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ROTATING MOVING D-BRANES WITH BACKGROUND FIELDS IN SUPERSTRING THEORY

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Using the boundary state formalism, we study rotating and moving Dp-branes in the presence of the Kalb–Ramond field, $U(1)$ gauge potential and tachyon background fields. The rotation and motion are in the brane volumes. The interaction amplitude of two Dp-branes is studied, and especially the contribution of the superstring massless modes is segregated. Because of the tachyon fields, rotations and velocities of the branes, the behavior of the interaction amplitude reveals obvious differences from what is conventional.

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1. INTRODUCTION

D-branes, as essential ingredients of superstring theory [1], have important applications in different aspects of theoretical physics. These objects are classical solutions of the low-energy string effective action and hence can be described in terms of closed strings. Besides, D-branes with nonzero background internal fields have shown several interesting properties [2–7]. For example, these fields affect the emitted closed strings of the branes and therefore modify the brane interactions.

On the other hand, we have the boundary state formalism for describing the D-branes [2, 8–14], which is a useful tool in many complicated situations. This is because the boundary state encodes all relevant properties of the D-branes, and that is why the formalism has been widely used recently in studying properties of D-branes in string theory. A boundary state can describe creation of a closed string from the vacuum, or equivalently it can be interpreted as a source for a closed string, emitted by a D-brane. Among achievements in this formalism is its extension to superstring theory and the analysis of the contribution of conformal and super-conformal ghosts. The overlap of two boundary states corresponding to two D-branes, via a closed-string propagator, gives the amplitude of interaction of the branes. So far, this method has been suitably applied to various configurations in the presence of different background fields, including stationary branes, moving branes with constant velocities, angled branes [15–19], various configurations in a compact spacetime [15], in the presence of the tachyon field [19, 20], a bound state of two D-branes [13], and so on.

Previously, we studied a general configuration of rotating and moving Dp-branes in bosonic string theory in the presence of the following background fields: the Kalb–Ramond field, $U(1)$ gauge potentials that live in the D-branes worldvolumes, and tachyon fields [20]. In this paper, the same setup is considered in superstring theory. The novelty of the results is considerable. Our procedure is as follows. For this setup, we obtain the boundary state associated with the brane and then compute the interaction between two such Dp-branes as a closed superstring tree-level diagram in the covariant formalism. The generality of the setup strongly recasts the feature of boundary states and interaction of the branes. We observe that the interaction amplitude and its long-range part, which occurs between distant branes, exhibit some appealing behaviors.

We note that we consider rotation of each brane in its volume and its motion along the brane directions. Due to the various fields inside the brane, there are preferred directions, which indicates the breaking of the Lorentz symmetry, and hence such rotation and motion are meaningful.

This paper is organized as follows. In Sec. 2, the boundary state of a closed superstring, correspond-

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ing to a rotating moving Dp-brane with various background fields is constructed. In Sec. 3, interaction of two Dp-branes in the NS–NS and R–R sectors of the superstring is calculated. In Sec. 4, the long-range force of the interaction is extracted. Section 5 is devoted to the conclusions.

2. BOUNDARY STATE ASSOCIATED WITH A ROTATING MOVING D-BRANE WITH BACKGROUND FIELDS

We use the following sigma-model action for a closed string to describe a rotating and moving Dp-brane in the presence of the Kalb–Ramond, photonic, and tachyonic fields:

$$S = -\frac{1}{4\pi\alpha'}\int_{\Sigma} d^2\sigma(\sqrt{-g}g^{ab}G_{\mu\nu}\partial_a X^\mu \partial_b X^\nu + \\ + \varepsilon^{ab}B_{\mu\nu}\partial_a X^\mu \partial_b X^\nu) + \\ + \frac{1}{2\pi\alpha'}\int_{\partial\Sigma} d\sigma(A_\alpha \partial_\sigma X^\alpha + \omega_{\alpha\beta}J_\tau^{\alpha\beta} + T(X^\alpha)), \quad (1)$$

where Σ is the worldsheet of a closed string emitted (absorbed) by the brane, and $\partial\Sigma$ is the boundary of the worldsheet. Besides, "α" and "β" are indices along the brane worldvolume and "i" is used for the directions perpendicular to it. In addition, the background fields $G_{\mu\nu}$, $B_{\mu\nu}$, A_α, T and the antisymmetric variables $\omega_{\alpha\beta}$ and $J_\tau^{\alpha\beta}$ are respectively the spacetime metric, a Kalb–Ramond (antisymmetric tensor) field, a gauge field, a tachyon field, the angular velocity, and the angular momentum density of the brane. We consider the following forms for these variables:

$$\begin{aligned} G_{\mu\nu} &= \eta_{\mu\nu} = \mathrm{diag}(-1,1,\ldots,1), \\ B_{\mu\nu} &= \mathrm{const}, \quad B_{\alpha i} = 0, \\ A_\alpha &= -\frac{1}{2}F_{\alpha\beta}X^\beta, \quad F_{\alpha\beta} = \mathrm{const}, \\ T(X) &= \frac{1}{2}U_{\alpha\beta}X^\alpha X^\beta, \quad U_{\alpha\beta} = U_{\beta\alpha} = \mathrm{const}, \\ U_{\alpha i} &= U_{ij} = 0, \\ \omega_{\alpha\beta}J_\tau^{\alpha\beta} &= 2\omega_{\alpha\beta}X^\alpha \partial_\tau X^\beta. \end{aligned} \quad (2)$$

The last equation indicates the rotation and motion of the brane. The components $\{\omega_{0\bar\alpha}|\bar\alpha = 1,2,\ldots,p\}$ denote the velocity of the brane, while the elements $\{\omega_{\bar\alpha\bar\beta}|\bar\alpha,\bar\beta = 1,2,\ldots,p\}$ represent its rotation.

We note that in the presence of a Dp-brane, the 10-dimensional $U(1)$ gauge field A_μ is decomposed into a longitudinal $U(1)$ gauge field A_α, which lives in the worldvolume of the Dp-brane, and a transverse part A_i associated with the $9-p$ scalar fields, from the worldvolume point of view. These scalars represent coordinates of the brane. We keep them fixed, that is, assume that the branes do not execute transverse motion. For the gauge field A_α, we have chosen the special gauge in the third equation in (2).

