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HOLOGRAPHIC THERMALIZATION IN A QUARK CONFINING BACKGROUND

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We study holographic thermalization of a strongly coupled theory inspired by two colliding shock waves in a vacuum confining background. Holographic thermalization means a black hole formation, in fact, a trapped surface formation. As the vacuum confining background, we considered the well-know bottom-up AdS/QCD model that provides the Cornell potential and reproduces the QCD β-function. We perturb the vacuum background by colliding domain shock waves that are assumed to be holographically dual to heavy ions collisions. Our main physical assumption is that we can make a restriction on the time of trapped surface formation, which results in a natural limitation on the size of the domain where the trapped surface is produced. This limits the intermediate domain where the main part of the entropy is produced. In this domain, we can use an intermediate vacuum background as an approximation to the full confining background. We find that the dependence of the multiplicity on energy for the intermediate background has an asymptotic expansion whose first term depends on energy as $E^{1/3}$, which is very similar to the experimental dependence of particle multiplicities on the colliding ion energy obtained from the RHIC and LHC. However, this first term, at the energies where the approximation of the confining metric by the intermediate background works, does not saturate the exact answer, and we have to take the nonleading terms into account.

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1. INTRODUCTION

QCD, which is the currently accepted theory of strong interactions, still has the well-known problems with describing a strong-coupling phenomena. The physics of heavy-ion collisions, in particular, a quark-gluon plasma (QGP) formation, involves real-time strongly coupled phenomena, which makes these phenomena difficult to study within the standard QCD methods. In the recent years, a powerful approach to QGP is explored, based on a holographic duality between the strong-coupling quantum field in d-dimensional Minkowski space and classical gravity in $(d+1)$-dimensional anti de Sitter (AdS) space [1–3]. In particular, there is considerable progress in the holographic description of equilibrium QGP [4]. The holographic approach is also applied to nonequilibrium QGP. Within this holographic approach, thermalization is described as a process of formation of a black hole in the AdS space.

The AdS/CFT (conformal-field theory) correspondence is based on string theory and perfectly works for the $\mathcal{N} = 4$ SUSY Yang–Mills theory, while the dual description of real QCD is unknown. Much effort has been invested into the search for holographic QCD from string theory (see, in particular, [5–7]). This approach is known as the "top-down" approach. An other approach, known as the "bottom-up" approach, is supposed to propose a suitable holographic QCD models from experimental data and lattice results [8–15]. The main idea of this approach is to use natural prescriptions of the general AdS/CFT correspondence to try to recover nonperturbative QCD phenomena, in particular, nonperturbative vacuum phenomena, finite-temperature, high-density, and nonzero chemical potential phenomena.

The 5-dimensional metrics that reproduce the Cornell potential [16], as well as ρ-meson spectrum, etc., have been proposed [10, 13, 14]. A so-called improved holographic QCD (IHQCD) that is able to reproduce

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the QCD β-function has been constructed [15]. Thermal deformations of these backgrounds are intensively studied in the last years (see [4] for a review).

The problem of QGP formation is the subject of intensive study within holographic approach in last years (see [17, 18] and the references therein). There is considerable progress in the understanding of the thermalization process from the gravity side as BH formation. Initially, this process has been considered starting from the AdS background [19–25]. However, the pure AdS background is unable to describe the vacuum QCD with quark confinement, nor is it able to reproduce the QCD β-function. There are backgrounds that solve one, or even two of these problems. The first was solved in [10] (see also [13, 14]), where a special version of a soft wall was proposed, and the β-function was reproduced from IHQCD [12, 15].

To describe thermalization, it is natural to study deformations of these backgrounds. Suitable deformations of IHQCD by shock waves have been studied in [26, 27], and it has been shown that without additional assumptions, the IHQCD metric does not reproduce the experimental multiplicity dependence on energy. It was noted in [28] that holographic realization of the experimental multiplicity requires an unstable background.

The goal of this paper is to close this gap and to show that the model that reproduces the Cornell potential can at the same time be used as a gravity background to give the correct energy dependence of multiplicities produced in a finite time. As a bonus of our approach, we obtain a reasonable estimation of the thermalization time.

