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RELATING QUANTUM DISCORD WITH THE QUANTUM DENSE CODING CAPACITY

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We establish the relations between quantum discord and the quantum dense coding capacity in $(n+1)$-particle quantum states. A necessary condition for the vanishing discord monogamy score is given. We also find that the loss of quantum dense coding capacity due to decoherence is bounded below by the sum of quantum discord. When these results are restricted to three-particle quantum states, some complementarity relations are obtained.

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1. INTRODUCTION

The protocols of quantum dense coding [1], quantum teleportation [2], and quantum key distribution [3] are viewed as the beginning of discoveries of quantum communication strategies. These protocols can be effectively used to transmit classical or quantum information in a way that cannot be realized with their classical counterparts. Thus, they have created a very substantial change in the attitude to modern communication schemes. Such protocols are initially introduced for a single sender and a single receiver, and have been realized experimentally in several physical systems such as photons, trapped ions, atoms in optical lattices, nuclear magnetic resonance, etc. [4–9]. However, fruitful applications and commercialization of these protocols require the implementations of these protocols in a multipartite scenario [10]. For example, quantum dense coding, which is used to transmit classical information, has already been introduced in multipartite systems [11, 12].

Quantum correlations occupy a central position in the quest for understanding and harnessing the power of quantum mechanics. Previously, entanglement has been successfully employed to interpret several phenomena that cannot be understood within classical physics [13]. It has also been identified as the vital element for the success of quantum communication protocols [10] and the essential ingredient of quantum computational tasks [14]. Therefore, entanglement is regarded as a unique quantum mechanics trait and considered synonymous with quantum correlations. Conversely, several recent studies have found that separable (i. e., not entangled) states may retain some signatures of quantumness with potential applications to quantum technology [15–20]. Quantum discord [21, 22] is one of these signatures. The dynamics of quantum discord [23–31] and its physical meaning [32, 33] are extensively studied. Experiments on quantum discord are also implemented [34, 35]. Recently, generalization of quantum discord to multipartite systems has received much attention [36, 37, 38].

To answer the question of whether quantum discord is merely a mathematical construct or has a definable physical role in information processing, the link between quantum discord and actual quantum tasks has been investigated [32, 33, 39–42]. An operational meaning of geometric quantum discord is given in terms of teleportation fidelity [40]. For three-qubit pure states, a complementarity relation is established between the capacity of multiport classical information transmission via quantum states and multiparty quantum correlation measures [41]. Inspired by the question, we relate quantum discord to the quantum dense coding capacity in this paper. Moreover, the understanding of quan-

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tum discord of multipartite systems, i. e., systems of more than two particles, is still limited, due to their structural complexity. Therefore, we consider the relation between quantum discord and quantum dense coding capacity for $(n+1)$-particle quantum states. In the scenario of a single sender and n receivers, we establish a necessary condition of a vanishing discord monogamy score based on the quantum dense coding capacity. Furthermore, in the same and the contrary scenarios, the relations between quantum discord and the loss of quantum dense coding capacity due to decoherence are given. The contrary scenario means that there are n senders and only a single receiver.

The paper is organized as follows. We begin with reviews of quantum dense coding capacity and the definition of quantum discord in Sec. 2. In Sec. 3, we give a necessary condition for the vanishing discord monogamy score. In Sec. 4, we establish the relation between quantum discord and the loss of quantum dense coding capacity due to decoherence. We present a conclusion in Sec. 5.

2. QUANTUM DENSE CODING CAPACITY AND QUANTUM DISCORD

Quantum dense coding is a quantum communication protocol by which classical information can be transmitted beyond the classical capacity of a quantum channel. The quantum channel together with a shared quantum state is the available resources for the transmission. Let the sender, called Alice, and the receiver, called Bob, share a bipartite quantum state ρ_{AB}. The amount of classical information that the sender can send to the receiver is given by [11, 12, 43–47]

$$C(A,B) \equiv C(\rho_{AB}) = \log_2 d_A + S(\rho_B) - S(\rho_{AB}), \quad (1)$$