In the literature, the tachyon field is usually nonzero just in one dimension and its effects are studied on a space-filling brane, while we here consider a Dp-brane with an arbitrary value of p. Besides, the square form of the tachyon profile is used to produce a Gaussian integral. Hence, the tachyon field has components along all directions of the brane worldvolume. The gauge and tachyon fields are in the spectrum of open strings attached to the Dp-brane.

In fact, in the presence of an antisymmetric field and a local gauge field, there are preferred alignments in the brane, and hence the rotation and motion of the brane in its volume is sensible.

2.1. Bosonic part of the boundary state

In the closed-string operator formalism, the D-branes of the type-IIA and type-IIB theories can be described by boundary states. These are closed-string states that insert a boundary on the closed-string worldsheet and enforce appropriate boundary conditions on it. We now extract the corresponding boundary state in our setup. Requiring the vanishing of the variation of the action with respect to the closed string coordinates $X^\mu(\sigma,\tau)$, we obtain the boundary-state equations

$$\begin{aligned} &[(\eta_{\alpha\beta} + 4\omega_{\alpha\beta})\partial_\tau X^\beta + \mathcal{F}_{\alpha\beta}\partial_\sigma X^\beta + \\ &\quad + U_{\alpha\beta}X^\beta]_{\tau=0}|B_{bos}\rangle = 0, \\ &(\delta X^i)_{\tau=0}|B_{bos}\rangle = 0, \end{aligned} \quad (3)$$

where $\mathcal{F}_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha - B_{\alpha\beta}$ is the total field strength.

It is worthwhile to show that the Lorentz symmetry is broken along the worldvolume of the brane. Equations (3) leads to

$$J_{bos}^{\alpha\beta}|B_{bos}\rangle = \\ = \int_0^\pi d\sigma\Big[(\mathbf{A}^{-1}\mathcal{F})^\alpha{}_\gamma X^\beta \partial_\sigma X^\gamma - (\mathbf{A}^{-1}\mathcal{F})^\beta{}_\gamma X^\alpha \partial_\sigma X^\gamma + \\ + (\mathbf{A}^{-1}U)^\alpha{}_\gamma X^\beta X^\gamma - (\mathbf{A}^{-1}U)^\beta{}_\gamma X^\alpha X^\gamma\Big]|B_{bos}\rangle, \quad (4)$$

where $\mathbf{A}_{\alpha\beta} = \eta_{\alpha\beta} + 4\omega_{\alpha\beta}$. We observe that for restoring the Lorentz invariance, all elements of the tachyon matrix $U_{\alpha\beta}$ and the total field strength $\mathcal{F}_{\alpha\beta}$ must vanish. We demonstrated this for the bosonic part of the boundary state. This procedure can also be applied to the total boundary state, which includes the bosonic and fermionic parts, to prove the breakdown of the Lorentz invariance along the worldvolume of the brane.

Introducing the closed-string mode expansion

$$X^\mu(\sigma,\tau) = x^\mu + 2\alpha' p^\mu \tau + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{1}{n} \left(\alpha_n^\mu e^{-2in(\tau-\sigma)} + \tilde{\alpha}_n^\mu e^{-2in(\tau+\sigma)} \right)$$

in Eq. (3) gives

$$\begin{aligned} &\left[\left(\eta_{\alpha\beta} + 4\omega_{\alpha\beta} - \mathcal{F}_{\alpha\beta} + \frac{i}{2m} U_{\alpha\beta} \right) \alpha_m^\beta + \left(\eta_{\alpha\beta} + 4\omega_{\alpha\beta} + \mathcal{F}_{\alpha\beta} - \frac{i}{2m} U_{\alpha\beta} \right) \widetilde{\alpha}_{-m}^\beta \right] \times \\ &\times |B_{bos}\rangle^{(osc)} = 0, \\ &\left[2\alpha'(\eta_{\alpha\beta} + 4\omega_{\alpha\beta}) p^\beta + U_{\alpha\beta} x^\beta \right] |B_{bos}\rangle^{(0)} = 0, \\ &(\alpha_m^i - \widetilde{\alpha}_{-m}^i)|B_{bos}\rangle^{(osc)} = 0, \\ &(x^i - y^i)|B_{bos}\rangle^{(0)} = 0, \end{aligned} \tag{5}$$

where the set $\{y^i | i = p{+}1, \ldots, 9\}$ indicates the position of the brane. Besides, for the boundary state

$$|B_{bos}\rangle = |B_{bos}\rangle^{(0)} \otimes |B_{bos}\rangle^{(osc)},$$

the components $|B_{bos}\rangle^{(0)}$ and $|B_{bos}\rangle^{(osc)}$ respectively represent boundary states for the zero modes and oscillating modes.

The solution of the oscillating part, which can be found by the coherent state method, is given by

$$|B_{bos}\rangle^{(osc)} = \prod_{n=1}^{\infty} [\det Q_{(n)}]^{-1} \times \exp\left[-\sum_{m=1}^{\infty} \frac{1}{m} \alpha_{-m}^\mu S_{(m)\mu\nu} \widetilde{\alpha}_{-m}^\nu \right] |0\rangle_\alpha \otimes |0\rangle_{\widetilde{\alpha}}, \tag{6}$$

where the matrices are defined as

$$\begin{aligned} &Q_{(m)\alpha\beta} = \eta_{\alpha\beta} + 4\omega_{\alpha\beta} - \mathcal{F}_{\alpha\beta} + \frac{i}{2m} U_{\alpha\beta}, \\ &S_{(m)\mu\nu} = \left(\frac{1}{2} \left[\Delta_{(m)} + \left(\Delta_{(-m)}^T \right)^{-1} \right]_{\alpha\beta}, -\delta_{ij} \right), \\ &\Delta_{(m)\alpha\beta} = (Q_{(m)}^{-1} N_{(m)})_{\alpha\beta}, \\ &N_{(m)\alpha\beta} = \eta_{\alpha\beta} + 4\omega_{\alpha\beta} + \mathcal{F}_{\alpha\beta} - \frac{i}{2m} U_{\alpha\beta}. \end{aligned} \tag{7}$$

Because the mode-dependent matrix $\Delta_{(m)}$ is not orthogonal in general, the matrix $\left(\Delta_{(-m)}^T\right)^{-1}$ also appears in the definition of $S_{(m)\mu\nu}$. In Eq. (6), the normalization factor $\prod_{n=1}^{\infty} [\det Q_{(n)}]^{-1}$ can be deduced from the disk partition function.