This paper is organized as follows. In Sec. 2.1, we recall the confining metrics that reproduce the Cornell potential. In Sec. 2.2, we recall the previous results concerning the dependence of multiplicities on energy. In Sec. 2.3, we present the main formula for the size of trapped surfaces formed in collision of domain walls. In Sec. 3, we consider a special metric that is far away from the confining metrics, but gives a suitable entropy. We also note that a restriction of the size of the trapped surface permits determining the thermalization time. In Sec. 4, we show that the confining metric in [14] can be approximated at intermediate values of the holographic coordinate z by the metric considered in Sec. 3. As a result, for the entropy produced during a short time $\tau_{term} \approx 0.25$ fm[1)], this gives an asymptotic expansion with the leading term that depends on energy as $E^{1/3}$. The same is true for the metric in [10].

[1)] Here and below the light velocity $c = 1$.

2. SETUP

2.1. Confining backgrounds

It is well known that the AdS space does not reproduce the quark confinement. To reproduce quark confinement, in particular, the appropriate glueball spectrum, Polchinski and Strassler [8] imposed a cut-off in the AdS space, a "hard wall model". Another modification of the AdS space, a "soft wall models" [9], is related to the dilaton. In the bottom-up approach, the metric is usually taken to be

$$ds^2 = b^2(z)(-dt^2 + dz^2 + dx_i^2), \tag{2.1}$$

where $b^2(z)$ is some function usually taken to be the AdS metric in the UV zone (this leads to the Coulomb potential in the UV) and is a deformed AdS metric in the IR. The deformation in the IR should be taken in such a way that the quark–antiquark potential exhibit confinement.

The experimental model of the potential that is used to fit lattice and experimental data [16] is usually taken to be the Cornell potential. In principle, this potential should reproduce the quarkonium spectrum, interpolating between one-gluon exchange in the UV and linear confinement in the IR.

The model proposed in [10] uses the warp factor

$$\begin{gathered} b^2(z) = \frac{L^2 h(z)}{z^2}, \quad h_{AZ} = \exp\left(\frac{az^2}{2}\right), \\ a = 0.42 \text{ GeV}^2. \end{gathered} \tag{2.2}$$

In [11], it was shown that this factor reproduces the static interquark potential obtained from $SU(3)$ lattice calculations [16].

In [14], the modification

$$\begin{gathered} h(z) = \frac{\exp\left(-\sigma z^2/2\right)}{[(z_{IR} - z)/z_{IR}]^{c_0}}, \quad \sigma = 0.34 \text{ GeV}^2, \\ c_0 = 1, \quad z_{IR} = 2.54 \text{ GeV}^{-1} \end{gathered} \tag{2.3}$$

was considered. This modification is in fact very close to the model in [13] for $0 < z < 2$ fm and reproduces the Cornell potential and the β-function.

In this paper, we consider modification (4.1) (see below) of factor (2.2), which also fits the Cornell potential well.

2.2. Multiplicities

The experimental data for multiplicities in heavy-ion collisions at the RHIC and LHC indicate that [29]

$$\mathcal{M}_{exp} \propto E^{0.3} + \ldots \tag{2.4}$$

The multiplicities obtained for the simplest holographic calculation in a conformal background with the AdS_5 metric [19–25],

$$\mathcal{M}_{AdS_5}(E) \propto E^{2/3}, \tag{2.5}$$

are in fact worse than the Landau bound

$$\mathcal{M}_{Landau}(E) \propto E^{1/2}. \tag{2.6}$$

To improve the energy dependence of multiplicities, Kiritsis and Taliotis [26] proposed to use modifications of the b-factor. They considered b-factors corresponding to conformal and nonconformal backgrounds. More precisely, they considered collision of holographic point-like sources in dilaton models and obtained estimations for a variety of models (depending on the dilaton potential)

$$\mathcal{M}_{a>1/3} \propto E^{(3a+3)/(3a+2)}, \tag{2.7}$$

$$\mathcal{M}_{a\le -1/3} \propto E^{(3a+1)/3a}. \tag{2.8}$$

We note that a perturbative QCD-inspired UV cut-off was also used in [20]. This modification provides logarithmic corrections. Following [24], where an energy-dependent cut-off in the high-energy limit was proposed, Kiritsis and Taliotis [26] have shown that this cut-off reduces the powers in (2.7) and (2.8) as

$$\mathcal{M}_{a>1/3} \propto E^{2/(3a+1)}, \tag{2.9}$$

$$\mathcal{M}_{a\le -1/3} \propto E^{2/[3(1-a)]}. \tag{2.10}$$

Later, in [28], we confirmed the results in (2.7) and (2.8) by considering the domain-wall collision models that generalized the Lin–Shuryak model [30, 31] to nonconformal cases. In [28], we also noted that the model with the b-factor $b(z) = L_{eff}/z$ gives a more realistic bound

$$\mathcal{M}_{ph-dilaton}(E) \propto E^{1/3}, \tag{2.11}$$

which is closer to (2.4). But the price for this modification is the phantom kinetic term for the dilaton. We note that we have not performed any UV cut-off in this model to obtain estimation (2.11).

