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IS THE STANDARD MODEL SAVED ASYMPTOTICALLY BY CONFORMAL SYMMETRY?

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It is pointed out that the top-quark and Higgs masses and the Higgs VEV with great accuracy satisfy the relations $4m_H^2 = 2m_T^2 = v^2$, which are very special and reminiscent of analogous ones at Argyres–Douglas points with enhanced conformal symmetry. Furthermore, the RG evolution of the corresponding Higgs self-interaction and Yukawa couplings $\lambda(0) = 1/8$ and $y(0) = 1$ leads to the free-field stable point $\lambda(M_{Pl}) = \dot{\lambda}(M_{Pl}) = 0$ in the pure scalar sector at the Planck scale, also suggesting enhanced conformal symmetry. Thus, it is conceivable that the Standard Model is the low-energy limit of a distinct special theory with (super?) conformal symmetry at the Planck scale. In the context of such a "scenario", one may further speculate that the Higgs particle is the Goldstone boson of (partly) spontaneously broken conformal symmetry. This would simultaneously resolve the hierarchy and Landau pole problems in the scalar sector and would provide a nearly flat potential with two almost degenerate minima at the electroweak and Planck scales.

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1. INTRODUCTION

Scalar theory, unless it is free, suffers from two severe problems: the Moscow zero (Landau pole) problem [1], well established in lattice calculations [2] and constructive field theory [3], and the hierarchy problem. This could cast a dark shadow on the Standard Model (SM), which depends crucially on the scalar Higgs field. The most popular ways to avoid them propose serious modifications of the SM at the TeV regime, either by adding super-partners to known elementary particles or by making some of them composite, or both. However, increasing attention is received recently by an alternative paradigm [4–7], according to which there can be no new physics beyond the SM all the way up to or around the Planck scale, that the above problems of the scalar sector are red herrings, and that the apparent fine-tuning of the Higgs potential is in fact an inescapable consequence of its distinct form in a healthy fundamental theory defined at Planck energies.

The main arguments in favor of this scenario are based on the very special values of the Higgs and top-quark masses m_H and m_T, or, equivalently, of

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the low-energy values of the scalar self-coupling $\lambda(0)$, most relevant to our discussion, the three gauge couplings $\mathbf{g}(0) \equiv (g_1(0), g_2(0), g_3(0))$, and the top Yukawa coupling constant $y(0)$, which in the conventional approaches are considered "accidental coincidences", while in the alternative one, very important evidence. Specifically, this "scenario" builds upon the following *experimental* facts and aims at providing alternative resolutions of the corresponding puzzles:

Fact 1: The values $\lambda(0)$, $\mathbf{g}(0)$, $y(0)$ are fine-tuned such that at the Planck scale, i.e., for $t = \log \mu^2 \sim \sim (0.5\text{–}1) \log M_{Pl}$, we simultaneously have

$$\begin{aligned} \dot{\lambda}(M_{Pl}) &= 0, \\ \lambda(M_{Pl}) &= 0. \end{aligned} \tag{1}$$

Puzzle 1: This seems to suggest that Nature started at the Planck level at a very distinguished point, where λ is stable and vanishing (free scalar theory), and after that the RG evolution, mainly due to the evolution of the gauge couplings, which were not stable at M_{Pl}, brought the scalar field to its present state with a very concrete potential.

Reverting the statement, is the $\lambda\phi^4$ sector of the standard model fine-tuned to be "asymptotically secure", instead of exhibiting unhealthy Landau pole behavior?

Fact 2 (perhaps, related to 1): According to [8, 9]: it seems that the values $\lambda(0)$, $\mathbf{g}(0)$, $y(0)$ are fine-tuned such that the effective potential for the scalar field, in addition to the SM Higgs vacuum expectation value $\langle\phi\rangle = v$, has another local minimum at $\langle\phi\rangle \approx M_{Pl}$ and *nearly degenerate* with the standard one. Perhaps, our minimum at v is even slightly metastable, since the SM parameters may be lying in a very narrow metastability region.

Puzzle 2: Does this form of the effective potential, which seems quite special, suggest something important about the fundamental theory of Nature, or is it just a coincidence?