The boundary state for the zero modes is given by

$$\begin{aligned} |B_{bos}\rangle^{(0)} = \int_{-\infty}^{\infty} \exp\Bigg\{ i\alpha' \Bigg[\sum_{\alpha=0}^{p} \left(U^{-1}\mathbf{A} \right)_{\alpha\alpha} (p^\alpha)^2 + \sum_{\alpha,\beta=0,\alpha\neq\beta}^{p} \left(U^{-1}\mathbf{A} + \mathbf{A}^T U^{-1} \right)_{\alpha\beta} p^\alpha p^\beta \Bigg] \Bigg\} \times \\ \times \left(\prod_\alpha |p^\alpha\rangle dp^\alpha \right) \otimes \prod_i \delta(x^i - y^i) |p^i = 0\rangle. \end{aligned} \tag{8}$$

The integration over the momenta indicates that the effects of all values of the momentum components have been taken into account. As we see, unlike the oscillating part, the total field strength does not enter Eq. (8).

We note that for calculating the interaction amplitude, the contribution of the conformal ghosts b, c, $\tilde{b}$ and $\tilde{c}$ in the bosonic boundary state must also be taken into account.

2.2. Fermionic part of the boundary state

The supersymmetric version of action (1) is invariant under the global worldsheet supersymmetry

$$\delta X^\mu = \bar{\epsilon}\psi^\mu,$$

$$\delta\psi^\mu = -i\rho^a \partial_a X^\mu \epsilon, \quad a \in \{\tau, \sigma\},$$

where ϵ is an infinitesimal constant anticommuting spinor. Because we need the explicit forms of the components of the worldsheet fermions

$$\psi^\mu = \begin{pmatrix} \psi_-^\mu \\ \psi_+^\mu \end{pmatrix},$$

we write them here:

$$\begin{aligned} \psi_-^\mu &= \sum_{n\in\mathbf{Z}} d_n^\mu e^{-2in(\tau-\sigma)}, \quad (\mathrm{R}), \\ \psi_-^\mu &= \sum_{r\in\mathbf{Z}+1/2} b_r^\mu e^{-2ir(\tau-\sigma)}, \quad (\mathrm{NS}), \\ \psi_+^\mu &= \sum_{n\in\mathbf{Z}} \tilde{d}_n^\mu e^{-2in(\tau+\sigma)}, \quad (\mathrm{R}), \\ \psi_+^\mu &= \sum_{r\in\mathbf{Z}+1/2} \tilde{b}_r^\mu e^{-2ir(\tau+\sigma)}, \quad (\mathrm{NS}), \end{aligned}$$

where the integer and half-integer modes respectively represent the Ramond (R) and Neveu–Schwarz (NS) solutions.

Now we can apply supersymmetry transformations to bosonic boundary-state equations (3) and transform them into their fermionic partners. We can use the replacements

$$\partial_+ X^\mu(\sigma,\tau) \to -i\eta\psi_+^\mu(\sigma,\tau), \quad \partial_- X^\mu(\sigma,\tau) \to \psi_-^\mu(\sigma,\tau), \tag{9}$$

where $\eta = \pm 1$ has been introduced for the GSO (Gliozzi–Scherk–Olive) projection of the boundary-state. As it was seen in the bosonic boundary-state equations, due to the presence of the tachyon field, a replacement for X^μ in terms of the fermionic components is also needed. To obtain that, by using the replacements (9) and $\partial_\pm = (\partial_\tau \pm \partial_\sigma)/2$ and integration, we receive

$$X^\mu(\sigma,\tau) \to \to \sum_k \frac{1}{2k}\left(i\psi_k^\mu \; e^{-2ik(\tau-\sigma)} + \eta\widetilde{\psi}_k^\mu \, e^{-2ik(\tau+\sigma)}\right). \tag{10}$$

Now, by introducing replacements (9) and (10) in Eqs. (3), for the closed string boundary at $\tau = 0$, we obtain

$$\begin{aligned} &\left[\left(\eta_{\alpha\beta} + 4\omega_{\alpha\beta} - \mathcal{F}_{\alpha\beta} + \frac{i}{2k}U_{\alpha\beta}\right)\psi_k^\beta - \right.\\ &\left. - i\eta\left(\eta_{\alpha\beta} + 4\omega_{\alpha\beta} + \mathcal{F}_{\alpha\beta} - \frac{i}{2k}U_{\alpha\beta}\right)\widetilde{\psi}_{-k}^\beta\right] \times \\ &\qquad \times |B_{ferm}^{(osc)}, \eta\rangle = 0, \\ &(\psi_k^i + i\eta\widetilde{\psi}_{-k}^i)|B_{ferm}^{(osc)}, \eta\rangle = 0, \end{aligned} \tag{11}$$

for the oscillating parts of the R–R and NS–NS sectors, and

$$\begin{aligned} &[(\eta_{\alpha\beta} + 4\omega_{\alpha\beta} - \mathcal{F}_{\alpha\beta})\psi_0^\beta - \\ &- i\eta(\eta_{\alpha\beta} + 4\omega_{\alpha\beta} + \mathcal{F}_{\alpha\beta})\widetilde{\psi}_0^\beta]|B,\eta\rangle_R^{(0)} = 0, \\ &(\psi_0^i + i\eta\widetilde{\psi}_0^i)|B,\eta\rangle_R^{(0)} = 0, \end{aligned} \tag{12}$$

for the zero-mode part of the R–R sector. As we see, the tachyon is omitted from the zero-mode boundary state in this sector. The importance of this part is to be revealed in the R–R sector of the boundary state. Equations (11) and (12) can be rewritten as

$$(\psi_k^\mu - i\eta\, S_{(k)}^\mu{}_\nu\, \widetilde{\psi}_{-k}^\nu)|B_{ferm}^{(osc)}, \eta\rangle = 0 \tag{13}$$

for the oscillating parts of both sectors, and

$$(d_0^\mu - i\eta\, \bar{S}^\mu{}_\nu\, \widetilde{d}_0^\nu)|B,\eta\rangle_R^{(0)} = 0 \tag{14}$$

for the zero-mode part of the R–R sector. The matrix $\bar{S}^\mu{}_\nu$ is defined as

$$\begin{aligned} &\bar{S}_{\mu\nu} = (\bar{\Delta}_{\alpha\beta}\;,\; -\delta_{ij}), \\ &\bar{\Delta}_{\alpha\beta} = (\bar{Q}^{-1}\bar{N})_{\alpha\beta}, \\ &\bar{Q}_{\alpha\beta} = \eta_{\alpha\beta} + 4\omega_{\alpha\beta} - \mathcal{F}_{\alpha\beta}, \\ &\bar{N}_{\alpha\beta} = \eta_{\alpha\beta} + 4\omega_{\alpha\beta} + \mathcal{F}_{\alpha\beta}. \end{aligned} \tag{15}$$

