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RAYLEIGH SURFACE WAVE INTERACTION WITH THE 2D EXCITON BOSE–EINSTEIN CONDENSATE

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We describe the interaction of a Rayleigh surface acoustic wave (SAW) traveling on the semiconductor substrate with the excitonic gas in a double quantum well located on the substrate surface. We study the SAW attenuation and its velocity renormalization due to the coupling to excitons. Both the deformation potential and piezoelectric mechanisms of the SAW–exciton interaction are considered. We focus on the frequency and excitonic density dependences of the SAW absorption coefficient and velocity renormalization at temperatures both above and well below the critical temperature of Bose–Einstein condensation of the excitonic gas. We demonstrate that the SAW attenuation and velocity renormalization are strongly different below and above the critical temperature.

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1. INTRODUCTION

The gas of bound electron–hole pairs, excitons, being a gas of Bose-like particles, can exhibit Bose–Einstein condensation (BEC) at extremely low temperatures. This phenomenon was theoretically predicted a long time ago [1–5] and was intensively studied in the Cu_2O system (see recent review article [6]). Recently, BEC of excitons in low-dimensional systems was confirmed in various experiments [7–9].

The experimental evidence of the exciton BEC existence is mainly based on optical arguments. The general idea is the narrowing of the luminescence line when the exciton gas is cooled to below the critical temperature.

The main aim of this paper is to theoretically demonstrate that the SAW experimental technique widely used in earlier studies of the two-dimensional electron gas [10] may yield an alternative method for studying the exciton BEC. We show that the SAW velocity renormalization $\Delta c/c$ and the SAW attenuation coefficient behave differently above and below the critical BEC temperature, and this may be used as an experimental confirmation of exciton BEC.

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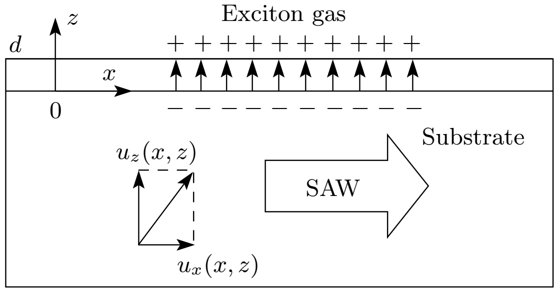

Fig. 1. A sketch of the system under study

We consider the double quantum well (DQW) structure depicted in Fig. 1. An electron and a hole are located in different QWs interacting via the Coulomb potential forming an exciton with the dipole moment **p** directed along the normal to the DQW plane. We consider the excitonic gas when the exciton Bohr radius a_B and the distance between QWs d satisfy the inequalities $na_B^2 \ll 1$ and $nd^2 \ll 1$. It was shown that the excitonic gas in the dilute limit $na_B^2 \ll 1$ has an excitation energy dispersion as in the Bogoliubov theory of weakly interacting Bose gas [4]. We use the Bogoliubov theory to calculate the SAW absorption and SAW velocity renormalization due to the interaction with excitons.

We regard an exciton as a rigid dipole particle with a dipole moment along direction z only, $\mathbf{p} = (0, 0, -ed)$. Here, e is the electron charge. Such a model ignores the internal motion of particles and the motion in the z-direction. This model is good enough to describe the system under study while the internal degrees of freedom are not excited. We assume that $e^2/a_B \gg \max[T, \omega]$, where ω is the SAW frequency and T is the temperature. In this limit, neither the SAW nor temperature can excite the internal degree of freedom of the exciton.

Nevertheless, the dipoles, as a whole, are free to move in the (x, y) plane. A SAW can interact with the excitonic gas via either the deformation potential or piezoelectric mechanisms. Acoustic and electric SAW fields are assumed to be the perturbations disturbing the excitonic gas from equilibrium. The response of the excitonic gas to the SAW perturbation depends on whether it is in the BEC state, resulting in different behaviors of the SAW velocity renormalization and attenuation coefficient. We consider the Rayleigh wave and start with the deformation potential mechanism.