Facts 3 & 4 (two relations): It is an experimental fact that the Higgs mass, the top-quark mass, and v satisfy the relations

$$4m_H^2 = 2m_T^2 = v^2 \tag{2}$$

with miraculous accuracy, i.e., there seems to be a clear conspiracy between the Higgs, the top-quark, and the W/Z-boson masses. More precisely,

$$\begin{aligned} \frac{\sqrt{2}m_T}{v} &= 0.9956 \pm 0.0044, \\ \frac{\sqrt{2}m_H}{m_T} &= 1.0252 \pm 0.0073. \end{aligned} \tag{3}$$

These are the pole (hence, not running) masses, and the Higgs field vacuum expectation value is defined from the value of the Fermi constant:

$$\begin{aligned} m_H &= 125.66 \pm 0.34 \text{ GeV}, \\ m_T &= 173.34 \pm 0.76 \text{ GeV}, \\ v = \frac{1}{2^{1/4}\sqrt{G_F}} &= 246.21817 \pm 0.00006 \text{ GeV}. \end{aligned} \tag{4}$$

Using (4), we obtain the Yukawa (y) and Higgs self-couplings (λ) as

$$\begin{aligned} y &= 1 \quad \left(m_T = \frac{v}{\sqrt{2}}\right), \\ \lambda &= \frac{1}{8} \quad \left(m_H = \frac{v}{2}\right). \end{aligned} \tag{5}$$

Puzzles 3 & 4: Is it possible that these special values of the couplings and the corresponding mass relations point to some hidden symmetry underlying the SM, which should further enhance a conformal-like symmetry at the Planck scale, as is strongly suggested by (1)? What this symmetry could be? Have we ever before encountered a similar situation? We point out in Sec. 4 that such relations are reminiscent of the Argyres–Douglas point known to exist in certain theories with enhanced symmetry. In that context, such mass relations are consequences of the symmetries of the theory and should be stable under the RG flow. In this connection, the following is a very welcome additional fact.

Fact 5: The difference $\xi = |\frac{1}{8}y^2 - \lambda| < 0.05$ remains small all along the RG-evolution region, and hence the Argyres–Douglas-like relation is RG-stable with relatively satisfactory accuracy.

Puzzle 5: However, this statement is sensitive to the exact value of the top-quark mass (which is so far obtained with good accuracy only by combining the results of four collaborations [10]). Stability of the above difference is pronounced especially well (see Fig. 3 below) if the parameters of the SM are chosen such that relations (1) are *exact*, as is *expected* in the context of an alternative paradigm speculated here. Does this adjustment actually take place when improved by higher-loop corrections and more precise measurements?

Assuming that it does, this choice of the SM parameters leads to another interesting bonus:

Fact 6: For the values of the parameters of the Standard Model that lead to relations (1), the 1-loop effective potential has a second almost degenerate minimum at a field value practically equal to the Planck scale (see Fig. 4 below).

Puzzle 6: Thus, the Planck scale, which is not present in the Lagrangian of the Standard Model, is nevertheless hidden in the actual values of its parameters and the conjectured property (1) of the fundamental theory at M_{Pl}.

All these puzzling facts seem to imply that the parameters of the Standard Model are not at all accidental. Instead, they may be fully determined by an assumption that the Standard Model is a low-energy limit of a very special fundamental theory defined naturally at the Planck scale, which is the next fundamental threshold in particle physics. Moreover, these relations imply that there is some additional symmetry, which underlies the Standard Model and the deeper fundamental theory. This symmetry should automatically protect the vacuum expectation value of the Higgs field (in order to protect relations like (5)) and, hence, solve the hierarchy problem (in the spirit of [11, 12]). Clearly, one could not hope for more, but unfortunately, we cannot be more concrete at this stage.

In the rest of this paper, we elaborate briefly on the above facts and speculate about the nature of a theory in the framework of the less conventional scenario sketched here.

2. RG FLOW TO (OR FROM) A VERY SPECIAL UV POINT

2.1. RG flow in the Standard Model

In Fig. 1, we plot the curves describing the one-loop RG evolution of the five couplings of the Standard Model (they are actually the same as those in [13, 14]). Our notation and initial values of coupling constants coincide with those in [9]. Nowadays, these results are enhanced to include two- and three-loop corrections [8, 9], but these only improve the level of the fine-tuning apparent already at one loop.

We can therefore see that the actual values of $\lambda(0)$ and $y(0)$ in particular are such that there is a region with properties (1) at approximately $\log_{10}\mu \geq 8.3$. With a three-loop accuracy [8, 9] it is shifted only slightly to $\log_{10}\mu \geq 8.5$, with the value of $\lambda(M_{Pl})$ even a bit closer to zero. This means that the one-loop approximation is quite reliable and we can recover the Standard Model at low energies starting from the theory with this very special property at the Planck scale.

Schematically, the one-loop RG equations of the SM have the well-known form (of the three gauge couplings, only $g_3 \equiv g$ is kept, being the most important one):

