi energy, $E \ll E_F$, such that the spectrum can be linearised. The pseudo spin $|L, R\rangle$ thus signifies a left/right moving electron with the Fermi wave vector $\mp k_F$, where $\sigma_z = |L\rangle\langle L| - |R\rangle\langle R|$. In the linearized regime, the current density operator is represented as $j = -v_F\sigma_z$. The Bogoliubov-de Gennes Hamiltonian is then given as ($\hbar = 1$)

$$H = [-iv_F\partial_x\sigma_z + V(x)\sigma_x]\,\tau_z - ev_F A(x)\sigma_z + \Delta\tau_x, \quad (1)$$

where the Pauli matrices τ_i represent the Nambu space. The potential V provides the reflection, as $\sigma_x = |L\rangle\langle R| + |R\rangle\langle L|$. Both V and A are real functions and have a finite support in the interval $x \in [0, L]$.

Let us first deal with a stationary phase ϕ. We diagonalize the Hamiltonian, Eq. (1), in the limit of a short constriction, $E, \Delta \ll v_F/L$. There is one Andreev bound state solution $|\varphi_A(x)\rangle$, with a subgap eigenenergy $E_A = \Delta\sqrt{1 - T_0\sin^2(\phi/2)}$, T_0 being the normal state transmission coefficient characterizing the transport channel under consideration. The Andreev bound state is responsible for the supercurrent in the constriction. Since the levels are spin-degenerate, the Andreev level can host $n = 0, 1, 2$ quasiparticles, the supercurrent being $I_s(1-n)$, where $I_s \equiv -2e\partial_\phi E_A$. In addition, there are the extended scattering eigenstates $|\varphi^{out}_{\alpha\eta}\rangle$ with eigenenergies $E > \Delta$. They have the BCS density of states $\nu(E) = \theta(E-\Delta)E/\sqrt{E^2 - \Delta^2}\nu_0$, where ν_0 is the density of states in the normal metal. The indices $\alpha = L, R$ and $\eta = e, h$ indicate the scattering state with an η-like quasiparticle outgoing to the contact α. The outgoing scattering states correspond to the solution of the advanced propagator. This set of states is related to the incoming scattering states (retarded propagator) via the scattering matrix $S_{\alpha\eta}^{\alpha'\eta'} = \langle \varphi_{\alpha'\eta'}^{out} | \varphi_{\alpha\eta}^{in} \rangle$. Our scattering matrix coincides with the one found in Ref. [17].

To describe an a.c. driven system, we assume $\phi(t) = \phi + \delta\phi \sin(\Omega t)$ and treat the phase modulation amplitude as a perturbation. We compute the rates of various processes in the lowest order when they are proportional to $(\delta\phi)^2$.

In addition, the constriction may be subject to quantum phase fluctuations, i.e., the phase modulation becomes an operator, $\delta\phi(t) \to \hat{\phi}$, whose dynamics is determined by the electromagnetic environment of the junction. The phase noise spectrum is $S(\omega) = \int dt\ e^{-i\omega t} \langle \delta\hat{\phi}(0)\, \delta\hat{\phi}^\dagger(t) \rangle_{env}$, where the expectation value is with respect to the environment degrees of freedom. If the environment is in thermal equilibrium, the noise can be related to the impedance $Z(\omega)$ felt by the constriction via the fluctuation dissipation theorem, $S(\omega) = 4\pi G_Q Z(\omega)/\omega$, where $\omega > 0$, $G_Q \equiv e^2/\pi\hbar$. The rates of the inelastic processes are readily computed with this.