where d_A is the dimension of Alice's Hilbert space, $\rho_B = \mathrm{Tr}_A(\rho_{AB})$, and $S(\rho) = -\mathrm{Tr}\,\rho\log_2\rho$ is the von Neumann entropy of its argument. The conditional entropy $S(\rho_{A|B}) = S(\rho_{AB}) - S(\rho_B)$ in the equation can have any sign. In the case where the conditional entropy is negative, the sender can transmit classical information beyond the "classical limit", i. e., $\log_2 d_A$, bits to the receiver. For example, when a maximally entangled state is shared between Alice and Bob, the capacity $C(A,B)$ reaches the maximal value. On the contrary, in the case where the conditional entropy is positive, the sender must use a noiseless quantum channel without using a shared quantum state, which is usually referred to as the "classical protocol", to transfer $\log_2 d_A$ bits of classical information.

We now pass to a brief review of the definition of quantum discord. Quantum discord, defined as the minimum difference between two expressions of mutual information extended from a classical to a quantum system, is introduced to characterize all the nonclassical correlations presented in a bipartite system [21, 22]. The von Neumann mutual information $\mathcal{I}$ for a bipartite system is given as

$$\mathcal{I}(A,B) \equiv \mathcal{I}(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}). \quad (2)$$

The mutual information is used to quantify the total correlations.

Conditioned on a complete set of von Neumann measurement Π_i^B (or, more generally, positive-operator valued measures (POVMs)) performed on subsystem B, the alternative version of quantum mutual information is

$$\mathcal{J}(A,B) \equiv \mathcal{J}(\rho_{AB}) = S(\rho_A) - \tilde{S}_{\{\Pi_i^B\}}(\rho_{A|B}) = \\ = S(\rho_A) - \min_{\{\Pi_i^B\}} p_i S(\rho_{A|i}). \quad (3)$$

In the equation, the probability of outcome i is

$$p_i = \mathrm{Tr}_{AB}(I_A \otimes \Pi_i^B \rho_{AB} I_A \otimes \Pi_i^B),$$

and the corresponding post-measurement state for the subsystem A is

$$\rho_{A|i} = \mathrm{Tr}_B(I_A \otimes \Pi_i^B \rho_{AB} I_A \otimes \Pi_i^B)/p_i$$

with I_A being the identity operator on the Hilbert space of subsystem A. Generally, $\mathcal{J}(A,B)$ is used to measure the classical correlations in bipartite systems.

Even though the two definitions of mutual information are equivalent for classical systems, their quantum generalizations $\mathcal{I}$ and $\mathcal{J}$ do not coincide in general, and quantum discord is defined as their discrepancy

$$\mathcal{D}(A,B) \equiv \mathcal{D}(\rho_{AB}) = \mathcal{I}(\rho_{AB}) - \mathcal{J}(\rho_{AB}). \quad (4)$$

Quantum discord measures the quantum nature of correlations between two subsystems, and it is always nonnegative. Moreover, quantum discord is in general asymmetric with respect to A and B.

In the subsequent sections, we use $S(A,B)$ to denote $S(\rho_{AB})$, and similarly for other quantities.

3. NECESSARY CONDITION FOR THE VANISHING DISCORD MONOGAMY SCORE

In multipartite quantum states, the sharing of quantum correlations among subsystems is often constrained by the concept of monogamy. More precisely, a bipartite quantum correlation measure $\mathcal{Q}$ is said to be monogamous for a $(n+1)$-particle state $\rho_{AB_1B_2\ldots B_n}$ if

$$\mathcal{Q}(\rho_{A|B_1B_2\ldots B_n}) \geq \mathcal{Q}(\rho_{AB_1}) + \\ + \mathcal{Q}(\rho_{AB_2}) + \ldots + \mathcal{Q}(\rho_{AB_n}). \quad (5)$$

Here, A is used as the "nodal observer",

$$\mathcal{Q}(\rho_{AB_1}) = \mathcal{Q}(\mathrm{Tr}_{B_2\ldots B_n}(\rho_{AB_1B_2\ldots B_n}))$$

denotes the quantum correlation (with respect to the measure $\mathcal{Q}$) between the subsystems A and B_1, and similarly for others, and $\mathcal{Q}(\rho_{A|B_1B_2\ldots B_n})$ measures quantum correlation of the state in the $A|B_1B_2\ldots B_n$ bipartite split. When entanglement is quantified by concurrence, such a relation is indeed satisfied, which indicates that two parties cannot have a large amount of entanglement shared with the third party if they are highly entangled [48–51]. As regards quantum discord, Bai et al. [52] proved that the monogamy relation is only satisfied for three-qubit pure states.