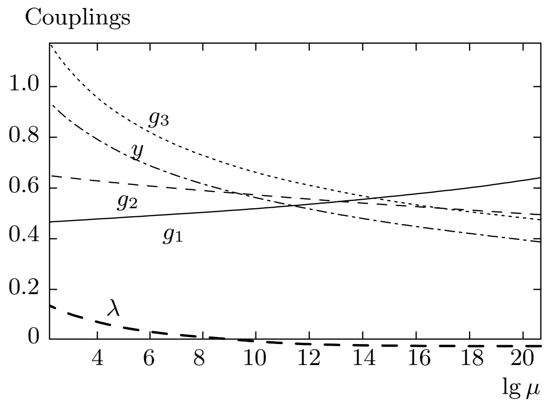

Fig. 1. RG flow of the coupling constants in a one-loop approximation as a function of $\lg\mu$ with the RG scale μ expressed in GeV. Note the well-known fact of a g_1, g_2, and g_3 "unification" at around 10^{15} GeV. Note also that the asymptotic behavior of the Higgs self-coupling is in good agreement with (1), given that the initial low-energy values used are the experimental central values of the couplings

$$\begin{aligned} &\frac{d}{dt}\,g^2 = -\beta g^4, \\ &\frac{d}{dt}\,y^2 = \alpha y^4 - \gamma g^2 y^2, \\ &\frac{d}{dt}\,\lambda = \lambda(a\lambda + by^2 - cg^2) - \mu y^4 + \nu g^4. \end{aligned} \tag{6}$$

Two remarks are in order about these equations:

First, the *signs* of the various terms of the β-functions are, of course, not accidental, reflecting basic properties of the SM, e. g.,

$$\begin{aligned} \beta > 0:&\quad \text{asymptotic freedom} \\ \alpha/\gamma > 0:&\quad \text{attraction/repulsion} \\ &\quad \text{in scalar/vector exchange} \\ a > 0:&\quad \text{Landau pole for the scalar} \\ &\qquad\qquad \text{self-coupling} \\ \alpha > 0:&\quad \text{Landau pole for the Yukawa} \\ &\qquad\qquad \text{coupling.} \end{aligned} \tag{7}$$

However, $\alpha > 0$ does not necessarily imply the existence of a Landau pole in the Yukawa coupling. Surprisingly, this depends not only on the coefficients of the differential equations but also on the initial values. Indeed, the first two equations do not depend on λ, and their solution is

$$\begin{aligned} &\frac{1}{g^2} = \frac{1}{g_0^2} + \beta t, \\ &\frac{1}{y^2} = \frac{\alpha}{\gamma-\beta}\,\frac{1}{g^2} + C\left(\frac{1}{g^2}\right)^{\gamma/\beta}. \end{aligned} \tag{8}$$

In practice, $\gamma/\beta > 1$ and C is defined by the initial condition for the gauge and Yukawa couplings. The presence or absence of the Landau pole in $y(t)$ depends on the sign of C. Finally, we can now solve the third equation for λ using the above solutions for $g(t)$ and $y(t)$ to complete the RG flows.

Second, we note that system (6) has a triangular property, allowing solutions to avoid chaotic behavior that could in principle lead to conclusions very different [15] from the one discussed here. Thus, this triangular structure of (6) in and of itself can serve as an argument in support of the idea that the above set of RG equations is very special and encodes important properties of the SM.

2.2. Asymptotically secure Higgs

If we want to "secure" the UV behavior of the scalar sector at the Planck scale in the way explained in the Introduction, then at $t = \log M \approx \log M_{Pl}$ we should require that $\lambda = 0$ and $\dot{\lambda} = 0$[1]. This requirement fixes the initial low-energy value $\lambda(0)$ of the Higgs self-coupling. Indeed, given the RG equation for λ,

$$\dot{\lambda} = a\lambda^2 + \lambda f(\mathbf{g}, y) + h(\mathbf{g}, y) \tag{9}$$

and the evolution laws $\mathbf{g}(t)$ and $y(t)$, we can find the scale μ at which $\lambda = 0 = \dot{\lambda}$ from the equation $h\Big(\mathbf{g}(\mu), y(\mu)\Big) = 0$, and then use (9) to solve for $\lambda(t)$ with $\lambda(\mu) = 0$. This gives the asymptotically secure fine-tuned value for $\lambda(0)$ at low energies. The fact that this procedure, when applied to the full one-loop RG equations of the SM, gives $\mu \approx M_{Pl}$ and for $\lambda(0)$ almost precisely the measured value of the Higgs coupling at the TeV scale (within the experimental error bar of one standard deviation) cannot in our opinion be considered plain coincidence.

3. EFFECTIVE POTENTIAL

The running coupling $\lambda(t)$ is also relevant to the computation of the (RG-improved) Coleman–Weinberg effective potential [17] for ϕ. At one loop, this effective potential is just[2]

$$V_{eff}(\phi) = \lambda(\phi)\,\phi^4. \tag{10}$$

[1] We call this situation *asymptotically secure* and not "asymptotically safe", because the latter usually refers to a nontrivial fixed point, while in our case the coupling is supposed to vanish. An example of the asymptotically safe Higgs theory can be found in [16].

[2] For large values of ϕ and with the quadratic divergences fine-tuned away, this is a very good approximation of the SM one-loop effective potential.