2. SAW–EXCITON INTERACTION VIA THE DEFORMATION POTENTIAL

We assume that the substrate is an isotropic elastic medium. The Rayleigh wave traveling along the surface is characterized by transverse, c_t, and longitudinal, c_l, sound velocities. Moreover, a typical SAW wavelength is much larger than the distance d between QWs. In this case, the influence of the excitonic gas on the SAW propagation can be described by changing the boundary conditions for the stress tensor σ_{ij} on the surface $z = 0$. The substrate displacement vector $\mathbf{u}$ satisfies the equation

$$\ddot{\mathbf{u}} = c_t^2 \Delta \mathbf{u} + (c_l^2 - c_t^2) \operatorname{grad} \operatorname{div} \mathbf{u} \tag{1}$$

and, in the case of a Rayleigh wave, it has z and x components $u_x(x, z) = u_x(z) e^{ikx - i\omega t}$ and $u_y = 0$, $u_z(x, z) = u_z(z) e^{ikx - i\omega t}$ [11], where

$$\begin{gathered} u_z(z) = -i\kappa_l B e^{\kappa_l z} - ikA e^{\kappa_t z}, \\ u_x(z) = kB e^{\kappa_l z} + \kappa_t A e^{\kappa_t z}, \\ \kappa_l = \sqrt{k^2 - \omega^2/c_l^2}, \quad \kappa_t = \sqrt{k^2 - \omega^2/c_t^2}. \end{gathered} \tag{2}$$

Arbitrary amplitudes A and B are found from the boundary conditions $\sigma_{ij}\tau_j = f_i$, where f_i is a surface force (per unit area) acting from the excitons on the substrate surface and τ_j is a unit vector normal to the surface $z = 0$. The surface force $\mathbf{f}$ arises due to the exciton density deviation from equilibrium,

$$\mathbf{f} = \lambda \operatorname{grad} n. \tag{3}$$

Here, $\lambda = \lambda_e + \lambda_h$ is the sum of electron and hole deformation constants. They can depend on the temperature and momenta of the particles. To simplify our consideration, we ignore such dependences below; $n = n_{k\omega} e^{ikx - i\omega t}$ is the exciton density fluctuation.

Thus, the boundary conditions on the surface $z = 0$ yield

$$\begin{gathered} \rho c_t^2 \left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} \right) = \lambda \frac{\partial n}{\partial x}, \\ c_l^2 \frac{\partial u_z}{\partial z} + (c_l^2 - 2c_t^2) \frac{\partial u_x}{\partial x} = 0. \end{gathered} \tag{4}$$

The exciton density fluctuation amplitude $n_{k\omega}$ can be found using the standard linear response theory as $n_{k\omega} = S_{k\omega} W_{k\omega}$, where $W_{k\omega} = \lambda (\operatorname{div} \mathbf{u})_{z=0} = \lambda (iku_x + \partial_z u_z)|_{z=0}$ is the potential energy of the exciton in the SAW deformation field. The structure of the response function $S_{k\omega}$ depends on the exciton gas state. Substituting $n_{k\omega}$ in boundary conditions (4) and taking Eqs. (2) into account, we obtain the dispersion equation

$$\begin{gathered} f_{k\omega} = \frac{2(\lambda k \omega)^2 \kappa_t}{\rho (c_l c_t)^2} S_{k\omega}, \\ f(k, \omega) = (\kappa_t^2 + k^2)^2 - 4\kappa_l \kappa_t k^2. \end{gathered} \tag{5}$$

In the absence of SAW–exciton interaction, $\lambda = 0$, the SAW dispersion $\omega(k)$ is given by $f(k, \omega) = 0$ and is linear in k: $\omega = c_t \xi_0 k$. Here, ξ_0 is a solution of the equation $f(k, \omega = c_t \xi_0 k) = k^4 f(\xi_0) = 0$, where [11]

$$f(\xi_0) = (2 - \xi_0^2)^2 - 4\sqrt{1 - \xi_0^2} \sqrt{1 - \frac{c_t^2}{c_l^2} \xi_0^2}. \tag{6}$$

Because of the interaction, ξ_0 has a $\delta\xi$ correction due to the presence of the r.h.s. in Eq. (5); $\delta\xi$ is a complex value, whose real part describes the SAW velocity renormalization and imaginary part gives the SAW attenuation coefficient Γ

$$\frac{\Delta c}{c} = \operatorname{Re} \left(\frac{\delta\xi}{\xi_0} \right), \quad \Gamma = -2k \operatorname{Im} \left(\frac{\delta\xi}{\xi_0} \right), \tag{7}$$

where $c = c_t \xi_0$ and $\omega_0 = ck$.