The concept of quantum monogamy score, which is independent of whether the given bipartite quantum correlation measure is monogamous, is defined as

$$\delta_{\mathcal{Q}} \equiv \mathcal{Q}(A|B_1B_2\ldots B_n) - \mathcal{Q}(A,B_1) - \\ - \mathcal{Q}(A,B_2) - \ldots - \mathcal{Q}(A,B_n).$$

For quantum discord, the discord monogamy score is given as

$$\delta_{\mathcal{D}} = \mathcal{D}(A|B_1B_2\ldots B_n) - \mathcal{D}(A,B_1) - \\ - \mathcal{D}(A,B_2) - \ldots - \mathcal{D}(A,B_n). \quad (6)$$

Now, we present a condition for a vanishing discord monogamy score based on the quantum dense coding capacity. We consider a pure or mixed $(n+1)$-particle state $\rho_{AB_1B_2\ldots B_n}$ in which the particles can have arbitrary dimensions; a necessary condition for the discord monogamy score to vanish is

$$\mathcal{D}(A|B_1B_2\ldots B_n) + \mathcal{J}(A,B_1) + \\ + \mathcal{J}(A,B_2) + \ldots + \mathcal{J}(A,B_n) \leq \\ \leq C(A,B_1) + C(A,B_2) + \ldots + C(A,B_n). \quad (7)$$

The condition can be obtained easily. Based on the definition of quantum discord

$$\mathcal{D}(A,B_i) = \mathcal{I}(A,B_i) - \mathcal{J}(A,B_i) = \\ = S(A) + S(B_i) - S(A,B_i) - \mathcal{J}(A,B_i),$$

a vanishing discord monogamy score implies that

$$\mathcal{D}(A|B_1B_2\ldots B_n) + \mathcal{J}(A,B_1) + \\ + \mathcal{J}(A,B_2) + \ldots + \mathcal{J}(A,B_n) = \\ = S(A) + S(B_1) - S(A,B_1) + S(A) + S(B_2) - \\ - S(A,B_2) + \ldots + S(A) + S(B_n) - S(A,B_n), \quad (8)$$

where we note that $S(A) \leq \log_2 d_A$, and substitute it into the above equation. From the expression for quantum dense coding capacity in Eq. (1), we then obtain the required condition.

This result is not only a necessary condition for the vanishing discord monogamy score but also indicates that the quantum discord between a single sender and the whole n receivers together with the total classical correlations between the sender and each receiver are bounded above by the sum of quantum dense coding capacities between the sender and each receiver.

Because the definitions of quantum discord and quantum dense coding capacity are suitable for bipartite states in arbitrary dimensions, it is worth noting that the condition of a vanishing discord monogamy score in Eq. (7) is independent of the dimensions of the particles involved. Actually, the results that we obtain here and in the subsequent sections work for particles of arbitrary dimensions.

In the particular case of three-particle states $\rho_{AB_1B_2}$, the condition reduces to

$$\mathcal{D}(A|B_1B_2) + \mathcal{J}(A,B_1) + \mathcal{J}(A,B_2) \leq 2\log_2 d_A. \quad (9)$$

To obtain the result, we note that

$$C(A,B_1) + C(A,B_2) = 2\log_2 d_A + S(B_1) - \\ - S(A,B_1) + S(B_2) - S(A,B_2)$$

and

$$S(B_1) - S(A,B_1) + S(B_2) - S(A,B_2) \leq 0$$

according to the strong subadditivity of von Neumann entropy [53]. Equation (9) is the necessary condition of a vanishing discord monogamy score for three-particle quantum states. Moreover, the complementarity relation established above clearly indicates that much more total classical correlations between the sender and each receiver decrease the quantum discord between the sender and all the receivers.