2.3. Trapped surface for domain-wall shock waves

The equation for the domain-wall wave profile $\phi^w(z)$ in the space with a b-factor is

$$\left(\partial_z^2 + \frac{3b'}{b}\partial_z\right)\phi^w(z) = -C\frac{\delta(z-z_*)}{b^3(z)}, \tag{2.12}$$

where

$$C = \frac{16\pi G_5 E}{L^2} \tag{2.13}$$

is a dimensionless variable, G_5 is the 5-dimensional gravitational constant, E is an energy, and L is a scale parameter. The solution of (2.12) is given by

$$\phi^w(z) = \phi_a\theta(z_* - z) + \phi_b\theta(z - z_*), \tag{2.14}$$

where

$$\phi_a = C_a\int_{z_a}^{z} b^{-3}dz, \quad \phi_b = C_b\int_{z_b}^{z} b^{-3}dz. \tag{2.15}$$

The constants C_a and C_b can be represented in the form (see [28, 30])

$$\begin{aligned} C_a &= C\int_{z_b}^{z_*} b^{-3}dz\Big/\int_{z_b}^{z_a} b^{-3}dz, \\ C_b &= C\int_{z_a}^{z_*} b^{-3}dz\Big/\int_{z_b}^{z_a} b^{-3}dz. \end{aligned} \tag{2.16}$$

As has been mentioned in the Introduction, we consider the collision of two shock domain walls in 5-dimensional space–time as a holographical model of heavy-ion collisions in real 4-dimensional space-time. The shock-wave profile ϕ^w satisfies Eq. (2.12). The trapped surface formed in the wall-on-wall collision obeys Eq. (2.12) and special boundary conditions. From these boundary conditions, we can find that the trapped surface is located in z-direction from some point z_a to a point z_b such that $z_a < z_* < z_b$. The points z_a and z_b can be found from the relations

$$\begin{aligned} \frac{C}{2}b^{-3}(z_a)\int_{z_b}^{z_*} b^{-3}dz\Big/\int_{z_b}^{z_a} b^{-3}dz &= 1, \\ \frac{C}{2}b^{-3}(z_b)\int_{z_a}^{z_*} b^{-3}dz\Big/\int_{z_b}^{z_a} b^{-3}dz &= -1. \end{aligned} \tag{2.17}$$

Relations (2.17) guaranty that the trapped surface forms and is located between z_a and z_b, and the collision point z_* is located between z_a and z_b, $z_a < z_* < z_b$ (see the details in [28, 30]).

From (2.17), we obtain

$$\frac{C}{2} = b^3(z_a) + b^3(z_b), \tag{2.18}$$

$$F(z_*) = \frac{b^{-3}(z_a)F(z_b) + b^{-3}(z_b)F(z_a)}{b^{-3}(z_a) + b^{-3}(z_b)}, \tag{2.19}$$