3. THE TRANSITION RATES AND MANIPULATION

To compute the rates, we apply Fermi's Golden rule. The advantage of the model and the gauge in use is that the matrix elements of the perturbation only depend on the wave functions $\varphi(x)$ at the origin. For instance, the rate of ionization from the bound state A to delocalized quasiparticle states n with energy $E = E_A + \Omega$ reads

$$\Gamma_I \equiv \Gamma_{A\to n} = \frac{\pi}{8}(\delta\phi)^2 \nu(E) \left| \langle \varphi_A(0) | j | \varphi_n(0) \rangle \right|^2. \quad (2)$$

The rate of the refill process whereby the quasiparticles occur in the state A and n reads

$$\Gamma_R \equiv \Gamma_{0\to An} = \frac{\pi}{8}(\delta\phi)^2 \nu(E) \left| \langle \varphi_n(0) | j | \tilde{\varphi}_A(0) \rangle \right|^2, \quad (3)$$

with $|\tilde{\varphi}_A(0)\rangle = i\tau_y \sigma_x |\varphi_n(0)\rangle^*$ and the energy of the emitted quasiparticle $E = \Omega - E_A$.

For the moment, let us assume that all the extended quasiparticle states are empty. In this regime, the only "natural" process changing the population of the Andreev level is the annihilation of two quasiparticles in the same Andreev bound state. The corresponding rate reads

$$\Gamma_A \equiv \Gamma_{2A\to 0}^{fluct} = S_\phi(2E_A) \left| \langle \varphi_A | j | \tilde{\varphi}_A \rangle \right|^2. \quad (4)$$

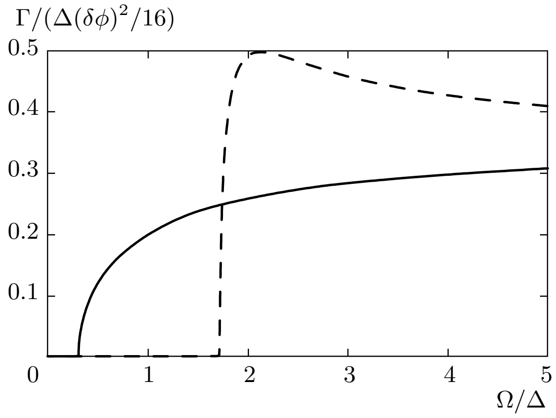

Fig. 2. Ionization and refill rates for $T_0 = 0.5$ and $\phi = \pi$, when $E_A \approx 0.7\Delta$. The ionization rate appears at the threshold $\Omega \approx 0.3\Delta$, while the threshold for the refill is $\approx 1.7\Delta$

Substituting the wave functions into Eqs. (2), (3), and (4), we arrive at the following expressions:

$$\Gamma_I = \frac{T_0(\delta\phi)^2}{16} \theta(\Omega + E_A - \Delta) \frac{\sqrt{\Delta^2 - E_A^2}}{E_A} \times \sqrt{(\Omega + E_A)^2 - \Delta^2} \frac{E_A\Omega + \Delta^2[\cos(\phi) + 1]}{(\Omega + E_A)^2 - E_A^2}, \quad (5)$$

$$\Gamma_R = \frac{T_0(\delta\phi)^2}{16} \theta(\Omega - E_A - \Delta) \frac{\sqrt{\Delta^2 - E_A^2}}{E_A} \times \sqrt{(\Omega - E_A)^2 - \Delta^2} \frac{E_A\Omega - \Delta^2[\cos(\phi) + 1]}{(\Omega - E_A)^2 - E_A^2}, \quad (6)$$

$$\Gamma_A = \frac{S_\phi(2E_A)}{4} \left(1 - \frac{E_A^2}{\Delta^2}\right) \times \left(\Delta^2 - E_A^2 - 4\left[\frac{\partial E_A}{\partial \phi}\right]^2\right). \quad (7)$$

We see that the ionization and refill rates at $T_0 \approx 1$ are of the order of $(\delta\phi)^2\Delta$ and, at sufficiently large phase modulation amplitudes, are restricted by Δ only. Thus the population of the Andreev bound state can be changed quickly. As to the annihilation rate, it may be estimated as $\Gamma_A \approx \langle\langle \phi^2 \rangle\rangle_q \Delta$, $\langle\langle \phi^2 \rangle\rangle_q \approx Z G_Q$ being the quantum fluctuation of the phase. For typical electromagnetic environments, Z is of the order of the vacuum impedance and $\langle\langle \phi^2 \rangle\rangle_q \approx 10^{-3}$. This implies that at sufficiently large a.c. modulations, $(\delta\phi)^2 \gg \langle\langle \phi^2 \rangle\rangle_q$, the annihilation rate can be neglected in comparison with the a.c.-induced rates.