We note that in the fermionic parts, we should also consider the boundary states associated with the superconformal ghosts, which are needed in calculating the interaction amplitude.

2.2.1. The Neveu–Schwarz sector

Similarly to the bosonic case, with the help of the coherent state method, the oscillating part of the fermionic boundary state including both sectors can be calculated. Thus, Eq. (13) implies that the NS–NS-sector boundary state has the form

$$|B_{ferm},\eta\rangle_{NS} = \prod_{r=1/2}^{\infty}[\det Q_{(r)}] \times \times \exp\left[i\eta\sum_{r=1/2}^{\infty}(b_{-r}^\mu S_{(r)\mu\nu}\widetilde{b}_{-r}^\nu)\right]|0\rangle_{NS}. \tag{16}$$

When the path integral is computed, the determinant is reversed compared to the bosonic case in Eq. (6). This is due to the Grassmannian property of the fermionic variables [2].

2.2.2. The Ramond–Ramond sector

Solving Eqs. (13) and (14) in the R–R sector yields the boundary state

$$|B_{ferm},\eta\rangle_R = \prod_{n=1}^{\infty}[\det Q_{(n)}] \times \times \exp\left[i\eta\sum_{m=1}^{\infty}(d_{-m}^\mu S_{(m)\mu\nu}\widetilde{d}_{-m}^\nu)\right]|B,\eta\rangle_R^{(0)}. \tag{17}$$

The explicit form of the zero-mode state in both type-IIA and type-IIB theories is

$$|B,\eta\rangle_R^{(0)} = = \left[C\Gamma^0\Gamma^1\ldots\Gamma^p\left(\frac{1+i\eta\Gamma_{11}}{1+i\eta}\right)\Omega\right]_{AB}|A\rangle\otimes|\widetilde{B}\rangle, \tag{18}$$

where A and B are the 32-dimensional indices for spinors and Γ-matrices in the 10-dimensional space-time, $|A\rangle\otimes|\widetilde{B}\rangle$ is the vacuum of the zero modes d_0^μ and $\widetilde{d}_0^\mu$, C is the charge-conjugation matrix, and

$$\Omega = *\exp\left(\frac{1}{2}\Phi_{\alpha\beta}\Gamma^\alpha\Gamma^\beta\right)*,$$
$$\Phi_{\alpha\beta} = \left((\bar{\Delta}-1)(\bar{\Delta}+1)^{-1}\right)_{\alpha\beta}. \qquad (19)$$

The notation "$*\ldots*$" implies that one should expand the exponential and then antisymmetrize the indices of the Γ-matrices. Therefore, because all terms in the expansion with repeated Lorentz indices are dropped, there are a finite number of terms for each value of p. As an example, for the D3-brane, the matrix Ω takes the form

$$\Omega = 1 + \frac{1}{2}\sum_{\alpha,\beta=0}^{3}\Phi_{\alpha\beta}\Gamma^\alpha\Gamma^\beta + (\Phi_{01}\Phi_{23} - \Phi_{02}\Phi_{13} + \Phi_{03}\Phi_{12})\Gamma^0\Gamma^1\Gamma^2\Gamma^3.$$

In fact, this convention implies that the matrix $\bar{\Delta}$ should be orthogonal, which gives the restriction that the matrices ω and $\mathcal{F}$ should anticommute with each other. For the D1-brane, there is an electric field along the brane. Hence, according to this restriction, the only element of the matrix ω, i.e., the speed of the brane along itself, vanishes. This is an expected result, because of the direction of the electric field, motion of the D-string along itself is not sensible. Other branes can be involved in both rotation and motion.

3. INTERACTION OF THE BRANES

Unbroken supersymmetry ensures that the Casimir energy of open superstrings is zero. Therefore, D-branes in supersymmetric configurations exert no net force on each other. A rotating/moving brane can generically break all the supersymmetries and leads to orientation/velocity-dependent forces.

In this section, we calculate the interaction of two rotating and moving parallel Dp-branes, equipped with background fields, via a closed-string exchange. For both the NS–NS and R–R sectors, the complete boundary state can be written as the product

$$|B,\eta\rangle_{NS,R} = \frac{T_p}{2}|B_{bos}\rangle \otimes |B_{gh}\rangle \otimes |B_{ferm},\eta\rangle_{NS,R} \otimes |B_{sgh},\eta\rangle_{NS,R},$$

where the overall normalization factor T_p is the Dp-brane tension. We note that the ghost and superghost boundary states are not affected by the rotation, motion, and background fields. The explicit expressions for $|B_{gh}\rangle$ and $|B_{sgh}\rangle_{NS,R}$ can be found in the literature, and hence we do not write them here.

