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OBSERVATIONAL LIMITS ON GAUSS–BONNET AND RANDALL–SUNDRUM GRAVITIES

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We discuss the possibilities of experimental search for the new physics predicted by the Gauss–Bonnet and the Randall–Sundrum theories of gravity. The effective four-dimensional spherically symmetrical solutions of these theories are analyzed. We consider these solutions in the weak-field limit and in the process of the primordial black hole evaporation. We show that the predictions of the discussed models are the same as of general relativity. Hence, current experiments are not applicable for such search, and therefore different methods of observation and higher accuracy are required.

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1. INTRODUCTION

A set of multidimensional gravity models beginning from the Kaluza–Klein one [1] result from the attempts to construct a unified field theory. Because we live in a space-time with four noncompact dimensions, any multidimensional theory needs an appropriate effective four-dimensional limit consistent with the predictions of general relativity (GR) and the results of observations and experiments.

String theory [2] along with loop quantum gravity [3] is currently a promising candidate for a quantum theory of gravity. Lovelock gravity [4] appeared to be a ghost-free four-dimensional low-energy effective limit of string theory [5, 6]:

$$L = \sqrt{-g}\left(R + \alpha_2 S^2 + \alpha_3 S^3 + \ldots\right), \tag{1}$$

where S^n is the Euler characteristic of the nth order. The leading and the most studied among them is the second-order curvature correction given by the Gauss–Bonnet term

$$S^2 = S_{GB} = R_{ijkl}R^{ijkl} - 4R_{ij}R^{ij} + R^2.$$

The effective four-dimensional limit of string theory also includes a scalar field, the projection of the $g_{10\ 10}$ component of the ten-dimensional string metric to the four-dimensional manifold. The Gauss–Bonnet term coupled to the scalar (dilatonic) field [7–11] describes the influence of compact extra dimensions on the four-dimensional space-time. Therefore, the Gauss–Bonnet theory with the dilaton scalar field serves as an effective four-dimensional limit of string theory.

Unlike string theory, the Randall–Sundrum model allows the only extra dimension to be large and even infinite [12, 13]. This model considers four-dimensional branes with tension embedded into a five-dimensional space-time (bulk) that is assumed to have an AdS_5 geometry. All matter and the three fundamental interactions are localized on this brane, but gravity is

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allowed to propagate into the bulk along the extra noncompact dimension. Thus, the Randall–Sundrum model contains the description of the four-dimensional space-time from the very beginning and therefore does not need any special theory serving as its effective four-dimensional limit. The Randall–Sundrum I (RSI) model includes two branes with different properties, helping to solve the hierarchy problem [12]. Moving the second brane to the infinity led to the Randall–Sundrum II (RSII) model with one brane [13]. In this paper, we deal with RSII only.

The black-hole solution is a basic one for any theory of gravity. First of all, it describes a compact object into which a very massive star collapses at the end of its life cycle. It also features the space-time curvature produced by the presence of matter and specific for the considered gravity model. Any extended theory of gravity should be consistent with the predictions of GR and the observational results, and therefore the existence of black holes and their properties are important indicators of the theory's viability.

The Gauss–Bonnet solution has been studied explicitly in recent years [7, 9–11]. On the other hand, there are several different solutions for the Randall–Sundrum model [14–18]. It was argued that static black holes cannot exist in RSII with a radius much greater than the AdS length ℓ [19–21] and that even very small RSII static black holes do not exist [22, 23]. Therefore, RSII solutions for large black holes, which have been found independently by Figueras and Wiseman [24, 25] and Abdolrahimi, Cattoën, Page, and Yaghoobpour-Tari (ACPY) [26] are an important improvement of the Randall–Sundrum model, interesting for further consideration. Abdolrahimi, Cattoën et al. [26] compare the obtained black-hole solution to the one in [25] and show that these solutions agree closely. In this paper, we use the ACPY solution [26] because it contains the necessary details. The Figueras–Wiseman solution [24] is considered separately.