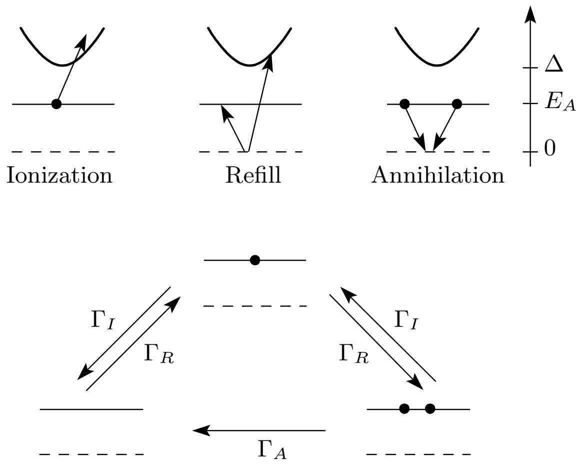

Fig. 3. Transitions causing changes in the Andreev bound state occupation

We illustrate the frequency dependence of the ionization and refill rates in Fig. 2. In the limit of large frequency, both rates saturate at the same value. We stress, however, that the practical frequencies for the manipulation of the Andreev bound state are most likely restricted by 2Δ: higher frequencies would cause massive generation of quasiparticle pairs at the constriction and in the bulk of the superconductor.

Let us determine the distribution of the bound state populations under constant driving. The processes causing transitions between $n = 0, 1, 2$ are summarized in Fig. 3. The master equation for the probabilities P_n, $n = 0, 1, 2$, reads

$$\dot{P}_0 = -2\Gamma_R P_0 + \Gamma_I P_1 + \Gamma_A P_2, \quad (8)$$

$$\dot{P}_1 = -(\Gamma_I + \Gamma_R) P_1 + 2\Gamma_R P_0 + 2\Gamma_I P_2, \quad (9)$$

$$\dot{P}_2 = -(\Gamma_A + 2\Gamma_I) P_2 + \Gamma_R P_1. \quad (10)$$

The factors 2 in this equation are due to the double spin degeneracy of the single quasiparticle state. In stationary state, the probabilities are given by

$$P_0 = \frac{\Gamma_A(\Gamma_I + \Gamma_R) + 2\Gamma_I^2}{\Gamma_I(\Gamma_A + 2\Gamma_I + 4\Gamma_R) + \Gamma_R(2\Gamma_R + 3\Gamma_A)}, \quad (11)$$

$$P_1 = \frac{2\Gamma_R(\Gamma_A + 2\Gamma_I)}{\Gamma_I(\Gamma_A + 2\Gamma_I + 4\Gamma_R) + \Gamma_R(2\Gamma_R + 3\Gamma_A)}, \quad (12)$$

$$P_2 = \frac{2\Gamma_R^2}{\Gamma_I(\Gamma_A + 2\Gamma_I + 4\Gamma_R) + \Gamma_R(2\Gamma_R + 3\Gamma_A)}. \quad (13)$$

In the absence of a refill rate, $\Gamma_R = 0$, $\Gamma_I \neq 0$, the Andreev bound state is always emptied by the ionization processes. Therefore the a.c. phase modulation can be used for "purification" of the localized quasiparticle states in nanodevices. We stress that the opposite situation, $\Gamma_I = 0$, $\Gamma_R \neq 0$, is not achievable since the phase modulation responsible for refill processes also produces ionization. In this case, the constant a.c. modulation will cause a random distribution of the population.

4. CHARGE IMBALANCE

An effect which, to the best of our knowledge, has been overlooked so far is that the evacuation of quasiparticles from Andreev bound state is asymmetric with respect to electron- and hole-like states and can thus create charge imbalance of the quasiparticles in the leads.