To find $\delta\xi$, we substitute $\omega = c_t k(\xi_0 + \delta\xi)$ in Eq. (5), expand $f(\xi_0 + \delta\xi) \approx f(\xi_0) + f'(\xi_0)\delta\xi$, and solve the equation by successive approximation. The result is

$$\delta\xi = \frac{2(\lambda \xi_0)^2 \sqrt{1 - \xi_0^2}}{f'(\xi_0) \rho c_l^2} \, k S_{k, \omega = c_t \xi_0 k}. \tag{8}$$

Thus, we can see from (8) that the imaginary and real parts of $\delta\xi$ are determined by the response function $S_{k,\omega}$. To find it, we consider the cases $T > T_c$ and $T < T_c$ separately; T_c is the exciton gas condensation temperature.

3. $T > T_c$

At a high temperature and low density, the excitons can be regarded as a weakly interacting gas. The interaction potential is nothing but the repulsive exciton–exciton interaction. The Fourier transform of the interaction potential is

$$g(k) = \frac{4\pi e^2}{(\epsilon+1)k}\left(1 - 2e^{-kd}\right) + \frac{2\pi e^2}{\epsilon k}\left(1 + \frac{\epsilon - 1}{\epsilon + 1}e^{-2kd}\right), \quad (9)$$

where ϵ is the QW dielectric constant. Using the mean field approach, we find the response function

$$S_{k\omega} = \frac{\Pi_{k\omega}}{1 - g_k \Pi_{k\omega}},$$
$$\Pi_{k\omega} = \sum_{\mathbf{p}} \frac{f^B_{\mathbf{p}+\mathbf{k}} - f^B_{\mathbf{p}}}{\omega + E_{\mathbf{p}+\mathbf{k}} - E_{\mathbf{p}} + i\delta}, \quad (10)$$

where $f^B_{\mathbf{p}}$ is the Bose distribution function, $E_{\mathbf{p}} = p^2/2M$ is the exciton kinetic energy, and M is the exciton mass. To calculate the polarization operator $\Pi_{k\omega}$, we consider the long-wavelength limit $k \ll M v_T$, where $v_T = \sqrt{2T/M}$ is the exciton gas thermal velocity. Expanding all expressions in (10) for a small $\mathbf{k}$, we obtain

$$\operatorname{Re}\Pi_{k\omega} = \frac{M}{2\pi}\int\limits_0^\infty dx \left[1 - \frac{|\eta|\theta(\eta^2 - x)}{\sqrt{\eta^2 - x}}\right](f^B)'_x,$$
$$\operatorname{Im}\Pi_{k\omega} = \frac{M}{2\pi}\int\limits_{\eta^2}^\infty dx \frac{\eta}{\sqrt{x - \eta^2}}(f^B)'_x, \quad (11)$$

where $f^B = [\exp(x - \mu/T) - 1]^{-1}$, $\eta = \omega/v_T k$, and the prime means the derivative with respect to x. Moreover, we can also simplify g_k in (10) because $kd \ll 1$ for typical SAW wavelength, and we have $g_{k=0} \approx 4\pi e^2 d/\epsilon$.

The integration in (11) is not possible in general, and we consider two limit cases: $\eta = \omega/v_T k = c_t\xi_0/v_T \ll 1$ and $\eta \gg 1$. These inequalities compare the SAW velocity $c_t\xi_0$ with the exciton gas thermal velocity v_T. The simple calculations yield the following results. If $c_t\xi_0/v_T \ll 1$, then

$$\frac{\Delta c}{c} = -k\frac{\lambda^2 M}{\pi\rho c_l^2}\,\frac{\xi_0\sqrt{1-\xi_0^2}}{f'(\xi_0)} \times \frac{e^{2\pi N_0/MT} - 1}{\left[1 + (2d/a)\left(e^{2\pi N_0/MT} - 1\right)\right]^2},$$
$$\Gamma = -k^2\frac{2\lambda^2 M}{\pi\rho c_l^2}\,\frac{\xi_0\sqrt{1-\xi_0^2}}{f'(\xi_0)} \times \frac{B(T)c_t\xi_0/v_T}{\left[1 + (2d/a)\left(e^{2\pi N_0/MT} - 1\right)\right]^2}, \quad (12)$$