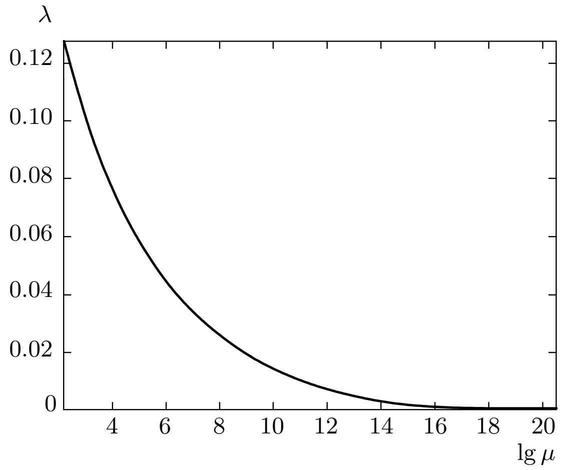

Fig. 2. The RG flow of the self-coupling $\lambda(\mu)$ in one-loop approximation as a function of $\lg\mu$ with the RG scale μ expressed in GeV. The "ideal" value of $y(m_T)$ that leads to relations (1) were used. The Planck scale is obtained automatically as the "touching point" of the curve to the abscissa axis. It is also instructive to look at the accuracy of the fine-tuning of $\lambda(\mu)$ and $y(\mu)$, needed to fit that special point, at higher energies, see Fig. 3

Normally, the zero of the beta-function of λ means nothing special for the effective potential. However, things are very different at a point like (1), since V_{eff} in (10) has a *minimum* at that point. Furthermore, this minimum is especially spectacular, because it occurs at large $\phi \sim \mu \sim M_{Pl}$, where the classical potential $\lambda(0)\phi^4$ is extremely large. This simple observation has recently been strengthened by a detailed analysis of the Standard Model [8, 9], which takes higher-loop corrections into account.

We first consider the possibility that the values of $\lambda(0)$ and $y(0)$ are such that relations (1) are satisfied exactly. With these parameters, the self-coupling λ behaves as in Fig. 2, and the shape of the effective potential implied by the Standard Model is shown in Fig. 4. We note that λ touches zero just at the Planck scale, as does the effective potential, and hence its second minimum is also at the Planck scale[3]. Still, this is not the result of a careful fine tuning. It is obtained simply by choosing the ratio of the Higgs-to-top masses such that $\lambda(t)$ just touches the horizontal axis and with the other SM parameters (e. g., the gauge couplings) taken from experiment. To within one standard deviation, the values of these masses satisfy this requirement, i. e., lead to

[3] This position of the minimum is, however, gauge dependent (see Ref. [18]).

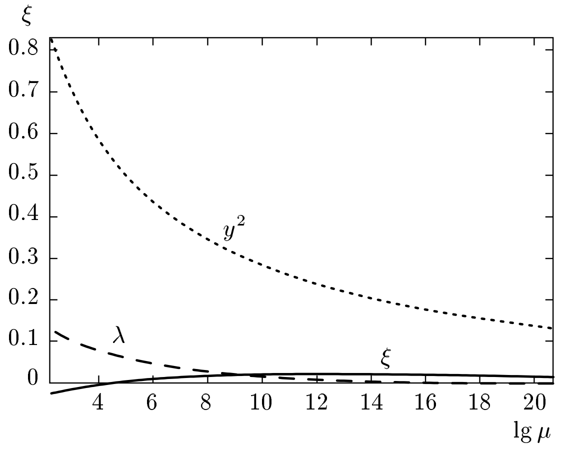

Fig. 3. Illustration of Fact 5. The difference $\xi \equiv \equiv y^2(\mu)/8 - \lambda(\mu)$ looks almost RG stable and is much smaller than the values of y and λ themselves. The picture is the one-loop approximation, and for "ideal" values of parameters for which relations (1) are exact

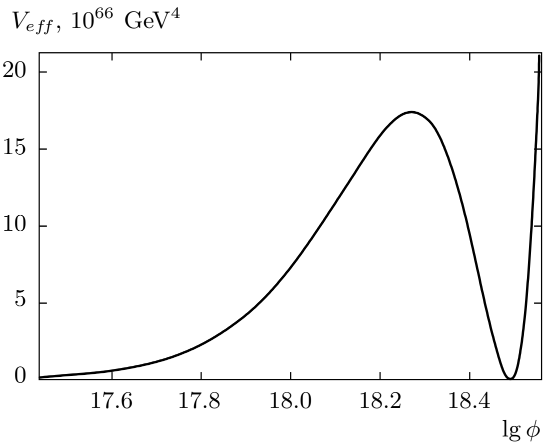

Fig. 4. The improved one-loop effective potential for the value of $y(m_Y)$ that leads to relations (1) (see Fig. 2), as a function of $\lg \phi$ with the field ϕ expressed in GeV. Note that, when the small uncertainty of the Standard Model parameters is fixed such that (1) are satisfied exactly, the second minimum of the potential is located at the Planck scale. This illustrates Fact 6 of the Introduction. Note also that the potential barrier is rather low, being seven orders of magnitude lower than its natural value M_{Pl}^4

$\lambda(t)$ which touches the axis, and miraculously, give the extra bonus that the "touching point" is obtained automatically at the Planck scale. To summarize, it seems that within one standard deviation, the parameters of the SM are such that relations (1) are satisfied and the special behavior of $\lambda(t)$ and $V_{eff}(\phi)$ given above is obtained, with the Planck scale arising automatically.