Namely, we find that the rate at which an outgoing electron-like quasiparticle is created is not equal to the one for outgoing hole-like quasiparticle in the same lead, $\Gamma_{\alpha e} \neq \Gamma_{\alpha h}$ for both refill and ionization processes, $\alpha = R, I$. In the limit of a short constriction where we can neglect the energy dependence of the transmission coefficients, there is a symmetry between the leads: $\Gamma_{\alpha e}$ ($\Gamma_{\alpha h}$) in one lead equals $\Gamma_{\alpha h}$ ($\Gamma_{\alpha e}$) in the opposite lead, so that the total number of quasiparticles emitted to each lead is the same on average.

As a consequence of the rate asymmetry, each quasiparticle excitation process is accompanied by an average charge transfer in the constriction,

$$q_\alpha(E) = \frac{\sqrt{E^2 - \Delta^2}}{E} \frac{\Gamma_{\alpha e} - \Gamma_{\alpha h}}{\Gamma_{\alpha e} + \Gamma_{\alpha h}}, \quad (14)$$

where the prefactor accounts for the energy-dependent quasiparticle charge at energy E.

Evaluating the rates, the concrete expressions are obtained as

$$q_I = -2\frac{\partial E_A}{\partial \phi}\sqrt{\frac{(\Omega + E_A)^2 - \Delta^2}{\Delta^2 - E_A^2}} \times \\ \times \frac{E_A\left(1 + \frac{E_A}{\Omega + E_A}\right)}{\Omega E_A + \Delta^2(1 + \cos\phi)}, \quad (15)$$

$$q_R = 2\frac{\partial E_A}{\partial \phi}\sqrt{\frac{(\Omega - E_A)^2 - \Delta^2}{\Delta^2 - E_A^2}} \times \\ \times \frac{E_A\left(1 - \frac{E_A}{\Omega - E_A}\right)}{\Omega E_A - \Delta^2(1 + \cos\phi)}. \quad (16)$$

In Fig. 4, q_I as a function of ϕ is plotted for several parameters. The plot for q_R would look similar. We see immediately that $q_\alpha(\phi) = -q_\alpha(-\phi)$, like the supercurrent. Indeed we see from the formulas that the charge transfer is proportional to the supercurrent carried by the Andreev bound state $\sim \partial_\phi E_A$. Inverting

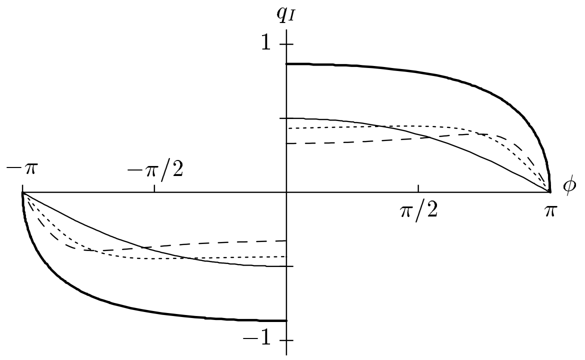

Fig. 4. The charge transfer q_I as a function of ϕ. The parameters are $T_0 = 0.5$ and $\Omega/\Delta = \{1/3, 1, 20\}$ (dashed, dotted, solid), and $T_0 = 1$, $\Omega/\Delta = 1$ (thick solid)

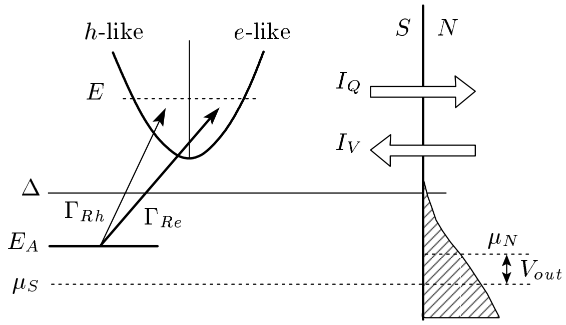

Fig. 5. Build-up of charge imbalance due to charge asymmetry of the quasiparticles emitted from the constriction. The charge imbalance is measured with a N–S tunnel junction voltage probe attached to the lead, V_{out} being the output signal

the stationary phase bias therefore inverts the charge transfer.