where $B(T) = \int_0^\infty (f^B)'_x dx/\sqrt{x} < 0$, $a = \epsilon/Me^2$, and N_0 is the equilibrium exciton density. In the opposite case $c_t\xi_0/v_T \gg 1$, we find

$$\frac{\Delta c}{c} = k\frac{2\lambda^2}{\rho c_l^2}\,\frac{\xi_0\sqrt{1-\xi_0^2}}{f'(\xi_0)}\,\frac{N_0 v_T^2}{2T(c_t\xi_0)^2},$$
$$\Gamma = k^2\frac{2\lambda^2 M}{\rho c_l^2\sqrt{\pi}}\,\frac{\xi_0\sqrt{1-\xi_0^2}}{f'(\xi_0)}\,\frac{c_t\xi_0}{v_T} \times \left(1 - e^{-2\pi N_0/MT}\right)e^{-(c_t\xi_0/v_T)^2}. \quad (13)$$

We discuss these results in the last section.

4. $T < T_c$

In this section, we consider the response function $S_{k\omega}$ in the presence of a Bose condensate. It is known that the elementary excitations of a Bose-condensed system are Bogoliubov quasiparticles. An explicit form of the dispersion law of Bogoliubov excitations depends on the model used to describe the interacting exciton system. In the case of a small exciton density $N_0 a_B^2 \ll 1$, where a_B is the Bohr exciton radius, an appropriate theoretical model is the Bogoluibov model of a weakly interacting Bose gas. In the framework of this model, the dispersion law of elementary excitations has the form

$$\varepsilon_k = \sqrt{\frac{k^2}{2M}\left(\frac{k^2}{2M} + 2g_0 n_c\right)},$$

where n_c is exciton density in the condensate. In the long-wavelength limit $k^2/2M \ll 2g_0 n_c$, elementary excitations are the sound quanta $\varepsilon_k \approx sk$, where $s = \sqrt{g_0 n_c/M}$ is the sound velocity. In a Bose-condensed state, most of the excitons are in the condensate, but there are also noncondensate particles, due to both the interaction and a finite temperature (thermal-excited particles). These three fractions can contribute to the response function $S_{k\omega}$. We consider the quantum limit $T \ll sk$ when quantum effects are the most important ones in the function of the system response to external excitation. In the quantum regime $T \ll sk$, thermal excitations are not important, and the theory can be developed for $T = 0$. This is the case we consider here. Due to the weak interaction between excitons, the density of noncondensate particles is sufficiently low enough, and we can neglect the interaction between fluctuations of the condensate and noncondensate densities. Thus, the response of condensate and noncondensate particles can be calculated independently, $S_{k\omega} = S^c_{k\omega} + S^n_{k\omega}$, where $S^c_{k\omega}$ and $S^n_{k\omega}$ are the respective response functions of condensate and noncondensate particles.

The response of condensate particles can be found using the Gross–Pitaevskii equation

$$i\partial_t \Psi(\mathbf{r},t) = \left(\mathbf{p}^2/2M - \mu + g_0|\Psi(\mathbf{r},t)|^2\right)\Psi(\mathbf{r},t) + W(\mathbf{r},t)\Psi(\mathbf{r},t). \quad (14)$$

The SAW deformation field $W(\mathbf{r},t)$ is treated here as a perturbation. Thus, the wave function of condensate particles $\Psi(\mathbf{r},t)$ is split into a stationary uniform value and a perturbed contribution, $\Psi(\mathbf{r},t) = \sqrt{n_c} + \psi(\mathbf{r},t)$. The response function of the condensate excitons is defined as $\delta n_c(k,\omega) = S^c_{k\omega} W_{k\omega}$, where $\delta n_c(k,\omega) = \sqrt{n_c}(\psi^*(\mathbf{r},t) + \psi(\mathbf{r},t))$ is a perturbation of the condensate particle density. Linearizing (14), we find

$$S^c_{k\omega} = \frac{n_c k^2/M}{(\omega + i\delta)^2 - \varepsilon_k^2}. \quad (15)$$