The outline of this paper is as follows. In Sec. 2, we discuss the weak-field and slow-motion approximation of the Gauss–Bonnet and Randall–Sundrum theories. Section 3 is devoted to the analysis of thermodynamical properties of these models and their influence on the primordial black hole mass spectra. In Sec. 4, we discuss the results obtained, offer conclusions, and outline the next steps.

2. WEAK-FIELD LIMIT

As a weak-field limit, we consider the dynamical conditions in the solar system, i. e., the post-Newtonian approximation. The metric tensor $g_{\mu\nu}$ can be represented as a perturbation $h_{\mu\nu}$ around the Minkowski space-time $\eta_{\mu\nu}$ [27]:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}. \tag{2}$$

In this paper, we consider only spherically symmetric solutions, and therefore the static gravity field at a distance r from its source. In the first post-Newtonian (PPN) order, the correction to the gravitational field $h_{\mu\nu}$ can be expressed by a series in the negative powers of the radial coordinate r up to the next-order terms:

$$h_{00} \sim \mathcal{O}(r^{-3}), \quad h_{0j} \sim \mathcal{O}(r^{-4}), \quad h_{ij} \sim \mathcal{O}(r^{-2}). \tag{3}$$

We use geometric units $\hbar = c = G = 1$, with non-dimensional masses expressed in units of the Planck mass.

The PPN limit is well tested by experiments [27, 28]. The better the experimental accuracy becomes [28], the more opportunities to test small gravitational effects predicted by currently viable theories should appear. We use expansion (3) to compare the magnitudes of the predicted effects in order to see if specific effects of the considered solutions can be tested. Because the PPN approximation requires the weak-field limit, we apply our results to the Solar system, where the PPN parameters are measured with high precision [29]. Our results are inapplicable to the strong-field limit.

2.1. Gauss–Bonnet gravity

We begin with exploring the weak-field limit of the Gauss–Bonnet theory (here and hereafter, when solving the equations, we use the dimensionless Planckian units, and only at the step of numerical estimation do we jump to the usual ones)

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \Big[-R + 2\partial_\mu\phi\partial^\mu\phi + \\ + \lambda e^{-2\phi} S_{GB} + \ldots \Big], \tag{4}$$

where ϕ is the potential of the dilatonic field, S_{GB} is the Gauss–Bonnet term, and λ is the string coupling constant. We construct a post-Newtonian parameterization of the static asymptotically flat spherically symmetric Gauss–Bonnet solution

$$ds^2 = \Delta dt^2 - \frac{\sigma^2}{\Delta}\, dr^2 - r^2 \Big(d\theta^2 + \sin^2\theta d\varphi^2 \Big), \tag{5}$$

$$\begin{gathered} \Delta = 1 - \frac{2M}{r} + \mathcal{O}\left(r^{-2}\right), \quad \sigma = 1 + \mathcal{O}\left(r^{-2}\right), \\ \phi = \phi_\infty + \frac{D}{r} + \mathcal{O}\left(r^{-2}\right), \end{gathered} \tag{6}$$

where t, r, θ, φ are the usual spherical coordinates and the functions Δ and σ depend on the radial coordinate r only, M is the Arnowitt–Deser–Misner (ADM) mass, D is dilatonic charge, i. e., the effective charge of the scalar field source, and ϕ_∞ is the asymptotic value of the dilatonic potential [7, 8]. As argued in [10], $D \propto 1/M$.