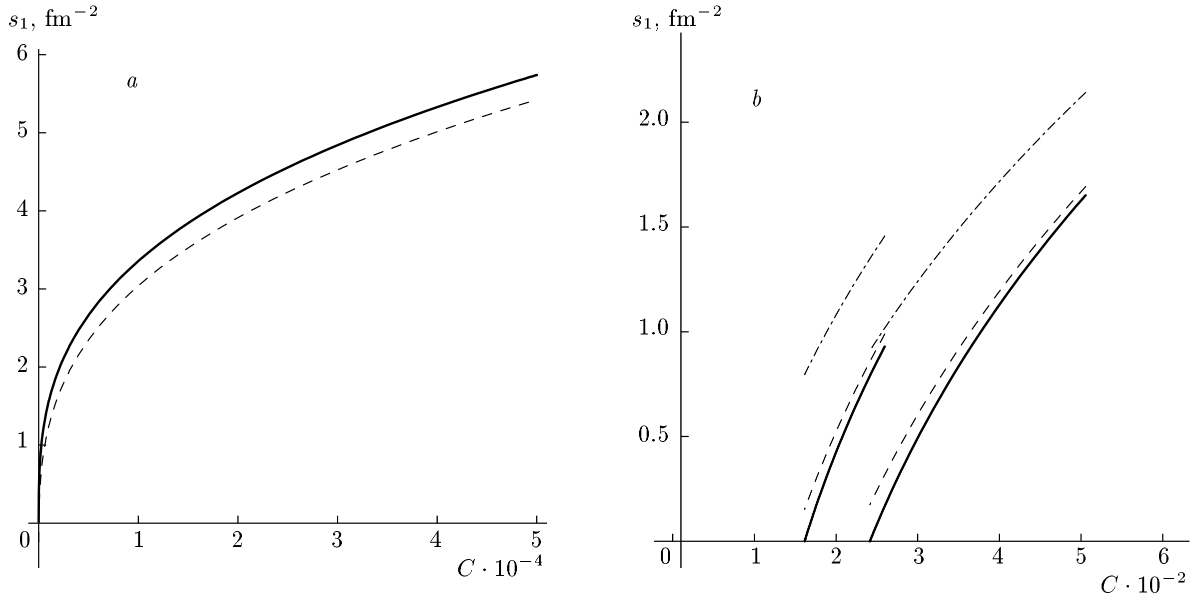

Fig. 1. (a) The thick line represents $s_1^{(0)}(C)$ and the thin one represents $s_1(C, z_b)$ at $z_b = 1.7$ fm. Here, we take $L_{eff} = 4.4$ fm and $G_5 = 44.83$ fm^3. (b) The dependence of the entropy $s_1(C, z_b)$ on C for $z_b = 1.7$ fm (solid thick line) and $z_b = 1.3$ fm (solid thin line). Approximations $s_1^{(1)}(C, z_b)$ (dot-and-dash lines) and $s_1^{(2)}(C, z_b)$ (dashed lines) for $z_b = 1.7$ fm (thick lines) and $z_b = 1.3$ fm (thin lines). Here, $L_{eff} = 20.7$ fm. For thick lines, z_a varies from 1.0 to 1.7 fm, for thin lines, z_a varies from 0.6 to 1.3 fm

where

$$\int_{z_i}^{z_j} b^{-3} dz = F(z_j) - F(z_i) \tag{2.20}$$

and

$$z_a < z_* < z_b. \tag{2.21}$$

There is the following formula for the entropy density [28] of the trapped surface:

$$s = \frac{S_{trap}}{\int d^2 x_\perp} = \frac{1}{2G_5} \int_{z_a}^{z_b} b^3\, dz. \tag{2.22}$$

3. INTERMEDIATE BACKGROUND

3.1. Entropy

In this section, we consider metric (2.1) with the b-factor

$$b = b_1 \equiv \left(\frac{L_{eff}}{z}\right)^{1/2}. \tag{3.1}$$

The entropy dependence on the energy can be read off from the formula

$$s_1 = \frac{L_{eff}}{G_5} \left[\left(\frac{L_{eff}}{z_a}\right)^{1/2} - \left(\frac{L_{eff}}{z_b}\right)^{1/2} \right], \tag{3.2}$$

where

$$\frac{z_a}{z_b} = \left[\frac{C}{2} \left(\frac{z_b}{L_{eff}}\right)^{3/2} - 1 \right]^{-2/3}. \tag{3.3}$$

Substituting (3.3) in (3.2), we obtain

$$s_1(C, z_b) = \frac{L_{eff}}{G_5} \left(\frac{L_{eff}}{z_b}\right)^{1/2} \times \\ \times \left\{ \left[\frac{C}{2} \left(\frac{z_b}{L_{eff}}\right)^{3/2} - 1 \right]^{1/3} - 1 \right\}. \tag{3.4}$$

We can perform the large-C expansion in formula (3.4) to obtain

$$s_1(C, z_b) = \frac{L_{eff}}{G_5} \left[\left(\frac{C}{2}\right)^{1/3} - \left(\frac{L_{eff}}{z_b}\right)^{1/2} - \\ - \frac{1}{3} \left(\frac{2}{C}\right)^{2/3} \left(\frac{L_{eff}}{z_b}\right)^{3/2} + \ldots \right], \tag{3.5}$$