Contrary to the supercurrent, the charge transfer exhibits a discontinuity at $\phi = 0$. The explanation of this rather counter-intuitive feature is that the wave function of the Andreev bound state is not a continuous function of ϕ at $\phi = 0$ since the state merges with the delocalized spectrum at this point. The charge transfers q_α are 2π-periodic and have a node at $\phi = \pi$, where thus the charge asymmetry vanishes.

The maximum charge transfer for a given ϕ is reached in the limit of a fully transparent constriction, see also the thick solid curve in Fig. 4. In this particular limit, the a.c. drive actually produces only a quasiparticle of one kind, namely, e-like (h-like) for $0 < \phi < \pi$ ($-\pi < \phi < 0$). In the opposite tunneling limit, $T_0 \ll 1$, the charge transfer vanishes as $q_\alpha \sim \sqrt{T_0}$. Likewise the charge transfer vanishes close to the threshold driving frequency, where $\delta\Omega = \Omega - \Delta + E_A \ll \Delta$ for $\alpha = I$ ($\delta\Omega = \Omega - \Delta - E_A \ll \Delta$ for $\alpha = R$), obeying the power law $q_\alpha \sim \sqrt{\delta\Omega/\Delta}$ in this limit. Far away from the threshold, $\Omega \gg \Delta$, the charge transfer saturates at $q_{I,R} \to \mp 2\partial_\phi E_A/\sqrt{\Delta^2 - E_A^2}$. For large driving frequencies, the maximal polarization value is $|q_\alpha|_{max} = \sqrt{T_0}$. Considering the thin curves in Fig. 4, where the driving frequency is varied, we observe that for large frequencies, $\Omega \gg \Delta$, the maximal value is at small stationary phase bias. For lower frequencies, $\Omega \le \Delta$, the polarization on the other hand may increase for $\pi > \phi > \pi/2$, where the bound state is deeper in the gap, before it drops to zero at $\phi = \pi$.

Under conditions of constant irradiation, the net charge transfer per unit time is computed from the master equation and reads

$$\dot{q} = q_I \Gamma_I (P_1 + 2P_2) + q_R \Gamma_R (2P_0 + P_1). \tag{17}$$

We see that the refill process is crucial for the net effect: otherwise the Andreev bound state will always be empty.

In principle, the charge transfer gives rise to an additional dissipative current $I_d = e\dot{q} \approx e\Gamma$ that is seen on the background of a generally much bigger supercurrent, $I_s \approx e\Delta$. Although it is possible to observe such a current, in the following we will concentrate on a more interesting manifestation of the effect.

If the thermalization of the quasiparticle distribution in the leads near the constriction is not immediate, the charge asymmetry gives rise to a build up of a net quasiparticle charge density ρ, also known as charge imbalance. The imbalance may be measured by a voltage probe connected to a lead at some distance from the constriction.

The idea of the measurement as introduced in Ref. [24] is depicted in Fig. 5. An unequal population of e-like and h-like quasiparticles gives rise to a current I_Q at the N–S tunnel junction that is proportional to the charge imbalance ρ near the junction. Applying a voltage $eV_{out} = \mu_N - \mu_S$ between the normal metal and superconducting contacts produces a counter-current I_V. The voltage V_{out} at which the net tunnel current in the probe vanishes, $I_Q + I_V = 0$, is a signal of the charge imbalance.