The calculation of the response function of noncondensate particles is more cumbersome [12, 13], and we present the result

$$S^n_{k\omega} = -\frac{g^2 n_c^2}{2(2\pi)^2}\int \frac{d\mathbf{p}}{\epsilon_{\mathbf{p}+\mathbf{k}}\epsilon_{\mathbf{p}}} \times \left[\frac{1}{i\omega_n + \epsilon_{\mathbf{p}+\mathbf{k}} + \epsilon_{\mathbf{p}}} - \frac{1}{i\omega_n - \epsilon_{\mathbf{p}+\mathbf{k}} - \epsilon_{\mathbf{p}}}\right]. \quad (16)$$

Calculating this integral at the zero temperature, we find

$$S^n_{k\omega} = -\frac{g^2 n_c^2}{4s^2}\left[\frac{\theta(s^2k^2 - \omega^2)}{\sqrt{s^2k^2 - \omega^2}} + i\frac{\theta(\omega^2 - s^2k^2)}{\sqrt{\omega^2 - s^2k^2}}\right]. \quad (17)$$

Substituting (15) and (17) in (8), we obtain the sound velocity renormalization and attenuation coefficient of the SAW in the presence of an exciton BEC:

$$\begin{aligned}\frac{\Delta c}{c} &= \frac{2\lambda^2\xi_0\sqrt{1-\xi_0^2}}{f'(\xi_0)\rho c_l^2} \times \\ &\times \left[\frac{n_c k/M}{c_t^2\xi_0^2 - s^2} - \frac{g^2 n_c^2 \theta(s^2 - c_t^2\xi_0^2)}{4s^2\sqrt{s^2 - c_t^2\xi_0^2}}\right], \\ \Gamma &= k\frac{4\lambda^2\xi_0\sqrt{1-\xi_0^2}}{f'(\xi_0)\rho c_l^2} \times \\ &\times \left[\frac{\pi n_c k}{2Ms}\delta(c_t\xi_0 - s) + \frac{g^2 n_c^2 \theta(c_t^2\xi_0^2 - s^2)}{4s^2\sqrt{c_t^2\xi_0^2 - s^2}}\right].\end{aligned} \quad (18)$$

Here, the first and second terms are respectively the condensate and noncondensate contributions, $\theta(x)$ is the Heaviside step function.

5. SAW–EXCITON INTERACTION VIA PIEZOELECTRIC COUPLING

To study the piezoelectric coupling, we must take the anisotropy of the substrate crystal lattice into account. Such an approach results in very cumbersome calculations and equations. To simplify the theoretical analysis, we follow paper [10] and consider the mechanical motion of the substrate as the motion of an isotropic medium and include anisotropy into the piezoelectric terms of the equations of motion. We assume that the substrate is made of a cubic crystal and the SAW travels along the piezo-active direction [110] (the x axis in Fig. 1) on the [001] surface (the $z = 0$ plane in Fig. 1). In this geometry, the motion of the medium and the electric field satisfy the equations

$$\begin{aligned}&(\omega^2 - c_l^2k^2)u_x + c_t^2u_x'' + (c_l^2 - c_t^2)iku_z' - 2i\beta k\phi'/\rho = 0, \\ &(\omega^2 - c_t^2k^2)u_z + c_l^2u_z'' + (c_l^2 - c_t^2)iku_x' + k^2\beta\phi/\rho = 0, \\ &\epsilon(z)(\phi'' - k^2\phi) + 8\pi\beta(iku_x' - u_zk^2/2) = 0,\end{aligned} \quad (19)$$

where the prime means the derivative with respect to z. Equations of motion (19) must be supplemented with boundary conditions for displacement vector components:

$$\begin{aligned}&\rho c_t^2(u_x' + iku_z) - i\beta k\phi = 0, \\ &(c_l^2 - 2c_t^2)iku_x + c_l^2u_z' = 0.\end{aligned} \quad (20)$$