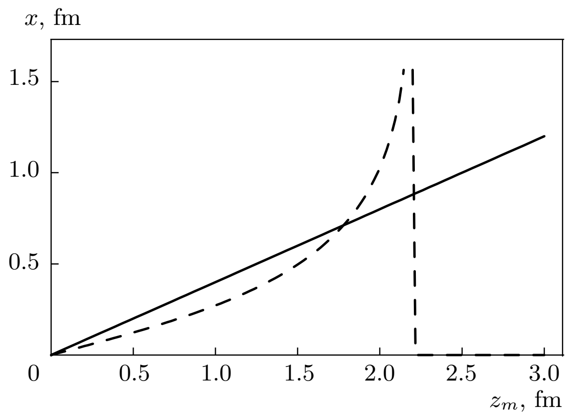

Fig. 2. Dependence of the interquark distance x on the string maximum holographic coordinate z_m for metric with the factor $b_1(z)$ (solid line) and for the metric $b_2(z)$ (dashed line)

whence the zeroth, first, and second approximations are given by

$$s_1^{(0)}(C) = \frac{L_{eff}}{G_5}\left(\frac{C}{2}\right)^{1/3},$$
$$s_1^{(1)}(C, z_b) = \frac{L_{eff}}{G_5}\left[\left(\frac{C}{2}\right)^{1/3} - \left(\frac{L_{eff}}{z_b}\right)^{1/2}\right], \quad (3.6)$$

$$s_1^{(2)}(C, z_b) = \frac{L_{eff}}{G_5}\left[\left(\frac{C}{2}\right)^{1/3} - \left(\frac{L_{eff}}{z_b}\right)^{1/2} - \frac{1}{3}\left(\frac{2}{C}\right)^{2/3}\left(\frac{L_{eff}}{z_b}\right)^{3/2}\right]. \quad (3.7)$$

The dependence of the entropy $s_1(C, z_b)$ on C at a fixed z_b is represented with the thin line in Fig. 1a. The approximation $s_1^{(0)}(C)$ at large C behaves as $C^{1/3}$ and is shown with the thick line in Fig. 1a. We note that due to relation (2.13), $C \propto E$. More precisely, taking $G_5 = 44.83$ fm^3 and $L_{eff} = 4.4$ fm, we have $C = 580E/\mathrm{GeV}$ and the range of C in Fig. 1a corresponds to the energy around 0.1 TeV. Therefore, we can say that at large energies, $s_1(E, z_b) \propto E^{1/3}$, which in fact is rather close to the experimental dependence $\propto E^{0.3}$.

However, relation (3.3) written in the form

$$\left(\frac{L_{eff}}{z_a}\right)^{3/2} + \left(\frac{L_{eff}}{z_b}\right)^{3/2} = \frac{C}{2} \quad (3.8)$$

may give a restriction on the possible variation range of energy. Indeed, by the construction, $z_a < z_b$, and to obtain a large value of C at a fixed z_b, we have to take a small z_a. In the case of an additional restriction on z_a, say, $z_a > z_{a,min}$, we obtain the restriction $C < C_{max}$. It may happen that in this domain of C, we cannot restrict ourself by the zeroth approximation to $s_1(C, z_b)$. A few examples of such a behavior are presented in Fig. 1b. In Fig. 1b, the energy dependences of the exact entropy $s_1(C, z_b)$ and approximated entropies $s_1^{(1)}(C, z_b)$ and $s_1^{(2)}(C, z_b)$ for different z_b are shown for relatively small values of C. Figure 1b shows that in the considered domains of C, we have to take the first three terms of approximation (3.5) into account. The choice of $L_{eff} = 20.7$ fm in Fig. 1b is clarified in Sec. 4.

3.2. Thermalization times

We estimate the thermalization time by a characteristic size of the trapped surface, i. e.,

$$\tau_{therm} \sim \frac{z_b - z_a}{2.4}. \quad (3.9)$$

We put the factor 2.4 taking the relation between the interquark distance x and the string maximum holographic coordinate z_m into account. The dependence of the interquark distance x on the maximum of the string z-coordinate, z_m, is given by

$$x = \int_0^{z_m} 2\left(\frac{b^4(z)}{b^4(z_m)} - 1\right)^{-1/2} dz \quad (3.10)$$

(see, e. g., [13] for the details). For metric (2.1) with b_1 given by (3.1), this dependence is represented in Fig. 2 by the solid line, and we see that $x = z_m/2.4$.