The Standard Model minimum, which is very close to the origin, is very shallow compared with the height of the barrier in Fig. 4. The location of the second minimum depends strongly on the parameters of the Standard Model. For the central values of the experimental SM parameters, the second minimum is deeper than the SM one, and may be located at energies even somewhat larger than the Plank mass. However, the barrier is still high enough to guarantee that the metastable Standard Model vacuum has a lifetime much longer than the age of the Universe [8, 19].

Furthermore, the flatness of the potential in Fig. 4, the barrier of which is about seven orders of magnitude lower than its "natural" scale M_{Pl}^4, fits nicely with the slow-roll requirement ($V''(\phi)/V(\phi) \ll M_{Pl}^{-2}$ and $V'(\phi)/V(\phi) \ll M_{Pl}^{-1}$ [20]) of scalar fields in inflation models and has inspired several authors to investigate the possibility that the Higgs itself plays the role of the inflaton in such models [21]. More recently, this possibility was studied, e. g., in [22] and [23].

Finally, it should be pointed out here that the form of potential (10) restricted to the scalar sector is to the leading order identical to the one obtained by Migdal and Shifman in [24, 25] in describing the low-energy dynamics of the dilaton field, a Goldstone-like field that arises as a consequence of the spontaneous breaking of conformal invariance in pure gluodynamics. The effective Lagrangian of the dilaton with this potential was constructed on general grounds and is exactly the one that guarantees the validity of the corresponding Ward identities. Furthermore, an analysis within the Standard Model of the Higgs particle as a dilaton has been performed in [26].

4. ARGYRES–DOUGLAS POINTS WITH ENHANCED SYMMETRY

If one asks whether there is any known situation when a conformal symmetry emerges in some sector of the theory at a given ratio of scalar/fermion masses, an immediate answer is the Argyres–Douglas point. The exact situation there differs from the Standard Model in many respects: the theory is supersymmetric (originally, it was $\mathcal{N} = 2$ SUSY, but actually $\mathcal{N} = 1$ is enough; see, however, [27]), the Higgs is hence in the adjoint representation, and it emerges in the infrared rather than in the ultraviolet region. Still, it illustrates the main fact: the emergence of an extra symmetry in one sector of the theory at some energies can be related to mysterious numerical relations observed in another

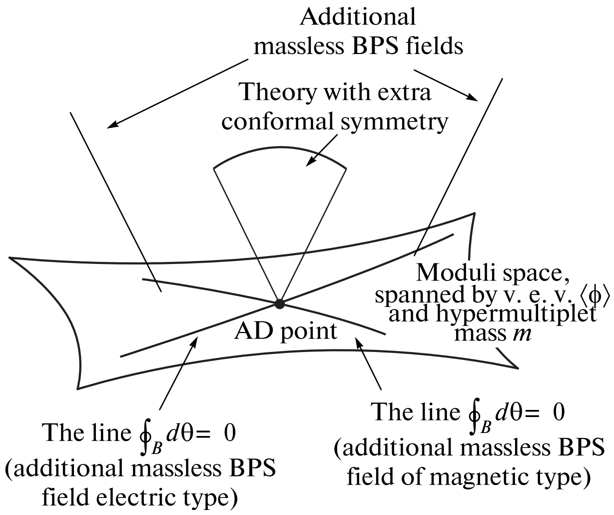

Fig. 5

sector. Further studies can easily make the analogy much stronger. Therefore, we briefly recall that old story.

4.1. AD point in SUSY chromodynamics

The low-energy sector of $\mathcal{N}=2$ SUSY theory is described by the Seiberg–Witten (SW) theory [28], where everything is encoded [29] in terms of a $(0+1)$-dimensional integrable system associated with a peculiar family of spectral curves Σ. In particular, masses of the BPS states are given by periods of the SW differential $d\theta = p\,dq$. Whenever a noncontractible cycle on the Riemann surface shrinks to zero, a BPS state becomes massless. This happens at particular points (hypersurfaces) in the moduli space of SW curves, i. e., at special values of the v.e.v. of the scalar (adjoint Higgs) field. At such points, there is in general a singularity in the moduli space but no additional symmetry.

At Argyres–Douglas (AD) points [30], two cycles simultaneously shrink to zero. At such points of the moduli space, pairs of massless BPS fields appear, which are mutually nonlocal, and the Coulomb branch is described by a very interesting nontrivial conformal theory [31] (Fig. 5).