Moreover, the Poisson equation in system (19) needs boundary conditions for the electric induction vector $\mathbf{D}$ and the electric potential ϕ. From the electrostatic standpoint, the exciton layer can be viewed as an electric double layer. To apply this model, conditions $kd \ll 1$ and $k/\kappa_{t,l} \ll 1$ must be satisfied. The boundary conditions for the double electric layer at the point $z = 0$ have the form

$$\begin{aligned} \phi'(+0) - \varepsilon\phi'(-0) = 4\pi\beta iku_x(-0),\\ \phi(+0) - \phi(-0) = 4\pi p n_{k\omega}, \end{aligned} \tag{21}$$

where $p = ed$ is the absolute exciton dipole moment value and $n_{k\omega} = S_{k\omega}[-pE_z(-0)]$ is the exciton density fluctuation caused by the piezoelectric field $E_z(-0) = (4\pi\beta/\varepsilon)iku_x(-0)$.

Solving Eqs. (19) in general is not a simple problem. To solve them, we use the fact that the piezoeffect is a small perturbation (the mathematical criterion is given below), and hence the β-dependent terms in Eqs. (19) can be considered a perturbation. The solutions of Eqs. (19) can be represented in the form

$$\begin{aligned} u_x(z) = u_x^0(z) + \delta u_x(z),\\ u_z(z) = u_z^0(z) + \delta u_z(z),\\ \phi(z) = \phi^0(z) + \delta\phi(z), \end{aligned} \tag{22}$$

where the unperturbed functions $u_x^0(z)$ and $u_z^0(z)$ are given by Eq. (2) and the unperturbed potential is

$$\begin{aligned} \phi^0(z) = Ce^{-kz}, \quad z > 0,\\ \phi^0(z) = De^{kz}, \quad z < 0. \end{aligned} \tag{23}$$

The corrections to the unperturbed solutions can be found from Eqs. (19) in the first order in β:

$$\begin{aligned} \delta\phi(z) = xAe^{\kappa_t z} + yBe^{\kappa_l z},\\ \delta u_x(z) = \eta De^{kz},\\ \delta u_y(z) = \xi De^{kz}, \end{aligned} \tag{24}$$

where the coefficients are given by

$$\begin{gathered} \begin{pmatrix} \eta \\ \xi \end{pmatrix} = \frac{\beta k^2}{\rho\omega^4} \begin{pmatrix} 2i\omega^2 + 3i(c_l^2 - c_t^2)k^2 \\ -\omega^2 + 3(c_l^2 - c_t^2)k^2 \end{pmatrix}, \\ \begin{pmatrix} x \\ y \end{pmatrix} = \frac{4\pi\beta}{\varepsilon} ik \begin{pmatrix} \dfrac{2\kappa_t^2 + k^2}{k^2 - \kappa_t^2} \\ \dfrac{3k\kappa_l}{k^2 - \kappa_l^2} \end{pmatrix}. \end{gathered} \tag{25}$$

Substituting Eq. (22) in boundary conditions (20) and (21), we arrive at the dispersion equation

$$f_{k\omega} = -\frac{4\pi p^2}{\varepsilon}\gamma k S_{k\omega} L_{k\omega}, \tag{26}$$

where $\gamma = 4\pi\beta^2/(1+\varepsilon)\rho c_t^2$ is the electromechanical coupling coefficient and

$$\begin{aligned} L_{k\omega} = \kappa_t(\kappa_t^2 - k^2)\left[1 - \frac{c_t^2k^2}{\omega^2}\left(1 + \frac{6k^2(c_l^2 - c_t^2)}{\omega^2}\right)\right] + \\ + \frac{c_l^2k^2}{\omega^2}k\left[\frac{\omega^2}{c_l^2} + (\kappa_t - \kappa_l)^2\right] \times \\ \times \left[3 - 4\frac{c_t^2}{c_l^2}\left(1 + \frac{3k^2(c_l^2 - c_t^2)}{2\omega^2}\right)\right]. \end{aligned} \tag{27}$$

In the right side of Eq. (26), we keep only the excitonic contribution and neglect the terms due to the piezoeffect in the absence of the excitonic gas.