We note that formula (3.9) is written using general causal arguments. We assume that the time of formation of an object extended along the holographic direction from z_a to z_b is the same as the formation time of an extended object along the x direction with a characteristic scale $\Delta x = (z_b - z_a)/2.4$. This is in accordance with (3.10). Formation of such an object can be performed no faster than Δx.

The dependence of the thermalization time on C for a given value of z_b can be estimated by substituting z_a from (3.3) in the r. h. s. of (3.9). In Fig. 3, the dependence of the thermalization time on C for different values of z_b is presented. To vary C, we vary z_a. In the plot, different domains of z_a are shown by lines with different thickness. Small values of z_a correspond to large energy values.

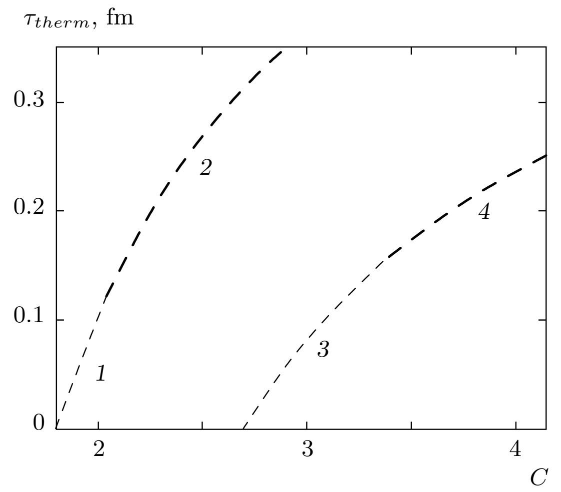

Fig. 3. The dependence of the thermalization time on C for different values of z_b ($z_b = 1.7$ fm — lines *1* and *2*, $z_b = 1.3$ fm — lines *3* and *4*). Different range of variation of z_a are shown by lines with different thickness: (*1*) 1.0 fm $< z_a <$ 1.3 fm; (*2*) 0.8 fm $< z_a <$ 1.0 fm; (*3*) 1.45 fm $< z_a <$ 1.6 fm; (*4*) 1.0 fm $< z_a <$ 1.45 fm

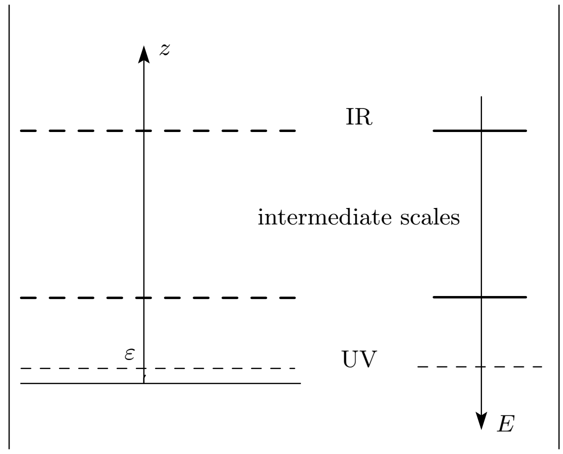

Fig. 4. Schematic picture of the bulk scales (ε is the UV regularization)

4. INTERMEDIATE BACKGROUND AS A PART OF THE CONFINING BACKGROUND

In this section, we consider metric (2.1) with the confining factor

$$b(z) = b_2(z) \equiv \frac{L}{z}\exp\left(\frac{az^2}{4}\right)\sqrt{1+gz}, \qquad g = -0.02 \text{ GeV}. \tag{4.1}$$

A schematic picture of the bulk scales is presented in Fig. 4.

Figure 5 shows that for $L = 4.4$ fm, in the range of intermediate holographic coordinate z, 1.3 fm $= z_{UV} < z < z_{IR} =$ 1.8 fm, the factor b_1 with $L_{eff} = 20.86$ fm coincides with b_2 up to 3 % and for 1.4 fm $< z <$ 1.7 fm, these factors are almost identical. Instead of (4.1), we can use the b-factor (2.2) from [10]. This leads to a slight variation of the parameters L_{eff}, z_{IR}, and z_{UV} (Fig. 6).

We note that the metric with the b-factor b_1 leads to a potential of interquark interaction of the form $V(r) \propto A\log(r/r_0)$, where A and r_0 are some constants. This form of the potential was suggested as a simple model to fit identical spin-averaged charmonium and bottomonium level splitting (see [16] and the references therein).