Concrete formulas behind this description in the simplest possible case, the $SU(2)$ theory with one fundamental matter hypermultiplet, are as follows:

$$\begin{gathered}\text{The theory: } \mathcal{N}=2 \text{ gauge supermultiplet } + \\ + \text{ fundamental matter hypermultiplet.} \\ \text{The family of curves:} \\ y^2 = (x^2-u)^2 - \Lambda^3(x-m_T),\end{gathered} \tag{11}$$

where the parameter Λ is associated with Λ_{QCD}, m_T with the mass of the fundamental hypermultiplet, and the modulus of the curve $u = \langle \mathrm{Tr}\Phi^2\rangle$ is related to the vacuum expectation v of the adjoint scalar field (from the gauge supermultiplet) by the Seiberg–Witten theory[4)]. At large v, $u = 1/2v^2$.

The curve describes the torus and is a Riemann surface with four ramification points, i. e., two independent cycles A and B. At the AD point in the moduli space,

$$\begin{aligned} u &= \frac{3}{4}\Lambda^2, \\ m_T^2 &= \frac{3}{4}u, \end{aligned} \tag{12}$$

the three of these ramification points merge, and the two cycles degenerate. This leads to the simultaneously emerging massless monopoles and charged states from the hypermultiplet and a nontrivial superconformal theory.

We note that supersymmetry requires that the superpotentital be of the form

$$m_T\tilde{\Psi}\Psi - \frac{1}{\sqrt{2}}\tilde{\Psi}\Phi\Psi + \text{H.c.} \tag{13}$$

After spontaneous symmetry breaking, the two components of the hypermultiplet have masses $m_\pm = m_T \pm \pm v/\sqrt{2}$. In order to have conformal invariance (i. e., a massless quark), we require that

$$m_T = \frac{v}{\sqrt{2}}. \tag{14}$$

Thus, after breaking the symmetry, one of the hypermultiplet components becomes massless, while the other acquires the mass $2m_T$, and this is a corollary of supersymmetry (unit Yukawa constant) and conformal invariance.

4.2. Breaking SUSY from $\mathcal{N}=2$ to $\mathcal{N}=1$

We can explicitly break $\mathcal{N}=2$ supersymmetry to $\mathcal{N}=1$ by adding a superpotential

$$\mathcal{W} = \sum_k g_k\,\mathrm{Tr}\,\Phi^k + \text{fermionic interactions.} \tag{15}$$

4) In terms of integrable systems, this model is [32] a degeneration of the XXX spin chain at two sites, and u is the Hamiltonian of the degenerated spin chain and v the action variable. The Hamiltonian interpretation of the AD points has been discussed recently in [33].

In the $SU(2)$ case described above, the superpotential contains the massive term of the adjoint scalar field $M\mathrm{Tr}\Phi^2$ and fermionic interactions.

The singular points of the Coulomb moduli space upon the perturbation become vacua in the $N = 1$ theory, where the AD point is the point where two vacua collide. It was shown in [34] that both the monopole and charge condensates vanish at this point, and the theory remains superconformal even after the strong breaking of $N = 2$ to $N = 1$. Therefore, physically, the critical behavior at AD point corresponds to the deconfinement phase transition.

We note that the condensates in this theory can be explicitly described within the technique developed in [35, 36]. At the AD point, they turn out to be related to parameters of the superpotential by simple relations. For instance, in the case described above, $v = \sqrt{2}m_T$ and $u = m_T^2 + 3/16\Lambda^2$ (i.e., in the limit of large m_T, we still have $v^2 = 2u$).

The AD points and domains have been studied in various examples in the SW theory, with different field contents [30, 31, 35–37].

5. RELATED IDEAS

5.1. The multicriticality principle by Froggatt and Nielsen

Perhaps, the first who attempted to make a strong case against an intermediate energy scale between Fermi and Planck scales on the basis of RG properties were Froggatt and Nielsen [4]. They used earlier results in [13], where the requirement of positivity of the scalar potential led to constraints on the Higgs mass. Instead, Froggatt and Nielsen demanded that the minima of the scalar potential be exactly degenerate and predicted the correct value of the Higgs mass, seventeen years before it was finally announced at CERN [38].

To justify from first principles why Nature chooses this degeneracy, it was noted in [4] that if in a multiphase thermodynamic system extensive parameters (like energy, the number of particles, and volume) are fixed instead of intensive ones (like temperature, chemical potential, and pressure), the system is automatically driven to the multicritical (say, triple) point, where all the phases coexist in thermodynamic equilibrium, and hence the intensive parameters are also fixed. Taking the multiverse for the system and the shape of the effective potential for intensive parameters makes the “multicriticality principle” (that the possible vacua of the effective potential should be degenerate) somehow justified and, perhaps, even attractive. It differs significantly from the anthropic principle [39], since it relies on ordinary fundamental physics without *a posteriori* assumptions like the existence of galaxies, planets, life, and consciousness.