We substitute metric (5) with expansions (6) in the field equations written in the computationally most convenient form [30]:

$$G_{\mu\nu} = 8\pi \left(T^m_{\mu\nu} + T^\phi_{\mu\nu} + T^{GB}_{\mu\nu}\right), \tag{7}$$

where $T^m_{\mu\nu}$ is the matter stress–energy tensor, and $T^\phi_{\mu\nu}$ and $T^{GB}_{\mu\nu}$ reflect the presence of the scalar field and the Gauss–Bonnet term:

$$T^\phi_{\mu\nu} = \frac{1}{8\pi}\left(\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\partial^\rho\phi\;\partial_\rho\phi\right),$$

$$T^{GB}_{\mu\nu} = \frac{1}{8\pi}\Big[(\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box)(e^{-2\phi}R) + \\ + 2\Big(\Box\delta^\sigma_\mu\delta^\sigma_\nu + \; g_{\mu\nu}\nabla^\rho\nabla^\sigma - \nabla^\rho\nabla_{(\mu}\delta^\sigma_{\nu)}\Big)(e^{-2\phi}\;R_{\rho\sigma}) - \\ - 2\nabla^\rho\nabla^\sigma(e^{-2\phi}\;R_{\mu\rho\nu\sigma})\Big].$$

Using the standard computational techniques [27], the leading order for the nontrivial correction to the Gauss–Bonnet metric tensor can be found as

$$\delta h^{GB}_{00} = 8\,\frac{DM}{r^4} + \mathcal{O}(r^{-5}). \tag{8}$$

Comparing this result with (3), we see that the correction term (8) lies beyond the PPN order, which should be proportional to $1/r^2$. Hence, the parameters of the Gauss–Bonnet model cannot be constrained by the Solar system tests. This result is consistent with conclusions in [30], where the cosmological limit of the discussed model was studied.

2.2. Randall–Sundrum gravity

The black hole solution of the Randall–Sundrum model was constructed in [24] using an associated five-dimensional anti-de Sitter space (AdS_5) and AdS_5–CFT_4 correspondence [31]. The Figueras–Wiseman solution describes a static black hole with a radius up to $\sim 20\ell$ and reproduces four-dimensional GR on the brane in the low-curvature and the low-energy limit. We intend to use the fact that the Schwarchild metric can be used not only as a black hole one but also as a description of a gravitational (stellar) system far from the central body (for example, for Solar system, with all the limitations and corrections taken into account).

The five-dimensional metric can be written near the conformal boundary $z = 0$ as

$$ds^2 = \frac{l^2}{z^2}\left[dz^2 + \tilde{g}_{\mu\nu}(z,x)\;dx^\mu dx^\nu\right], \tag{9}$$

where z is a coordinate of the brane along the extra dimension and $\tilde{g}_{\mu\nu}(z,x)$ is the metric on the brane determined by the Fefferman–Graham expansion [31]. The corresponding effective four-dimensional field equations [24] are

$$G_{\mu\nu} = 8\pi G_4 T^{brane}_{\mu\nu} + \epsilon^2\Big\{16\pi G_4\langle T^{CFT}_{\mu\nu}[g]\rangle + \\ + a_{\mu\nu}[g] + \log\epsilon\; b_{\mu\nu}[g]\Big\} + O(\epsilon^4\log\epsilon), \tag{10}$$

where G_4 is the usual four-dimensional gravitational constant, $T^{brane}_{\mu\nu}$ is the stress–energy tensor of matter localized on the brane, the tensors $\langle T^{CFT}_{\mu\nu}[g]\rangle$, $a_{\mu\nu}[g]$, and $b_{\mu\nu}[g]$ result from the extra dimension and depend on the metric tensor components, and ϵ is a small perturbation parameter indicating the deviation of the brane position from the equilibrium $z = 0$.

The additional term in the post-Newtonian expansion of the Figueras–Wiseman solution calculated in this paper is

$$\delta h^{FW}_{00} = \frac{121}{27}\,\frac{\epsilon^2}{\ell^2}\,\frac{M^2}{r^2}. \tag{11}$$

The obtained value (11) lies within the PPN limit (3) and points at a potentially observable effect. In the Randall–Sundrum model, gravity is allowed to propagate into the bulk along the extra dimension, and therefore the effect described by (11) most likely leads to a negative nonlinearity in gravitational superposition. In other words, the resulting gravitational field produced by two or more massive objects can be less than the direct vector sum of their contributions. The parameterized PPN parameter β is responsible for such an effect [27, 28]. Therefore, result (11) should be expressed as

$$\beta = 1 - \frac{\epsilon^2}{\ell^2}\,\frac{121}{108}\,M^2, \tag{12}$$

where M is the mass of the massive central object. In the considered case, it equals the solar mass. It is also expressed in Planck units of mass and is therefore dimensionless.