The dependence of the interquark distance x on the string maximum z coordinate z_m for metric (2.1) with the confining factor b_2 is presented above in Fig. 2 by a dashed line. We note that at $z \approx 2.2$ fm, there is a string breaking, which is in accordance with [32]. This point is out side our intermediate zone.

In formula (2.22) for the entropy, we take the usually accepted value $G_5 \approx 44.83$ fm^3 [19].

In Fig. 7*a*, the entropy dependence on the energy is presented for the confining metric and $b_1^2 = L_{eff}/z$. We see that the energy dependences of entropies are very close in this intermediate region 1.2 fm $< z <$ 1.8 fm. This intermediate region, according to (3.9), corresponds to the thermalization time $\tau_{therm} \approx 0.25$ fm.

We note that our assumption about the restriction of the area of trapped surface formation and the choice of the nonconfining metric with b-factor (3.1) instead of the confining metric with b-factor (4.1) (with the desired energy dependence of entropy in the asymptotic regime) is similar in some sense to the proposal to use an energy-dependent cut-off in the high-energy limit [24]. We can also say that our estimations give an analytic realization of this proposal.

However, if we consider a wider region for possible values of z_a and, for example, consider $z_a \to 0$, which corresponds to large energies, we obtain different entropy behaviors in these two models. The model with the factor b_2 has a typical behavior $s(C) \propto C^{2/3}$ [19, 31] and the model with b_1 has $s(C) \propto C^{1/3}$ at large C [27]. This means that without changing the asymptotic forms of the b-factor in the UV region, we cannot change the behavior of $s(C)$ at large energies. From another point of view, the UV asymptotic form

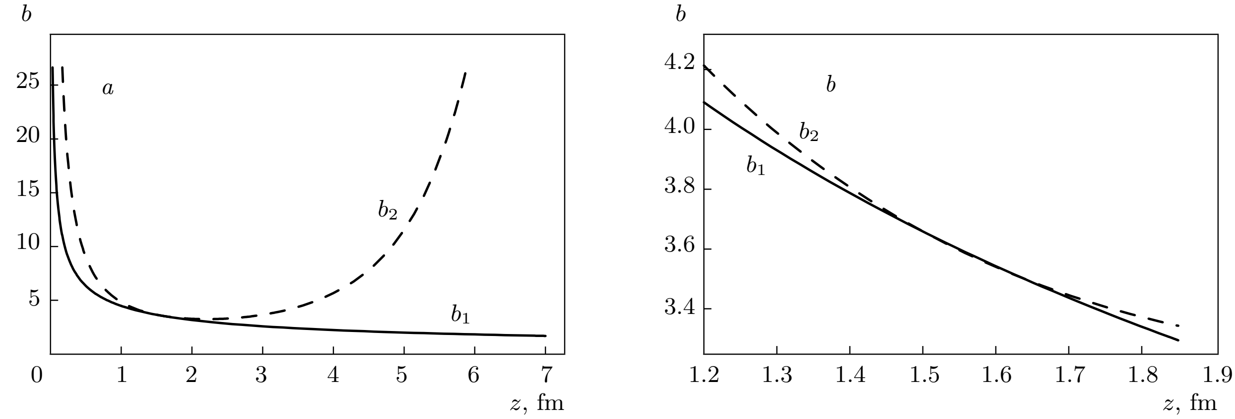

Fig. 5. (a) Factors $b_1(z)$ and $b_2(z)$. Here $L = 4.4$ fm and $L_{eff} = 20.86$ fm. (b) The same b-factors as in (a) in the intermediate region 1.2 fm $< z < 1.8$ fm. Solid lines correspond to $b_1 = b_1(z)$, dashed lines correspond to $b_2 = b_2(z)$

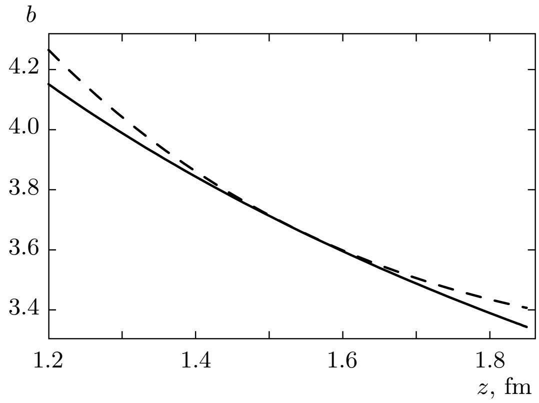

Fig. 6. Factors $b_1(z)$ with $L_{eff} = 20.7$ fm (solid line) and $b(z)$ given by (2.2) (dashed line) in the intermediate region 1.2 fm $< z < 1.8$ fm

of the b-factor is fixed by the Coulomb potential [1]. This leads us to a modification of the holographic scenario and consideration of an anisotropic background at small z, where we can expect that the most part of the entropy for large energy is produced [33].