The calculation of the velocity renormalization and attenuation coefficient is similar to the procedure described above. We present the results. For $c_t\xi_0 \ll v_T$, we have

$$\begin{aligned} \frac{\Delta c}{c} = \gamma k\frac{2Mp^2}{\varepsilon}\frac{L(\xi_0)}{\xi_0 f'(\xi_0)} \times \\ \times \frac{e^{2\pi N_0/MT} - 1}{\left[1 + (2d/a)\left(e^{2\pi N_0/MT} - 1\right)\right]^2}, \\ \Gamma = \gamma k^2\frac{4Mp^2}{\varepsilon}\frac{L(\xi_0)}{\xi_0 f'(\xi_0)} \times \\ \times \frac{B(T)c_t\xi_0/v_T}{\left[1 + (2d/a)\left(e^{2\pi N_0/MT} - 1\right)\right]^2}, \end{aligned} \tag{28}$$

and in the opposite case $c_t\xi_0 \gg v_T$,

$$\begin{aligned} \frac{\Delta c}{c} = -\gamma k\frac{4\pi p^2}{\varepsilon}\frac{L(\xi_0)}{\xi_0 f'(\xi_0)}\frac{N_0 v_T^2}{2T(c_t\xi_0)^2}, \\ \Gamma = -\gamma k^2\frac{4Mp^2}{\varepsilon\sqrt{\pi}}\frac{L(\xi_0)}{\xi_0 f'(\xi_0)}\frac{c_t\xi_0}{v_T} \times \\ \times \left(1 - e^{-2\pi N_0/MT}\right)e^{-(c_t\xi_0/v_T)^2}, \end{aligned} \tag{29}$$

where $L(\xi_0) < 0$.

To find the SAW attenuation and velocity renormalization in the presence of the excitonic BEC, we use response functions (15) and (17). Simple calculations yield

$$\begin{aligned} \frac{\Delta c}{c} = -\gamma\frac{4\pi p^2 L(\xi_0)}{\varepsilon\xi_0 f'(\xi_0)} \times \\ \times \left[\frac{n_c k/M}{c_t^2\xi_0^2 - s^2} - \frac{g^2n_c^2\theta(s^2 - c_t^2\xi_0^2)}{4s^2\sqrt{s^2 - c_t^2\xi_0^2}}\right], \\ \Gamma = -\gamma k\frac{8\pi p^2 L(\xi_0)}{\varepsilon\xi_0 f'(\xi_0)} \times \\ \times \left[\frac{\pi n_c k}{2Ms}\delta(c_t\xi_0 - s) + \frac{g^2n_c^2\theta(c_t^2\xi_0^2 - s^2)}{4s^2\sqrt{c_t^2\xi_0^2 - s^2}}\right]. \end{aligned} \tag{30}$$

6. DISCUSSION

We considered the SAW–exciton-gas interaction at a temperature both above and below the exciton gas condensation temperature T_c. It is shown that above T_c, the SAW absorbtion coefficient is a monotonic function of the exciton density for both deformation and piezoelectric interaction mechanisms, Eqs. (12), (13) and (28), (29). In the presence of the exciton condensate at zero temperature, the absorption coefficient also has a step-like dependence on the exciton density for both deformation and piezoelectric interaction mechanisms. Indeed, it is well known that the imaginary part of polarization operators $S^c_{k\omega}$ and $S^n_{k\omega}$ describes the absorption of an external perturbation in the system, in our case, the SAW absorption. We see from Eq. (15) that $\mathrm{Im}\, S^c_{k\omega} \propto \delta(\omega^2 - \epsilon_k^2)$; in other words, the perturbation damping is due to the direct transformation of the SAW phonon $\omega = c_t \xi k$ into a Bogoliubov excitation ϵ_k. In our case, $\epsilon_k \approx sk$. This is a well-known wave transformation mechanism (see Fig. 2). The noncondensate particle contribution to the damping is given by the imaginary part of $S^n_{k\omega}$ in Eq. (17). The microscopic origin of this decay is due to the transformation of quantum ω into a pair of excitations $\omega = \epsilon_{\mathbf{p}+\mathbf{k}} + \epsilon_{\mathbf{p}}$, the Beliaev mechanism (see Fig. 3). This mechanism produces the SAW damping (at zero temperatures) with threshold-like behavior. Such behavior can be understood from the general equation (16). At the zero temperatures, the Beliaev mechanism gives the condition $\omega = s|\mathbf{p} + \mathbf{k}| + sp$. Simple analysis shows that this equation has a solution only if $|\omega| > sk$ at any values of $\mathbf{p}$. In our case, $\omega = c_t \xi k$ is a SAW phonon; hence, we conclude that SAW absorption occurs at $c_t \xi > s$. The Bogoliubov quasiparticle velocity s depends on the exciton concentration of condensate particles n_c via the relation $s = \sqrt{g n_c / M}$. Thus, the inequality $c_t \xi_0 > s$ is equivalent to $n_c < n_c^0$, where the critical exciton density is given by $n_c^0 = M(c_t \xi_0)^2/g$, and the attenuation coefficient Γ has a step-like dependence on the exciton density, $\Gamma \propto \theta(n_c^0 - n_c)$. We can conclude that a SAW travels through the system without dissipation if the exciton density of condensate particles is less than some critical value n_c^0.