4. QUANTUM DISCORD BEING A LOWER BOUND OF THE LOSS OF QUANTUM DENSE CODING CAPACITY

In practice, implementation of a quantum information protocol is inevitably affected by loss and noise, and we consider the case where decoherence occurs only at the receiver's end. Physically, the environmental decoherence could be emulated by a particular quantum operation for which there is a unitary coupling between the receiver's qubit B_i and an ancillary environment system R_i, and then R_i is traced out.

First of all, the size of the Hilbert space should be expanded in order to model quantum measurement (or an other quantum operation) by coupling to the ancillary subsystem and then discarding it. The ancilla R_i is initially assumed in a pure state $|0\rangle_i$, while there is a unitary interaction U_i between B_i and R_i. We let primes denote the state of the system after U_i is applied. Because R_i acts on a product state with AB_i, we have $S(A,B_i) = S(A',B_i'R_i')$ and $\mathcal{I}(A,B_iR_i) = \mathcal{I}(A',B_i'R_i')$. Because the mutual information cannot increase by discarding the ancillary system, we obtain that $\mathcal{I}(A',B_i') \le \mathcal{I}(A',B_i'R_i')$.

We now consider quantum dense coding with a single sender and n receivers in an $(n+1)$-particle state in the presence of the R_i. The yield of quantum dense coding on system $AB_1\ldots B_n$ is the same as that of quantum dense coding on system $AB_1\ldots B_nR_1\ldots R_n$, in which B_i interacts coherently with R_i through a unitary interaction U_i. Here, each environment R_i is initially prepared in a pure state. Discarding the ancillary system leads to $\mathcal{I}(A',B_i') \le \mathcal{I}(A',B_i'R_i')$. At the same time, $\mathcal{I}(A',B_i'R_i') = \mathcal{I}(A,B_iR_i) = \mathcal{I}(A,B_i)$. Hence, $\mathcal{I}(A',B_i') \le \mathcal{I}(A,B_i)$, which indicates that $S(A'|B_i') \ge S(A|B_i)$.

With and without decoherence, the quantum dense coding capacity between the sender and the ith receiver is respectively expressed as $C(A,B_i)$ and $C(A',B_i')$. The difference between them is

$$D(A,B_i) = C(A,B_i) - C(A',B_i') = \\ = S(B_i) - S(A,B_i) - (S(B_i') - S(A',B_i')) = \\ = S(A'|B_i') - S(A|B_i).$$

Obviously, D quantifies the loss of quantum dense coding capacity due to environmental decoherence.

We now minimize D over all environmental operations by performing measurements [39, 42]. Based on the measurement model of quantum operations [54], the state ρ_{AB_i} changes to $\rho'_{AB_i} = \sum_j p_j\rho_{A|j}\otimes\Pi_j$ under measurement of subsystem B_i, where $\{\Pi_j\}$ are orthogonal projectors resulting from a Neumark extension of the POVM elements [55]. Therefore, we can obtain

$$S(A',B_i') = S(\rho'_{AB_i}) = S\left(\sum_j p_j\rho_{A|j}\otimes\Pi_j\right) = \\ = H(p_j) + \sum_j p_jS(\rho_{A|j}) \quad (10)$$

and

$$S(B_i') = S(\rho'_{B_i}) = S\left(\sum_j p_j\Pi_j\right) = H(p_j), \quad (11)$$

where $H(p_j) = -\sum_j p_j\log_2 p_j$. In obtaining the above equation, we note that the unconditioned post-measurement states of A and B_i are respectively given by

$$\rho'_A = \sum_j p_j\rho_{A|j} \quad \text{and} \quad \rho'_{B_i} = \sum_j p_j\Pi_j.$$