5. CONCLUSION

Our calculations show that within the holographic model of heavy-ion collisions using the confining vacuum background and colliding domain shock waves, the produced entropy has an asymptotic expansion whose first term provides a suitable dependence on the energy $s_2^{(0)} \propto E^{1/3}$. However, the entropy produced during a time about 0.25 fm after collision of two shock domain walls in the confining background cannot be saturated by the first term and contributions of nonleading terms have to be taken into account. This is related to the fact that to restrict our asymptotic expansion by the first term, we have to consider the asymptotic expansion at large energies. Large energies correspond to small values of the holographic coordinate z, where our approximation of the metric with b_2 factor (4.1) by the metric with b_1 factor (3.1) fails. On the other hand, as mentioned above, we cannot change the asymptotic form of b_2 because it is related to the Coulomb potential.

It seems that a possible resolution of the problem is a change of the scenario of isotropic holographic thermalization to an anisotropic short-time holographic thermalization scenario. This scenario assumes that the main part of multiplicity is produced in an anisotropic regime, and this part of multiplicity can be estimated by the trapped surface produced under a collision of the two shock waves in an anisotropic background. This scenario is accepted in recent paper [33], where collisions of shock waves in a Lifshitz-like background have been considered.

It would be interesting to compare our estimation of the thermalization time with thermalization time estimations given by the Vaidya confining bulk metric, as well as with the thermalization time obtained in the holographic hard-wall model using the homogeneous injection of energy [34].

We note that our discussion may have applications not only for heavy-ion collisions but also to studies of thermalization process in a broader class of strongly correlated multiparticle systems.

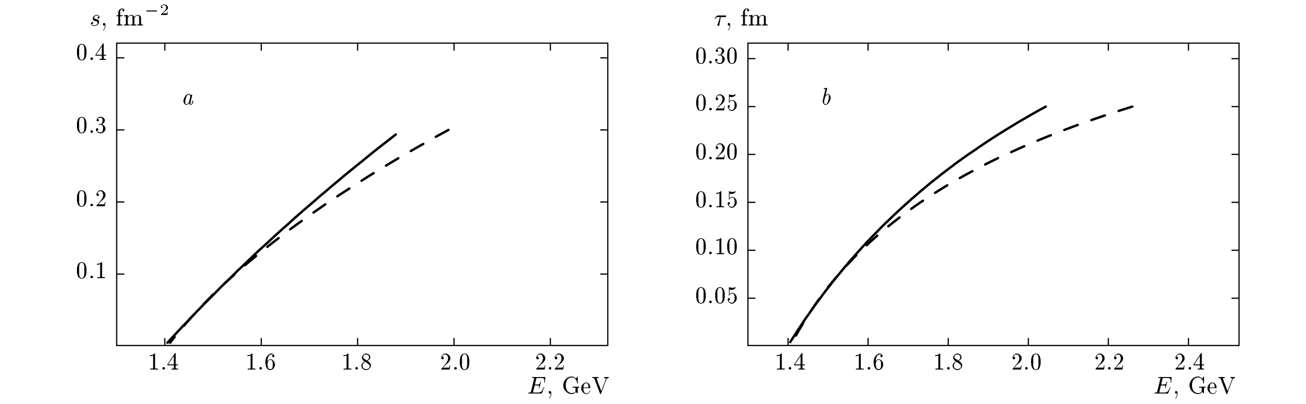

Fig. 7. (a) The entropy dependence on the energy for the confining metric (dashed line) and $b_1^2 = L_{eff}/z$ (solid line). For both lines, $z_b = 1.8$ fm and z_a varies from $z_a = 1.2$ fm to $z_a = 1.8$ fm. (b) The thermalization time dependence on the energy. The dashed line corresponds to the confining metric, the solid line corresponds to $b_1^2 = L_{eff}/z$. z_b and z_a are the same as in (a)

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