5.2. Models with t-quark condensates

A well-known scenario in which the masses of the Higgs boson and the top quark are related is based on the Nambu–Jona-Lasinio original ideas and is described in [40, 41]. A four-fermi on interaction is added to the SM action and the top-quark condensate is assumed to form, with the characteristic compositeness scale $\Lambda \sim 10^{15}$–10^{19} GeV. The Higgs boson emerges as a scalar excitation over the condensate and the top-quark mass turns out to be around $m_T \sim 200$ GeV. Finally, the masses of the scalar (Higgs) excitation and the top quark are shown to satisfy simple relations, like the so-called Nambu relation $m_H = 2m_T$, which, however, are model dependent.

The two basic features of this scenario that make contact with our discussion are: (i) the huge difference between the particle masses m_H, m_T and the compositeness scale Λ, which implies that the theory is “almost conformal”, a feature shared by the $\mathcal{N} = 1$ model discussed in Sec. 4 near the AD point; (ii) the initial condition used in [40] for the renormalization group at the compositeness scale Λ is the vanishing of the scalar self-coupling, which corresponds to one of the two conditions in (1).

5.3. Asymptotically safe gravity

The idea of asymptotically safe theories, put forward by Weinberg [42], has not so far attracted the attention it deserves, with the exception of asymptotically safe *gravity*, which is relatively well studied primarily by Reuter [43].

This is a radical idea with today’s standards, since it admits that there is no new physics beyond the Standard Model even at the Planck scale or above it[5]. In such a context, it is natural to unify the ideas of an asymptotically secure Higgs and asymptotically safe gravity, as was strongly advocated in [44].

5.4. Brane interpretation

Like any Yang–Mills theory, the Standard Model can be embedded in various brane backgrounds, and

[5] If strings do not show up there, there is no obstacle to go to higher scales, only in string theory the regions above and below Planck mass are dual to each other.

it is interesting to discuss their properties from the perspective of this paper. In a brane picture, all condensates and other moduli are interpreted as distances and fluxes in extra dimensions. One could speculate that the remarkable "numerical coincidences" described in the Introduction are needed for the stability of the whole brane configuration in a wide range of energies or, equivalently, values of the radial RG coordinate. The approximate "flatness" of the Higgs potential could imply that the brane configuration is nearly BPS, since the flatness of the potential requires cancelation of the interaction between the corresponding branes.

Another possible source of relations between parameters is the matching of theories on the "flavor" and "color" branes. The theories on these branes are essentially different (for example, one is Abelian in the case of one flavor, while the other is not), but all physical phenomena should be equally well described in terms of both branes. The familiar example of this phenomenon is the equivalent description of conventional QCD as a theory on the flavor branes (chiral Lagrangian) or as a theory on the color branes (QCD Lagrangian). Another example is the $2d/4d$ correspondence, where the $4d$ physics can be equivalently described by the $2d$ theory on the non-Abelian string. An interesting kind of matching condition is provided by the decoupling of a heavy flavor. The conformal anomaly implies that the condensate of the fermion field disappears as its mass increases: $m\langle\tilde{\Psi}\Psi\rangle = \langle\mathrm{Tr}\, G^2\rangle$. This relation turns out to be part of the stability condition of the brane geometry [45] and holds in all QCD-related backgrounds.

If the Standard Model is indeed at the borderline of metastability, an interesting question is to understand what becomes unstable in the brane picture. In the well-controlled supersymmetric context, the AD point lies at the marginal stability line/surface, where unstable in the $\mathcal{N} = 2$ case are BPS particles, but in the $\mathcal{N} = 1$ case unstable are instead the extended objects — domain walls [34]. It is much less clear what would happen when supersymmetry is completely broken, but we can imagine that the metastability of the Standard Model vacuum reflects a metastability of the "color brane" at an AD-like point in the parameter space.

6. DISCUSSION

Usually, the biggest obstacle to the idea that there is no new physics in between the Fermi and Planck scales is the hierarchy problem: one should explain why quadratic divergences do not generate a scalar mass of the Planck size (see [46] for a recent discussion). Together with the similar cosmological constant problem, it clearly implies that power divergences should be ignored in the Standard Model. Moreover, even supersymmetry does not help, because, being broken, it is not sufficient to explain the smallness of the cosmological constant. The idea of asymptotic safety also is not sufficient, because the fact that the theory is very nice in the ultraviolet does not guarantee that unwanted contributions are not generated by the RG evolution. Power divergences are automatically absent in dimensional regularization schemes, but it is unclear whether the possible existence of small extra dimensions could actually help. Whatever one thinks about this problem, it is phenomenologically clear that quadratic divergences have to be ignored in the Standard Model, and this is widely recognized in the literature: it suffices to mention that the RG-evolution plots in Refs. [13] (the early counterpart of our Fig. 1) and [8, 9] included evolution of the mass term, but only logarithmic corrections were taken into account and considered relevant to the "real" physics.