Combining Eqs. (10) and (11), we obtain

$$S(A'|B_i') = \sum_j p_jS(\rho_{A|j}). \quad (12)$$

Subsequently, $D(A,B_i)$ reduces to $\mathcal{D}(A,B_i)$, which is the quantum discord of system AB_i, by minimizing over all POVMs. Therefore, we can conclude that quantum discord quantifies the minimal loss in quantum dense coding due to decoherence, and we have

$$C(A,B_i) - C(A',B_i') \ge \mathcal{D}(A,B_i). \quad (13)$$

Applying the above equation to an quantum dense coding protocol with an $(n+1)$-particle quantum state $\rho_{AB_1B_2\ldots B_n}$, with one sender and n receivers, we obtain the relation between quantum discord and dense coding capacity

$$\sum_i C(A,B_i) - \sum_i C(A',B_i') \ge \sum_i \mathcal{D}(A,B_i). \quad (14)$$

The above equation indicates that due to the decoherence at the receivers' end, the loss of the sum of quantum dense coding capacities is not less than the sum of quantum discords between the sender and each receiver.

We consider a special case of a three-particle quantum state. We then obtain a much simpler result:

$$C(A,B_1) + C(A,B_2) \ge C(A',B_1') + C(A',B_2') + \\ + \mathcal{D}(A,B_1) + \mathcal{D}(A,B_2).$$

Because $C(A,B_1) + C(A,B_2) \le 2\log_2 d_A$ [53], we obtain

$$C(A',B_1') + C(A',B_2') + \mathcal{D}(A,B_1) + \mathcal{D}(A,B_2) \le \\ \le 2\log_2 d_A. \quad (15)$$

The complementarity relation indicates that a much more total quantum discord between the sender and each receiver decreases the sum of quantum dense coding capacities after the effect of decoherence.

Similarly, in the opposite case where there are n receivers and a single sender in an $(n+1)$-particle quantum state $\rho_{A_1 A_2 \ldots A_n B}$, according to the same procedure as that used in obtaining Eq. (14), we can obtain

$$\sum_i C(A_i, B) - \sum_i C(A_i', B') \geq \sum_i \mathcal{D}(A_i, B), \quad (16)$$

where we still assume that particle B distributed to the receiver is affected by the environment. From the equation, we note again that the loss of the sum of quantum dense coding capacities is bounded below by the sum of quantum discord between each sender and the single receiver. In particular, for the three-particle quantum state $\rho_{A_1 A_2 B}$, in which A_1 and A_2 belong to the senders and B belongs to receiver, the above relation can be simplified to

$$\begin{aligned} C(A_1', B') + C(A_2', B') + \mathcal{D}(A_1, B) + \mathcal{D}(A_2, B) \leq \\ \leq C(A_1, B) + C(A_2, B) \leq C(A_1 A_2 : B), \quad (17) \end{aligned}$$

where $C(A_1 A_2 : B)$ denotes the quantum dense coding capacity of $A_1 A_2$ to B, and

$$C(A_1 A_2 : B) = \log_2(d_{A_1} d_{A_2}) + S(B) - S(A_1, A_2).$$

The strong subadditivity of the von Neumann entropy

$$\begin{aligned} S(B) - S(A_1, B) + S(B) - S(A_2, B) \leq \\ \leq S(B) - S(A_1 A_2, B) \end{aligned}$$

should be used to obtain the second inequality.

5. CONCLUSION

Summarizing, we have established the relations between quantum discord and quantum dense coding capacity in $(n+1)$-particle quantum states, independent of the dimensions of the particles. Especially, a necessary condition for the vanishing discord monogamy score is given. When the result is restricted to three-particle quantum states, a complementarity relation between quantum discord and classical correlation is established. We also find that the loss of the sum of quantum dense coding capacities between the sender and every receiver is always bounded below by the sum of quantum discord in a distributed dense coding protocol with a single sender and n receivers. For the particular three-particle quantum states, the result is reduced to a complementarity relation between quantum discord and the quantum dense coding capacity involving decoherence. A similar result can also be obtained in a distributed dense coding protocol with n senders and a single receiver. In other words, between every sender and a single receiver, the sum of quantum discords is a lower bound of the loss of the sum of quantum dense coding capacities.

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