For eliminating unwanted states, e.g., the closed-string tachyon, and at the same time making the number of spacetime bosonic and fermionic physical excitations equal at each mass level, as is needed for supersymmetry, one should use the GSO projection. Therefore, the total boundary states that are used in calculating the interaction acquire the forms

$$|B\rangle_{NS} = \frac{1}{2}\left(|B,+\rangle_{NS} - |B,-\rangle_{NS}\right),$$
$$|B\rangle_{R} = \frac{1}{2}\left(|B,+\rangle_{R} + |B,-\rangle_{R}\right). \qquad (20)$$

The interaction amplitude of two D-branes can be obtained by either the open-string one-loop or the closed-string tree-level diagram. The former is a quantum process, while the latter is a classical process. In the closed-string picture, the interaction between two D-branes is viewed as an exchange of a closed string between two boundary states, geometrically describing a cylinder. From this standpoint, the interaction is computed with a tree-level diagram. In this process, a closed string is created by one D-brane; it propagates in the transverse space between the two D-branes, and then the other D-brane absorbs it. Therefore, the interaction amplitude between two D-branes in each sector is given by the overlap of the boundary states

$$\mathcal{A}_{NS-NS,R-R} = {}_{NS,R}\langle B_1|D|B_2\rangle_{NS,R},$$

where D is the closed-string propagator. In other words, we have

$$\mathcal{A}_{NS-NS,R-R} = 2\alpha'\int_0^\infty dt\ {}_{NS,R}\langle B_1| \times \exp(-tH_{NS,R})|B_2\rangle_{NS,R}.$$

The total closed superstring Hamiltonian $H_{NS,R}$ is the sum of the Hamiltonians of worldsheet bosons, fermions, conformal ghosts, and super-conformal ghosts in each sector. The complete interaction amplitude is given by the combination

$$\mathcal{A}_{total} = \mathcal{A}_{NS-NS} + \mathcal{A}_{R-R}.$$

According to this formula, boundary states are convenient tools for summing over all forces between two D-branes, which are mediated by the NS–NS and R–R states of a closed superstring.

3.1. The NS–NS sector interaction

For maintaining the generality, we consider the d-dimensional spacetime instead of $d = 10$. Using the

GSO-projected boundary states (20), we obtain the interaction amplitude between two parallel Dp-branes in the NS–NS sector as

$$\mathcal{A}_{NS-NS} = \frac{T_p^2 V_{p+1}\alpha'}{8(2\pi)^{d-p-1}} \times \prod_{m=1}^{\infty} \frac{\det[Q^\dagger_{(m-1/2)1} Q_{(m-1/2)2}]}{\det\left[Q^\dagger_{(m)1} Q_{(m)2}\right]} \times \int_0^\infty dt \left\{ \frac{1}{\sqrt{\det(R_1^\dagger R_2)}} \left(\sqrt{\frac{\pi}{\alpha' t}}\right)^{d-p-1} \times \exp\left(-\frac{1}{4\alpha' t}\sum_i \left(y_2^i - y_1^i\right)^2\right) \times \frac{1}{q}\left(\prod_{n=1}^{\infty}\left[\left(\frac{1-q^{2n}}{1+q^{2n-1}}\right)^{3+p-d} \times \frac{\det(\mathbf{1} + H^\dagger_{(n)1} H_{(n)2}\, q^{2n-1})}{\det(\mathbf{1} - H^\dagger_{(n)1} H_{(n)2}\, q^{2n})}\right] - \prod_{n=1}^{\infty}\left[\left(\frac{1-q^{2n}}{1-q^{2n-1}}\right)^{3+p-d} \times \frac{\det(\mathbf{1} - H^\dagger_{(n)1} H_{(n)2}\, q^{2n-1})}{\det(\mathbf{1} - H^\dagger_{(n)1} H_{(n)2}\, q^{2n})}\right]\right)\right\}, \quad (21)$$

where the indices "1" and "2" refer to the first brane or $|B_1\rangle$ and the second brane or $|B_2\rangle$, V_{p+1} is the common worldvolume of the two Dp-branes,

$$q = e^{-2t}, \quad H_{(n)a} = \frac{\Delta_{(n)a} + [\Delta^{-1}_{(-n)a}]^T}{2}$$

with $a = 1, 2$, and the symmetric matrices R_1 and R_2 contain nonzero elements only along the worldvolumes of the branes:

$$\begin{gathered}(R_a)_{\alpha\beta} = 2\alpha'(-i\mathcal{M}_a - iU_a^{-1}\mathbf{A}_a - - i\mathbf{A}_a^T U_a^{-1} + t\mathbf{1})_{\alpha\beta}, \quad a = 1, 2, \\ \mathcal{M}_a = \begin{pmatrix} \left(U_a^{-1}\mathbf{A}_a\right)_{00} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \left(U_a^{-1}\mathbf{A}_a\right)_{pp} \end{pmatrix}, \\ (\mathbf{A}_a)_{\alpha\beta} = \eta_{\alpha\beta} + 4(\omega_a)_{\alpha\beta}.\end{gathered} \quad (22)$$

In addition, we used the relations

$$\langle p^\alpha | p^\beta\rangle = 2\pi\delta(p^\alpha - p^\beta), \quad (2\pi)^{p+1}\delta^{(p+1)}(0) = V_{p+1}.$$

In this amplitude, the exponential is a damping factor with respect to the distance of the branes. In the last two products, the determinant in the denominators reflects the portion of the boson oscillators along the brane worldvolumes, and the determinants in the numerators are due to fermion oscillators, also along the branes worldvolumes. The other factors in the products are contributions of the boson and fermion oscillators perpendicular to the brane worldvolume, and also of the conformal and superconformal ghosts. Explicitly, the power $3 + p - d = 2 - (d - p - 1)$ is decomposed as follows: 2 in the numerators for the ghosts, 2 in the denominators for the superghosts, $-(d - p - 1)$ in the numerators for transverse oscillators of the bosons, and $-(d - p - 1)$ in the denominators for transverse oscillators of the fermions. The remaining part of the integrand of the amplitude is the overlap of the boundary states of bosonic zero modes, i. e. Eq. (8). This part is completely determined by the internal tachyon fields, the motion and rotation of the branes.

Contributions of all closed superstring states in the NS–NS sector that the two branes can emit are gathered in amplitude (21). One part of the strength of the interaction is given by the constant overall factor of this amplitude, i. e., two first lines in Eq. (21), which possesses contributions from the field parameters, linear and angular velocities, and the brane tensions.