In the case of low temperatures, $T \ll \Delta$, this measurement is extremely sensitive. This is because I_V is formed by the normal-metal excitations with energies $> \Delta$. At low temperatures the number of these excitations is exponentially small and therefore a large V_{out} is required to compensate I_Q. In the linear regime, the signal voltage reads

$$eV_{out} = T\frac{\rho}{c_0}, \tag{18}$$

$c_0 = \nu_0\sqrt{2\pi T\Delta}\exp(\Delta/T)$ being the (exponentially small) equilibrium quasiparticle density. Owing to this, even at moderately low $T = 0.05\Delta$ in aluminium, a charge imbalance of 0.001 elementary charges per cubic micrometer produces already a signal $\approx 0.1T/e$. The above relation is valid if $eV_{out} \ll T$, at larger imbalances the signal saturates at $T\ln(\rho/c_0)$.

To estimate ρ, we note that potential scattering does not lead to the relaxation of charge imbalance. This relaxation should involve inelastic processes and/or scattering on magnetic impurities. The charge imbalance lifetime τ_Q is therefore long and quasiparticles diffuse far away from the constriction. The net charge transfer $\dot{q}$, quasiparticle diffusion and relaxation are combined into a diffusion-relaxation equation for the charge-imbalance density $\rho(\mathbf{r})$,

$$\dot{\rho} - D\nabla^2\rho + \rho/\tau_Q = \dot{q}\delta(\mathbf{r}). \tag{19}$$

The charge imbalance is thus spread over the length scale $L_Q \approx \sqrt{\tau_Q D}$. We assume the N–S voltage probe to be placed within this scale. The created quasiparticles are spread over $\mathcal{V}$, the volume of the lead at the scale L_Q,

$$\rho \approx \dot{q}\frac{\tau_Q}{\mathcal{V}} \approx \dot{q}\frac{L_Q^2}{D\mathcal{V}}. \tag{20}$$

Let us note that the normal-state resistance of the piece at the scale L_Q can be estimated as $R_Q^{-1} = e^2\nu_0 D\mathcal{V}/L_Q^2$. This allows to represent the estimation in a compact form, independent on peculiarities of the geometry and disorder in the leads. Namely,

$$\rho \approx (R_Q G_Q)\nu_0\dot{q}. \tag{21}$$

Combining estimations for V_{out} and ρ, and estimating $\dot{q} \approx \Gamma$, we find

$$eV_{out} \approx (R_Q G_Q)\frac{\nu_0}{c_0}\Gamma \approx \approx \Gamma(R_Q G_Q)\sqrt{\frac{T}{\Delta}}\exp\left[\Delta/T\right]. \tag{22}$$

To get a rough estimate of achievable values, we take $R_Q \approx 1$ Ohm, $\Gamma \approx 10^{-3}\Delta \approx 1$ μeV, $T \approx 0.05\Delta$. Without the exponential factor, the value of V_{out} would be in the nano-volt range. However, the exponential factor yields nine orders of magnitude. Since such an estimation greatly exceeds T, the signal voltage in this case is already saturated at the value $\approx T \approx 10$ μV and is easy to measure.

5. CONCLUSIONS

We have investigated the processes of quasiparticle emission in a superconducting constriction subject to an a.c. phase modulation. We derived the rates involving the dynamics of the Andreev bound state occupation, and based on this, we proposed efficient schemes to control the occupation. In addition, we found an asymmetry of the rates of electron- and hole-like quasiparticle emission. We demonstrated that this asymmetry may lead to a measurable charge imbalance of the quasiparticles accumulated near the constriction.

The experiments can be performed on the same setup as in [20, 21] where high-frequency irradiation can be applied to the constriction and the Andreev state populations can be detected by measuring the supercurrent. The setup can be easily modified to measure the charge imbalance effect predicted. In this case, the additional normal-metal electrodes should be brought in contact with the superconducting leads at sub-micron distance from the constriction. We look forward to experimental confirmation of our findings.

Our results can be generalized to multi-channel superconducting constriction that can be fabricated much easier than the break junctions. Such a generalization is especially straightforward in case of a short junction, that is, shorter than the superconducting correlation length. In this case, the junction can be regarded as a collection of independent transport channels, and all the quantities discussed are thus contributed by each channel. This is the subject of our ongoing research.

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