We estimate the effect at $T > T_c$. The typical experimental values of the critical temperature are 3–5 K. Using the data $N_0 = 10^{10}$ cm^{-2}, $d = 0.5 \cdot 10^{-7}$ cm, $c_t = 3.35 \cdot 10^5$ cm/s, $c_l = 4.75 \cdot 10^5$ cm/s, $|\lambda| \sim 10$ eV, $M = 0.5 \cdot 10^{-28}$ g, and $T = 10$ K, we have $\Gamma \sim 1.4$ cm^{-1} for the deformation potential mechanism and $\Gamma \sim 2.78 \cdot 10^{-3}$ cm^{-1} for the piezoelectric mechanism. We can see that SAW absorption is less effective in the case of the exciton–SAW interaction via the piezoelectric mechanism. This is due to a small value of the distance d in comparison with the SAW wavelength, $\lambda \gg d$. We now estimate the absorption at $T = 0$. We confine ourselves to the deformation potential model because the deformation mechanism is more effective. Equation (18) can be rewritten in the form

$$\Gamma = \Gamma_n \delta(1 - s/c_t\xi_0) + \Gamma_c \theta(c_t^2 \xi_0^2 - s^2). \qquad (31)$$

Using the data given above, we obtain $\Gamma_n \sim 92$ cm^{-1} and $\Gamma_c \sim 0.6$ cm^{-1}.

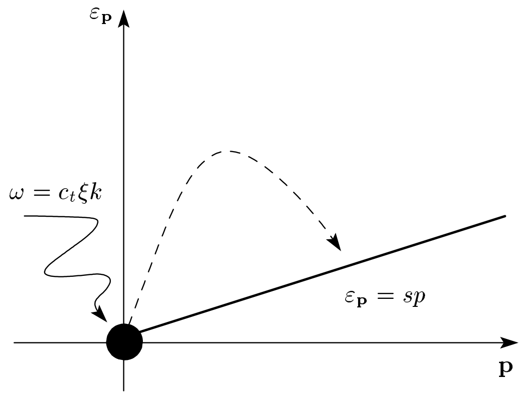

Fig. 2. Quasiparticle transitions under the wave-transformation damping mechanism. The wavy line represents the SAW phonon, the solid dot is a Bose–Einstein condensate at zero momentum, and the straight line is a linear domain of the Bogoliubov quasiparticle dispersion relation

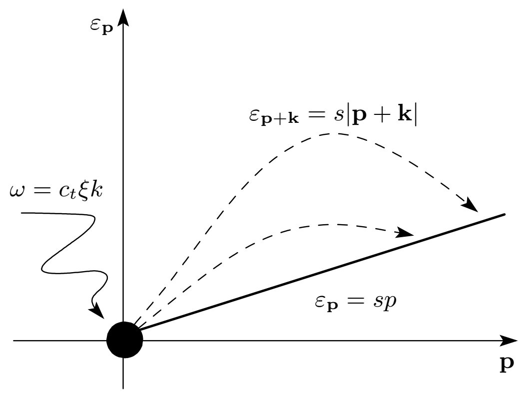

Fig. 3. Diagram of the quasiparticle transitions under the Beliaev damping mechanism. Labels are the same as in Fig. 2

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