3.2. The R–R sector interaction

Applying the total GSO-projected boundary states (20), we obtain the following interaction amplitude in the R–R sector:

$$\mathcal{A}_{R-R} = \frac{T_p^2 V_{p+1}\alpha'}{8(2\pi)^{d-p-1}} \times \int_0^\infty dt \left\{ \left(\kappa \prod_{n=1}^{\infty}\left[\left(\frac{1-q^{2n}}{1+q^{2n}}\right)^{3+p-d} \times \frac{\det(\mathbf{1} + H^\dagger_{(n)1} H_{(n)2} q^{2n})}{\det(\mathbf{1} - H^\dagger_{(n)1} H_{(n)2} q^{2n})}\right] + \kappa'\right) \times \frac{1}{\sqrt{\det(R_1^\dagger R_2)}} \left(\sqrt{\frac{\pi}{\alpha' t}}\right)^{d-p-1} \times \exp\left(-\frac{1}{4\alpha' t}\sum_i \left(y_2^i - y_1^i\right)^2\right)\right\}, \quad (23)$$

where

$$\begin{gathered}\kappa \equiv \frac{1}{2}(-1)^{p+1}\,\mathrm{Tr}[\Omega_1\, C^{-1}\, \Omega_2^T\, C], \\ \kappa' \equiv i(-1)^p\,\mathrm{Tr}[\Omega_1\, C^{-1}\, \Omega_2^T\, C\, \Gamma_{11}].\end{gathered} \quad (24)$$

In these relations, the matrices $\Omega_{1,2}$ are defined by Eq. (19) via the matrices $\omega_{1,2}$ and $\mathcal{F}_{1,2}$ for the first and second branes. We see that in the R–R-sector boundary state, and hence in the corresponding amplitude, the normalizing determinant factors of bosons and fermions cancel each other.

We are now interested in the total amplitude, i. e., the combination of the amplitudes in the NS–NS and R–R sectors. In the total amplitude of the described system, attraction due to the exchange of NS–NS states of a closed string is not compensated by repulsion of R–R states. Thus, we can conclude that our setup does not satisfy the BPS (Bogomol'nyi–Prasad–Sommerfield) no-force condition. This is because this configuration of two D-branes does not preserve enough value of the spacetime supersymmetries of the type-IIA and type-IIB theories. In fact, in the absence of background fields, motions, and rotations, the total amplitude vanishes because this setup of the branes preserves half of the supersymmetry.

A special feature of non-BPS branes is the presence of a tachyon field in their worldvolumes. In fact, it is not evident how the spacetime supersymmetry is realized with the tachyons, and the existence of broken supersymmetry in the presence of tachyons has never been explicitly proved [21]. However, setting the branes in relative motion (or rotating them) breaks all the supersymmetries generically, and leads to velocity- or orientation-dependent forces [22].

We observe that in the amplitudes of both sectors, for a system of two D$(d-3)$-branes, the effect of ghosts (superghosts) eliminates the contribution of the transverse oscillators of the bosons (fermions).

3.3. An example

To clarify our system, we study a special case, i. e., parallel D2-branes. We consider the ath brane ($a = 1,2$) with the linear velocity $\{(v_{\bar\alpha})_a|\bar\alpha = 1,2\}$, the angular velocity $(\omega_{12})_a = \varpi_a$, and the fields $(F_{0\bar\alpha})_a = (E_{\bar\alpha})_a$, $(F_{12})_a = B_a$, and $(U_{\alpha\beta})_a$. Therefore, the interaction amplitude for the NS–NS sector is given by

$$\begin{aligned}
\mathcal{A}_{NS-NS} &= \frac{T_2^2 V_3 \alpha'}{8(2\pi)^{d-3}} \times \\
&\times \prod_{m=1}^{\infty} \frac{\det[Q^\dagger_{(m-1/2)1} Q_{(m-1/2)2}]}{\det\left[Q^\dagger_{(m)1} Q_{(m)2}\right]} \times \\
&\times \int_0^\infty dt \left\{ \frac{1}{\sqrt{\det(R_1^\dagger R_2)}} \left(\sqrt{\frac{\pi}{\alpha' t}}\right)^{d-3} \times \right.\\
&\times \exp\left(-\frac{1}{4\alpha' t} \sum_{i=3}^{d-1} \left(y_2^i - y_1^i\right)^2\right) \times \\
&\times \frac{1}{q} \left(\prod_{n=1}^{\infty} \left[\left(\frac{1-q^{2n}}{1+q^{2n+1}}\right)^{5-d} \times \right.\right.\\
&\times \left. \frac{\det(\mathbf{1} + H^\dagger_{(n)1} H_{(n)2} q^{2n-1})}{\det(\mathbf{1} - H^\dagger_{(n)1} H_{(n)2} q^{2n})} \right] - \\
&- \prod_{n=1}^{\infty} \left[\left(\frac{1-q^{2n}}{1-q^{2n-1}}\right)^{5-d} \times \right.\\
&\times \left.\left.\left. \frac{\det(\mathbf{1} - H^\dagger_{(n)1} H_{(n)2} q^{2n-1})}{\det(\mathbf{1} - H^\dagger_{(n)1} H_{(n)2} q^{2n})} \right] \right) \right\}, \qquad (25)
\end{aligned}$$

where the matrix $H_{(n)a}$ is defined in terms of $Q_{(\pm n)a}$ and $N_{(\pm n)a}$, as before, with

$$Q_{(n)a} = \begin{pmatrix} -1 + \frac{iU_{00}}{2n} & 4v_1 - E_1 + \frac{iU_{01}}{2n} & 4v_2 - E_2 + \frac{iU_{02}}{2n} \\ -4v_1 + E_1 + \frac{iU_{10}}{2n} & 1 + \frac{iU_{11}}{2n} & 4\varpi - B + \frac{iU_{12}}{2n} \\ -4v_2 + E_2 + \frac{iU_{20}}{2n} & -4\varpi + B + \frac{iU_{21}}{2n} & 1 + \frac{iU_{22}}{2n} \end{pmatrix}_a, \quad a = 1,\ 2,$$

$$N_{(n)a} = \begin{pmatrix} -1 - \frac{iU_{00}}{2n} & 4v_1 + E_1 - \frac{iU_{01}}{2n} & 4v_2 + E_2 - \frac{iU_{02}}{2n} \\ -4v_1 - E_1 - \frac{iU_{10}}{2n} & 1 - \frac{iU_{11}}{2n} & 4\varpi + B - \frac{iU_{12}}{2n} \\ -4v_2 - E_2 - \frac{iU_{20}}{2n} & -4\varpi - B - \frac{iU_{21}}{2n} & 1 - \frac{iU_{22}}{2n} \end{pmatrix}_a, \quad a = 1,2. \qquad (26)$$