The constraint on the PPN parameter β obtained from the analysis of the lunar laser ranging data [32] is $|\beta - 1| \le 1.1\cdot 10^{-4}$ [29]. The admitted region of the AdS length is limited by the results of the Newton's law test $\ell < 10^{-5}$ m [33]. Therefore, the upper limit on the value of ϵ is

$$\epsilon \leq 5.7 \cdot 10^{-47}\ \mathrm{cm} \ll l_{Pl}. \tag{13}$$

Originally, the parameter ϵ was assumed to be negligibly small and the vanishing value found in (13) implies that in fact $\epsilon = 0$. Thus, the Figueras–Wiseman four-dimensional black hole solution is not only self-consistent but also well consistent with the solar system constraints. Therefore, this solution is eventually indistinguishable form GR in the PPN limit.

The other recent Randall–Sundrum solution obtained by Abdolrahimi, Cattoën, Page, and Yaghoobpour-Tari [26] is asymptotically conformal to the Schwarzschild metric and includes a negative five-dimensional cosmological constant Λ_5:

$$\begin{gathered}
ds^2 = -u(r)dt^2 + \frac{v(r)}{u(r)}\,dr^2 + \left[r^2 + \frac{F(r)}{-\Lambda_5}\right] d\Omega^2, \\
u(r) = 1 - 2M/r, \\
v(r) = 1 + \frac{r-2M}{r-3M/2}\left[\frac{F(r)}{-\Lambda_5 r}\right]', \\
F(r) = 1 - 1.1241\frac{2M}{r} + 1.956\left(\frac{2M}{r}\right)^2 - \\
- 9.961\left(\frac{2M}{r}\right)^3 + \ldots + 2.900\left(\frac{2M}{r}\right)^{11},
\end{gathered} \tag{14}$$

where $' \equiv d/dr$. The function $F(r)$ describes the perturbation caused by the bulk. The best fit for it was obtained in [26].

The field equations induced on the brane were derived in [34]:

$$G_{\mu\nu} = -\Lambda_4 g_{\mu\nu} + \frac{8\pi}{M_{Pl4}^2}\,T_{\mu\nu} + \frac{8\pi}{M_{Pl5}^3}\,S_{\mu\nu} - \mathcal{E}_{\mu\nu}, \tag{15}$$

where Λ_4 is the usual four-dimensional cosmological constant, $g_{\mu\nu}$ is the metric on the brane, $T_{\mu\nu}$ is the stress–energy tensor of matter localized on the brane, $S_{\mu\nu}$ is the local quadratic stress–energy correction, and $\mathcal{E}_{\mu\nu}$ is the four-dimensional projection of the five-dimensional Weyl tensor; M_{Pl4} is usual four-dimensional Planck mass and M_{Pl5} is the fundamental five-dimensional Planck mass.

The induced metric on the brane is flat and the bulk is an anti-de-Sitter space-time as in the original Randall–Sundrum scenario [13], whence $\mathcal{E}_{\mu\nu} = 0$ [16]. Therefore, the correction term due to the contribution from ACPY topology (14) that follows from (15) has the form

$$\delta h_{00}^{AP} = \frac{\ell^2 M^2}{96}\,\frac{1}{r^4} + \mathcal{O}(r^{-5}). \tag{16}$$

According to (3), the expansion term of the PPN order should be proportional to r^{-2}. The correction in (16) contains the next perturbation order, which lies beyond the PPN approximation, similarly to the Gauss–Bonnet case (8). Therefore, the obtained contribution (16) also cannot be observed in the solar system experiments. This conclusion on the Randall–Sundrum model predictions confirms the result for the Figueras–Wiseman solution and coincides with that in the Gauss–Bonnet case.