As for explanations, the hope may be that the "hidden symmetry" reflected in relations such as (5) could provide a new tool for the resolution of the hierarchy problem, since the symmetry would protect these relations, in particular, leaving no room for quadratic divergences. In fact, although not sufficiently well appreciated, the idea that the apparent conformal symmetry of the Standard Model at the classical level could forbid the generation of quadratic corrections at the quantum level has been discussed in the literature: it is best expressed in [11], where even a concrete quantization scheme was suggested. This idea is also studied in [24], which we mentioned in Sec. 3, or very recently in [47], and, in a context related to the neutrino mass mechanism, in [48].

We emphasize that these ideas receive additional support from our Fact 1. Usually, classical conformal symmetry of the Standard Model is broken softly by mass terms and seriously by (logarithmic) quantum corrections, giving rise to nonvanishing beta-functions. Our Fact 1 implies that the only role of the beta-functions is to drive the theory away from the UV point, but exactly *there* is the approximate conformal symmetry actually enhanced: in the scalar sector, the beta-function vanishes and the interaction also vanishes. The theory looks even more conformal than one could expect. And this is further supported by the extreme flatness of the effective potential (it is clear from Fig. 4 that the height of the barrier is seven orders of magnitude lower than the naive M_{Pl}^4, while the mass of the scalar mode at the Planckian minimum is instead higher by many orders of magnitude than the naive M_{Pl}, and hence can actually be ignored), and all this is just an *experimental fact* following from the well-established properties of the Standard Model itself (see also a discussion in [49]), with no reference to any kind of "new physics", to say nothing about quantum gravity and string theory (the Planck scale appears in Fig. 4 just from the study of RG evolution of the Standard Model itself(!)). The only assumption is to neglect the quadratic quantum corrections, but given not just the classical conformal symmetry of [11] but its further enhancement by (1) at the "starting point" in the ultraviolet, we can hardly be surprised that they *should* be neglected in an appropriate quantization scheme. In our view, it is now a clear challenge for string theory or whatever is the UV completion of the Standard Model to make such a scheme *natural*.

As we mentioned, one option within ordinary quantum field theory would be to look for a formulation where the Higgs scalars are actually Goldstones of spontaneously broken conformal symmetry, which acquire relatively small masses due to the explicit breaking of this symmetry by beta-functions, as implied by the analogy with a similar situation in [24]. However, in this general review, we prefer not to speculate further about particular realizations of this option.

7. CONCLUSION

Inspired by the old works of Froggatt–Nielsen–Takanishi [4, 5] on one side, and by the spectacular relations among the parameters of the Standard Model on the other, we reviewed the evidence that the Standard Model lies at a very special point of the parameter space. Namely, that it is connected by RG evolution to a theory with enhanced (conformal-like) symmetry at Planck energies, where it is supposed to be mixed with quantum gravity and, perhaps, string theory. If true, this implies the exciting possibility that the actual values of couplings, which may seem fine-tuned at our energies, may just reflect the fact that we are looking at the low-energy limit of a UV healthy theory, thus providing a kind of refinement of the renormalizability principle. In other words, it is possible that not only the low-energy theory is necessarily a gauge theory but also its scalar sector should be very special, just as a consequence of being a low-energy effective theory. This option, if actually realized, would resolve many puzzles about the Standard Model at once.

We also emphasized that the well-known interconnected Facts 1 & 2 about the Standard Model are complemented by Facts 3, 4 & 5. We mentioned that these two seemingly unrelated properties — the existence of an enhanced conformal-like symmetry at one scale (Facts 1 & 2) and remarkably special numerical relations at another (Facts 3 & 4), which in addition look RG stable (Fact 5) — may well be related with each other. At least one example with similar properties is already known: at the Argyres–Douglas point, conformal symmetry in the BPS sector emerges at the very special points in the original moduli space of vacuum expectation values and couplings. In the Standard Model, conformal symmetry (probably) emerges in the ultraviolet and not in the infrared, but this is rather an advantage, because this explains why we should wish to adjust the parameters of the moduli space to be at this special AD point.

To summarize:

• Problems of the Higgs sector (zero charge and hierarchy) could be naturally resolved by treating it as a low-energy limit of an especially nice theory at the Planck scale.

• That theory can be at least conformal, or, perhaps, even superconformal invariant. This not only seems to match nicely with expectations based on string theory, but also looks *phenomenologically* motivated by the actual features of the Standard Model.

• As a dream-like scenario, the Higgs sector could actually emerge as a Goldstone one, associated with spontaneous breaking of high-energy conformal invariance, and this could solve both the hierarchy and the Landau pole problems.

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