The matrix elements of the symmetric matrix R_a are given by

$$\begin{aligned}
(R_a)_{00} &= -2i\alpha'[(U^{-1})_{00} - 4v_1(U^{-1})_{01} - \\
&\quad - 4v_2(U^{-1})_{02} + it]_a, \\
(R_a)_{01} &= -2i\alpha'[2(U^{-1})_{01} - \\
&\quad - 4v_1\left((U^{-1})_{00} + (U^{-1})_{11}\right) - \\
&\quad - 4\varpi(U^{-1})_{02} - 4v_2(U^{-1})_{21} + it]_a, \\
(R_a)_{02} &= -2i\alpha'\,[2(U^{-1})_{02} - \\
&\quad - 4v_2\left((U^{-1})_{00} + (U^{-1})_{22}\right) + \\
&\quad + 4\varpi(U^{-1})_{01} - 4v_1(U^{-1})_{12} + it]_a, \qquad (27)\\
(R_a)_{11} &= -2i\alpha'[(U^{-1})_{11} - 4v_1(U^{-1})_{10} - \\
&\quad - 4\varpi(U^{-1})_{12} + it]_a, \\
(R_a)_{12} &= -2i\alpha'[2(U^{-1})_{12} + \\
&\quad + 4\varpi\left((U^{-1})_{11} - (U^{-1})_{22}\right) - \\
&\quad - 4v_2(U^{-1})_{10} - 4v_1(U^{-1})_{02} + it]_a, \\
(R_a)_{22} &= -2i\alpha'[(U^{-1})_{22} - 4v_2(U^{-1})_{20} + \\
&\quad + 4\varpi(U^{-1})_{21} + it]_a,
\end{aligned}$$

with $a = 1, 2$. Also, for the R–R-sector amplitude becomes

$$\begin{aligned}
\mathcal{A}_{R-R} &= \frac{T_2^2 V_3 \alpha'}{8(2\pi)^{d-3}} \times \\
&\times \int_0^\infty dt \left\{ \frac{1}{\sqrt{\det(R_1^\dagger R_2)}} \left(\sqrt{\frac{\pi}{\alpha' t}}\right)^{d-3} \times \right.\\
&\times \exp\left(-\frac{1}{4\alpha' t}\sum_{i=3}^{d-1}\left(y_2^i - y_1^i\right)^2\right) \times \\
&\times \left(\kappa \prod_{n=1}^{\infty}\left[\left(\frac{1-q^{2n}}{1+q^{2n}}\right)^{5-d} \times \right.\right.\\
&\left.\left.\left.\times \frac{\det(\mathbf{1} + H^\dagger_{(n)1}H_{(n)2}q^{2n})}{\det(\mathbf{1} - H^\dagger_{(n)1}H_{(n)2}q^{2n})}\right] + \kappa'\right)\right\}, \qquad (28)
\end{aligned}$$

where

$$\begin{aligned}
\kappa &= 16\left(-1 + \Phi_{(1)01}\Phi_{(2)01} + \right.\\
&\quad \left. + \Phi_{(1)02}\Phi_{(2)02} - \Phi_{(1)12}\Phi_{(2)12}\right), \\
\kappa' &= -\frac{1}{4}i \sum_{\alpha,\beta=0}^{2}\sum_{\alpha',\beta'=0}^{2} \Phi_{(1)\alpha\beta}\Phi_{(2)\alpha'\beta'} \times \qquad (29)\\
&\quad \times \mathrm{Tr}(\Gamma^\alpha\Gamma^\beta\Gamma^{\alpha'}\Gamma^{\beta'}\Gamma_{11}).
\end{aligned}$$

We note that we have used the relation $(\Gamma^\mu)^T = -C\Gamma^\mu C^{-1}$. In fact, the D2-brane is the simplest brane whose rotation and motion along its directions are sensible. We see that in this simple case, the interaction amplitudes are already very complicated.

4. INTERACTION BETWEEN DISTANT D-BRANES

For distant D-branes, only the closed-superstring massless states make a considerable contribution to the interaction. Technically, the contribution of these states to the interaction amplitude is obtained by taking the limit of the oscillators portions of Eqs. (21) and (23). This is because for a massive state with a mass "m", the interaction vanishes as $\exp(-2\pi\alpha' m^2 t)$ (see, e. g., the Coleman–Weinberg formula in the open-string channel [23]). Therefore, for a sufficiently long time, which is equivalent to a large distance of the branes, the contribution of the massive states vanishes, while the contribution of the massless ones becomes dominant. This procedure clarifies why we do not expand the interaction amplitude with respect to the length scale to separate the contribution of the massless closed superstrings.

Let

$$P_{(n)} \in \{-\mathbf{1},\ \mathbf{1},\ -H^\dagger_{(n)1}H_{(n)2},\ H^\dagger_{(n)1}H_{(n)2}\}$$

and

$$q_n \in \{q^{2n},\ -q^{2n},\ q^{2n-1},-\ q^{2n-1}\}.$$

By applying the relation

$$\begin{aligned}
&\prod_{n=1}^{\infty}\det(\mathbf{1} + q_n P_{(n)}) = \\
&\quad = \exp\sum_{k=0}^{\infty}\left[\frac{(-1)^k}{k+1}\sum_{n=1}^{\infty}\mathrm{Tr}(q_n P_{(n)})^{k+1}\right], \qquad (30)
\end{aligned}$$

in amplitudes (21) and (23) and sending q to zero, we can obtain the contribution of the massless states. Therefore, in the 10-dimensional spacetime, we obtain the amplitudes

$$\begin{aligned}
\mathcal{A}^{(massless)}_{NS-NS} &= \frac{T_p^2 V_{p+1}\alpha'}{4(2\pi)^{9-p}} \times \\
&\times \prod_{m=1}^{\infty}\frac{\det[Q^\dagger_{(m-1/2)1}Q_{(m-1/2)2}]}{\det\left[Q^\dagger_{(m)1}Q_{(m)2}\right]} \times \\
&\times \int^\infty dt \left\{\left(\sqrt{\frac{\pi}{\alpha' t}}\right)^{9-p}\frac{7-p+\mathrm{Tr}(H^\dagger_{(1)1}H_{(1)2})}{\sqrt{\det(R_1^\dagger R_2)}} \times \right.\\
&\left.\times \exp\left(-\frac{1}{4\alpha' t}\sum_i\left(y_i^2 - y_i^1\right)^2\right)\right\} \qquad (31)
\end{aligned}$$