3. THERMODYNAMICS AND PRIMORDIAL BLACK HOLES

It is conjectured that density fluctuations in the early Universe could have created black holes with arbitrarily small masses, even to the Planck scale [35]. These black holes are referred to as primordial black holes (PBHs) [36] and can be used to consider viable theories in cosmological conditions.

Hawking evaporation [37, 38] is one of the most significant properties of a black hole and can be described by the mass-loss rate equation [39]

$$-\frac{dM}{dt} = \frac{1}{256}\,\frac{k_B}{\pi^3 M^2}, \tag{17}$$

where M is the mass of the black hole and k_B is the Stefan–Boltzmann constant. Hawking evaporation is a quantum process forbidden in classical physics. An outgoing radiation has to cross a potential barrier of the black hole horizon [40], and hence the radiation surrounding the black hole is in thermal equilibrium and can be described as the black-body radiation. Therefore, black hole evaporation obeys the law

$$-\frac{dM}{dt} = k_B S T^4, \tag{18}$$

where S is its surface area. We use this formula to estimate the lifetime of black holes in the Gauss–Bonnet and Randall–Sundrum models.

According to (17), the black holes with stellar masses evaporate very slowly and do not lose mass through this process noticeably. On the other hand, PBHs with the initial masses smaller than

$$M_0 \approx 5.0 \cdot 10^{14}\ \mathrm{g} \tag{19}$$

have already evaporated and can contribute to the extragalactic background radiation [38]. PBHs with the initial mass greater than M_0 in (19) should be evaporating until now [41]. According to some models of black hole evaporation [7, 8, 41], the last stages of this process can be accompanied by bursts of high-energy particles [8], including gamma radiation with energy in the MeV–TeV range, occuring at the distances about $z \leq 9.4$ [42]. Such events should be rather rare and, on the other hand, the set of simpler explanations for most of gamma-ray bursts (GRB) exists. Nevertheless, PBHs at the last stage of evaporation can serve as additional candidates for GRB progenitors, and therefore the limit estimation for the black hole evaporation rate can be obtained in such a way.

Different theories of gravity predict different black hole evaporation rates and therefore different initial masses of the PBHs that fully evaporate for the Universe lifetime. In this paper, we compare the evaporation rates for the Gauss–Bonnet and RSII black hole solutions. According to the GRB data and the precision of the Fermi Large Area telescope (LAT), the closest distance d at which a telescope can detect the evaporation of primordial black holes is [36]

$$d \approx 0.04 \left(\frac{\Omega}{\mathrm{sr}}\right)^{-0.5} \left(\frac{E}{\mathrm{GeV}}\right)^{0.7} \left(\frac{T}{\mathrm{TeV}}\right)^{0.8} \mathrm{pc}, \qquad (20)$$

where Ω is the angular resolution of the telescope, E is the energy range of the telescope, and T is the temperature of the black hole. The same procedure reversed, using a telescope to detect gamma-ray bursts, leads to the observable difference of the PBH initial mass on its final evaporation stage, which can deviate from the GR predictions within the following limits:

$$\frac{M_{investigated\ \ theory}}{M_{GR}} > 10^5. \qquad (21)$$

We use this limit as the mass cutoff threshold in our calculations.

Using the method in [43], it is possible to rewrite the expression for the Gauss–Bonnet black hole temperature and then use (7). In the astrophysical case, the dilatonic charge is $D \approx 1/M$ [10]. Therefore, the right-hand side of (18) can be expanded in a series as

$$-\frac{dM}{dt} \approx \frac{1}{256}\frac{k_B}{\pi^3 M^2} + \frac{1}{512}\frac{k_B}{\pi^3 M^6} + \mathcal{O}\left(M^{-10}\right). \qquad (22)$$

The initial mass of the PBH that fully evaporates during the lifetime of the Universe in this case is

$$M_{GB} = 8 \cdot 10^{14}\ \mathrm{g}. \qquad (23)$$

The difference between the obtained value and the similar GR quantity in (19) is smaller than the cutoff threshold set by (21). Thus, the specific features of the Gauss–Bonnet evaporation rate are negligible at the current level of accuracy and the predictions of Gauss–Bonnet gravity for the Hawking evaporation are indistinguishable from those of GR.