for the NS–NS sector and

$$\mathcal{A}^{(massless)}_{R-R} = \frac{T_p^2 V_{p+1}\alpha'}{8(2\pi)^{9-p}}(\kappa+\kappa') \times \\ \times \int^{\infty} dt \left\{ \left(\sqrt{\frac{\pi}{\alpha' t}}\right)^{9-p} \frac{1}{\sqrt{\det(R_1^\dagger R_2)}} \times \right. \\ \left. \times \exp\left(-\frac{1}{4\alpha' t}\sum_i \left(y_i^2 - y_i^1\right)^2\right)\right\} \quad (32)$$

for the R–R sector. We did not put the limit on the exponential factors and the two other time-dependent parts $\left(\sqrt{\pi/\alpha' t}\right)^{9-p}$ and $1/\sqrt{\det(R_1^\dagger R_2)}$ in Eqs. (31) and (32). The exponential parts indicate the locations of the branes, while closed-string emission (absorption) does not depend on the positions of the branes. The other two factors originate in the zero modes, but not in the oscillators that define the closed string states. The provenance of the factor $1/\sqrt{\det(R_1^\dagger R_2)}$ is the tachyon fields, which for large time weaken the interaction amplitudes. Precisely, because the presence of the open string tachyon makes the system unstable, the tachyon will roll down toward its potential minimum after a sufficiently long time, which causes a decrease in the amplitude. In the absence of the tachyonic fields, this slowing down factor disappears.

We observe that for large-distance branes, the amplitude of the NS–NS sector depends on the total field strengths $\mathcal{F}_1$ and $\mathcal{F}_2$, while these fields are absent in the R–R sector. In other words, the internal electric and magnetic fields of the branes suppress the exchange of the graviton, dilaton, and Kalb–Ramond states, but do not modify the R–R repulsion force between the distant branes.

The total amplitude

$$\mathcal{A}^{(massless)} = \mathcal{A}^{(massless)}_{NS-NS} + \mathcal{A}^{(massless)}_{R-R} = \\ = \frac{1}{(\alpha')^{3(p+1)/2}}\frac{\bar{T}_p^2 V_{p+1}}{4(2\pi)^{9-p}} \times \\ \times \left[\frac{1}{2}(\kappa+\kappa') + \left(7-p+\mathrm{Tr}(H^\dagger_{(1)1}H_{(1)2})\right) \times \right. \\ \left. \times \prod_{m=1}^{\infty} \frac{\det\left(Q^\dagger_{(m-1/2)1}Q_{(m-1/2)2}\right)}{\det\left(Q^\dagger_{(m)1}Q_{(m)2}\right)}\right] \times \\ \times \int^{\infty} dt \left\{ \left(\sqrt{\frac{\pi}{t}}\right)^{9-p} \frac{1}{\sqrt{\det(\bar{R}_1^\dagger \bar{R}_2)}} \times \right. \\ \left. \times \exp\left(-\frac{L^2}{4\alpha' t}\right)\right\}, \quad (33)$$

exhibits a long-range force between the Dp-branes interaction, where $\bar{T}_p = T_p|_{\alpha'=1}$, $\bar{R}_{1,2} = R_{1,2}|_{\alpha'=1}$ and $L^2 = \sum_i \left(y_i^2 - y_i^1\right)^2$ is the square distance between the branes. We observe that α' appears only in the prefactor and in the exponential part, while the other factors are independent of α'. However, for very large time, we have $L^2/4\alpha' t \ll 1$, and hence

$$\exp\left(-\frac{L^2}{4\alpha' t}\right) \approx 1 - \frac{L^2}{4\alpha' t}.$$

The NS–NS part indicates the exchange of the graviton, dilaton and Kalb–Ramond fields, in which the dilaton and the graviton give an attraction force, while the Kalb–Ramond field gives a repulsion one. In the same way, the R–R part indicates the repulsive contribution of the $(p+1)$-form potentials in the R–R sector. The net force for the static branes with zero background fields vanishes, because the branes are BPS states. But when the branes have velocity, rotation, and background fields, the total force is nonzero, i. e., the various contributions are not balanced.

5. CONCLUSIONS

We have constructed a closed-superstring boundary state corresponding to a rotating and moving Dp-brane incorporating configurations of electric, magnetic, and tachyonic background fields. The bosonic boundary state includes an exponential factor that is absent in the conventional boundary states, i. e., that one without a tachyon. This factor originates from the bosonic zero modes, rotation motion, and tachyon terms in the boundary action.

It should be mentioned that in this article, we considered the rotation axis perpendicular to the branes. In addition, the branes move along their volumes. According to the background fields, we then have preferred directions in the branes, which break the Lorentz invariance. Therefore, these rotations and motions are meaningful.

According to the eigenvalues in the boundary-state equations, we deduce the constraint equation

$$p^\alpha = -\frac{1}{2\alpha'}[(\eta+4\omega)^{-1}U]^\alpha{}_\beta x^\beta.$$

This implies that along the worldvolume of the brane, the momentum of an emitted (absorbed) closed string depends on its center of mass position. The source of this relation is entirely the tachyon field. Thus, in the presence of the tachyon, a closed string feels an exotic potential affecting its evolution.

The boundary states enabled us to calculate the interaction amplitude of two moving and rotating Dp-branes with background fields. This amplitude exponentially decreases with the squared distance between the branes, but it is a very complicated function of the configuration parameters. The variety of adjustable parameters controls the treatment of the interaction. For example, for two D$(d-3)$-branes, which can have different background fields and different motions, the contribution of the (super)ghosts removes the effects of all transverse oscillators. It was shown that even for co-dimension parallel branes with similar fields, the total amplitude is nonzero. That is, our system does not satisfy the BPS no-force condition. This is due to the presence of rotations, velocities, and tachyonic and photonic fields on the branes.

The long-range part of the interaction was extracted. In this domain, the instability of the branes due to the background tachyon fields weakens the interaction. This decreasing behavior can be understood as dissipation of the branes to the bulk modes because of the rolling of the tachyon to its potential minimum in the long-time regime. Finally, we observed that the internal electric and magnetic fields of the branes do not suppress the R–R repulsion force between the distant branes.

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