One of the first and most studied black hole solutions of the Randall–Sundrum model was found in [14, 16], where an exact localized black hole solution was obtained that remarkably had the mathematical form of the Reissner–Nordström solution, but without the electric charge [14]:

$$-g_{tt} = g_{rr} = 1 - \frac{2M}{r} + \frac{q}{M_{Pl5}^2}\frac{1}{r^2}. \qquad (24)$$

The Reissner–Nordström-type correction to the Schwarzschild potential in (24) can be regarded as a dimensionless "tidal charge" parameter q, arising from the projection onto the brane of free gravitational field effects in the bulk transmitted via the bulk Weyl tensor [14]. The projected Weyl tensor, transmitting the tidal charge stresses from the bulk to the brane, is [14]

$$\mathcal{E}_{\mu\nu} = -\frac{q}{M_{Pl5}^2}\frac{1}{r^4}\left(u_\mu u_\nu - 2r_\mu r_\nu + h_{\mu\nu}\right),$$

where $h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$ projects orthogonally to the 4-velocity field u^μ, and r_μ is a unit radial vector.

The mass loss rate obtained similarly to the Gauss–Bonnet case is

$$-\frac{dM}{dt} = \frac{1}{216}\frac{k_B}{\pi^3 M^2} + \mathcal{O}\left(M^{-6}\right). \qquad (25)$$

The leading term in (25) cannot produce the needed fifth-order difference defined by threshold parameter (21). The initial mass of the Dadhich–Rezania black hole that evaporates completely during the lifetime of the Universe proves this fact:

$$M_{DR} = 5.3 \cdot 10^{14}\ \mathrm{g}. \qquad (26)$$

Because the obtained difference is much less than the cutoff threshold in (21), the "tidal charge" influence vanishes and cannot have experimentally verifiable consequences.

Black hole evaporation for the ACPY solution discussed in the preceding section was also considered in a similar manner. The evaporation rate of this solution has completely the same form as the original Hawking formula (17) up to M^{-10} terms, and hence the value of the initial mass is equal to that given by GR:

$$M_{AP} = 5.0 \cdot 10^{14}\ \mathrm{g}. \qquad (27)$$

The results for the Figueras–Wiseman solution are the same because of the form of the solution (11).

The obtained results (22), (23), (25)–(27) lead to the conclusion that the precision of the currently existing GRB data is not sufficient to distinguish the GR, Gauss–Bonnet, and Randall–Sundrum gravity theories from each other via the PBH consideration.

4. DISCUSSION AND CONCLUSIONS

In this paper, we discussed the possibilities to test the theories extending GR in different ways by the example of the Gauss–Bonnet and Randall–Sundrum models both in the weak field and in the cosmological limits. For this, the post-Newtonian expansion and the black hole evaporation in these theories were considered.

The Gauss–Bonnet term coupled to the scalar field does not influence the post-Newtonian limit (8), and therefore the nontrivial scalar hair generated by it [7, 11] does not contribute to the required order of the spherically symmetric solution expansion. This result agrees with the previous conclusions in [30], where the cosmological solution of action (4) was considered and the influence of the Gauss–Bonnet term was shown to be negligible at solar system scales. Combining these two results, we can state that the leading term of Lovelock expansion (1) describing a second-order curvature correction does not provide any visible deviation from GR predictions in the weak-field limit, and therefore such a theory of gravity fully agrees with GR.

This conclusion is also valid for any model with higher-order curvature corrections having a proper Newtonian limit. Because the Gauss–Bonnet term is the leading curvature correction of the Lovelock gravity, its contribution to the post-Newtonian expansion of the metric is also the largest. Taking other Euler characteristics (the next orders of curvature corrections) into account cannot change the picture because their influence is even less and obviously lies far beyond the PPN limit. Thus, the conclusions for the Gauss–Bonnet model can be generalized to the Lovelock gravity.

The theories with curvature power series are not the only method for geometrically extending GR. In the generic case, the Lagrangian can contain an arbitrary function of the Ricci scalar R. Such theories set up $f(R)$-gravity [44, 45], and the Lovelock gravity is its particular case. Many $f(R)$-gravity models, such as $\ln R$ or $1/R$ [44, 46], were originally introduced as attempts to explain dark energy or dark matter. They do not have a proper PPN limit [44] and are inapplicable to the solar system scale. Therefore, our conclusions for the Gauss–Bonnet theory in the weak-field limit are applicable for Lovelock gravity and $f(R)$-gravity of the Lovelock type.

The thermodynamical properties of the Gauss–Bonnet black hole solution were considered in detail previously [7, 8, 10], but only the black holes of Planck scales were investigated. For the black holes with larger masses, the influence of the Gauss–Bonnet term and the scalar field becomes negligibly small, and therefore the evaporation is predictably the same as in the GR case.

Since Randall and Sundrum proposed a theory of gravity with a noncompact extra dimension [12, 13], several black hole solutions have been found [14, 17, 24–26]. Analysis of the post-Newtonian expansion of the Figueras–Wiseman solution [24] reveals a possible effect of negative nonlinearity of gravitational superposition (12). It naturally results from the theory itself because gravity is allowed to propagate to the extra dimension in the Randall–Sundrum model. However, the breaking of gravitational superposition turns out to depend on a negligibly small parameter, and hence the predictions of the Figueras–Wiseman solution fully agree with GR and the present observations. This effect may influence the strong-field regime (close binary systems, black holes) as a consequence of curvature growth. Hence, the next step could be the search for such features of the Randall–Sundrum model in the strong-field limit. Fortunately, this investigation is feasible because large stable black hole solutions for RSII black holes have been found [24, 26].

The consideration of the black hole solution by Abdolrahimi, Page et al. [26] shows that the terms describing the bulk influence (16) greatly exceed the limits of the post-Newtonian approximation. As a result, both large Randall–Sundrum black holes solutions cannot be distinguished from the Schwarzschild metric at the solar system scales.

We have also examined the evaporation rate for the Randall–Sundrum black holes. The results for one of the first solutions obtained in [14] and the latest one in [26] are presented in Eqs. (22), (23), (25)–(27). The difference between the Dadhich–Rezania solution and GR is negligibly small and the Page solution coincides with GR completely.

As is easy to see, many extended gravity models cannot be distinguished from GR and from each other both at the solar system scales and by the black holes thermodynamic properties. Therefore, the coincidence of these extended theories with GR serves as a good argument in favor of their validity. However, this does not mean that no difference can be found by other verification methods. Besides the weak field and the cosmological tests, a strong-field approximation is widely used. It has a verification laboratory such as close binary systems, primarily those containing pulsars as one or even both of their components. A great amount of data has been obtained from these observations and it obviously should be used for testing the extended gravity models, although this method has its own shortcomings. If the orders of the post-Newtonian corrections of extended gravity models lie beyond the PPN order, it is natural to suggest that the parameters of the models should be limited via the second or the third post-Newtonian orders. The corresponding 2PN and 3PN formalisms do exist [47, 48]. These formalisms consider the gravitational radiation and its subtle effects on pulsar timing and orbit parameters. However, many calculations there are based on GR and are not suitable for comparing arbitrary extended theories of gravity as Will's formalism [27] is.

There are also other ways to test astrophysical predictions of extended theories of gravity, such as accretion onto massive objects and microlensing. After computing the accretion rate for some solution, the result can be compared with GR predictions and some other extended gravity cases. The investigation of the data of gravity lensing events is also a perspective method because these data become more and more complete. Verifying extended gravity models via studying binary systems and particularly the pulsar data requires special methods and approaches. Their construction is the subject of further considerations.

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