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HOLOGRAPHY BEYOND CONFORMAL INVARIANCE AND AdS ISOMETRY?

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We suggest that the principle of holographic duality can be extended beyond conformal invariance and AdS isometry. Such an extension is based on a special relation between functional determinants of the operators acting in the bulk and on its boundary, provided that the boundary operator represents the inverse propagators of the theory induced on the boundary by the Dirichlet boundary value problem in the bulk spacetime. This relation holds for operators of a general spin–tensor structure on generic manifolds with boundaries irrespective of their background geometry and conformal invariance, and it apparently underlies numerous $O(N^0)$ tests of the AdS/CFT correspondence, based on direct calculation of the bulk and boundary partition functions, Casimir energies, and conformal anomalies. The generalized holographic duality is discussed within the concept of the "double-trace" deformation of the boundary theory, which is responsible in the case of large-N CFT coupled to the tower of higher-spin gauge fields for the renormalization group flow between infrared and ultraviolet fixed points. Potential extension of this method beyond the one-loop order is also briefly discussed.

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1. INTRODUCTION

It is a great pleasure to write this paper dedicated to Valery Rubakov on the occasion of his sixties birthday. Our scientific careers have started simultaneously when we were students at the Moscow University and shared common interests in physics — classical and quantum gravity — and invariably pursued these interests, in our own ways and styles, throughout the years to come. In particular, the results of this work were conceived in the course of discussions, when Valery suggested to work out a covariant method for calculating radiative corrections in brane gravity models [1] as a means of establishing applicability limits of the perturbation theory. By the time this method has become ready for use, the peak of interest in brane models was basically over, and interests of scientific community have shifted to other areas, not the least of those being the idea of holographic duality and the AdS/CFT correspondence. Interestingly, that old method now seems to find application in this field, and, I hope, Valery will be amused to see how his suggestions are realized in this nonperturbative concept of high-energy physics.

The idea of holographic duality between a d-dimensional conformal field theory (CFT) and a theory in the $(d+1)$-dimensional anti-de Sitter (AdS) spacetime that initially began with supersymmetric models of $N \times N$-matrix valued fields [2–4] was later formulated for much simpler "vectorial" models without the need in supersymmetry [5]. These models have an infinite tower of nearly conserved higher-spin currents and in this way naturally lead to a corresponding tower of massless higher-spin gauge fields. Therefore, the holography concept implies that the dual theory should contain these fields in AdS spacetime, thus forming the Vasiliev theory of nonlinear higher-spin gauge fields [6, 7], which necessarily imply an infinite set of those, because the principle of gauge invariance for spins $s > 2$ cannot be realized for a finite tower of spins. In contrast to the original supersymmetric models in which the AdS/CFT correspondence was checked

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for supersymmetry-protected correlators, holographic duality in vectorial models underwent verification by numerous nontrivial calculations that go beyond simple kinematical or group-theoretical reasoning and extend from the tree level $O(N^1)$ to the "one-loop" order $O(N^0)$.

In particular, the calculation of the $U(N)$ singlet scalar CFT partition function on $S^1 \times S^2$ was shown to agree with the corresponding higher-spin partition function calculation in AdS_4 [8], a result extended to the $O(N)$ singlet sector of a scalar CFT [9]. Then these results were confirmed and extended to arbitrary dimensions in [10], including the comparison of thermal and Casimir energy parts of partition functions in CFT_d and AdS_{d+1} in [11]. The vanishing Casimir energy in odd-dimensional theory (associated with the absence of the conformal anomaly) implies the same on the AdS side, which is nontrivial because it implies an infinite summation over the tower of higher-spin gauge fields — the property that was observed in $d = 4$ on the AdS_5 side [12] and confirmed by an explicit summation of conformal anomaly coefficients a_s for conformal higher-spin fields on the S^4 side [13]. The list of similar results agreeing on both sides of the AdS_{d+1}/CFT_d correspondence was extended in [11].

A special class of holographic dualities is associated with the so-called double-trace deformations of the scalar CFT [14], which generates its renormalization group (RG) flow from the IR fixed point (free CFT) to the UV fixed point [15]. The associated holographic dual of this RG flow in the AdS spacetime is the transition between two different boundary conditions on the dual massless gauge fields of higher spins at the AdS boundary [12, 15].

The variety of these miraculous coincidences and the gradually extending area of validity of duality relations (from supersymmetric models to nonsupersymmetric ones, from lower spacetime dimensions and lower spins to higher ones, from divergent and Casimir energy parts of partition functions to their thermal parts, from bosons to fermions, etc) imply that there should be some deep functional reasons underlying all this and perhaps even allowing one to extend holographic duality beyond AdS isometry and conformal invariance. The goal of this paper is to show that this is indeed possible. Within the class of holographic dualities associated with the double-trace deformation of CFT, there exist universal relations for one-loop functional determinants of local and nonlocal operators on generic $(d+1)$-dimensional spacetime and its d-dimensional boundary [16] that guarantee this duality irrespective of the background geometry and conformal invariance. The only condition that relates $(d+1)$-dimensional and d-dimensional theories is that at the tree level, the boundary theory be induced from the bulk by a Dirichlet boundary value problem; then their one-loop quantum corrections dutifully match. The proof of this statement is based on linear algebra of (pseudo)differential operators and a sequence of Gaussian functional integrations. When the theory has a small parameter $1/N$ playing the role of a semiclassical Planck constant, this sequence of integrations might apparently be extended to holographic duality beyond the one-loop order $O(N^0)$.

2. DOUBLE-TRACE DEFORMATION OF CFT AND THE ADS/CFT CORRESPONDENCE

The double-trace deformation [14] of the large-N CFT of scalar fields $\Phi^i(x)$, $i = 1, \dots, N$, by the square of the $O(N)$ invariant single-trace scalar operator

$$J(x) = \Phi^i(x)\Phi^i(x),$$

$$S_{CFT}(\Phi) \to S_{CFT}(\Phi) - \frac{1}{2f}\int dx\, J^2(x),$$

leads to the renormalization group flow between the IR fixed point of the free CFT and its UV fixed point. In the limit of large N, this was clearly demonstrated by using the Habbard–Stratonovich transformation as follows [15].

We consider the generating functional $Z_{CFT}(\varphi)$ of the correlators of J for the perturbed theory with sources φ,

$$Z_{CFT}(\varphi) = \int d\Phi \times \\ \times \exp\left(-S_{CFT}(\Phi) + \frac{1}{2}\, J(\Phi)\mathbf{f}^{-1} J(\Phi) + \varphi J(\Phi)\right),$$

with

$$\frac{Z_{CFT}(\varphi)}{Z_{CFT}(0)} = \left\langle \exp\left(\frac{1}{2}\,\hat{J}\mathbf{f}^{-1}\hat{J} + \varphi\hat{J}\right)\right\rangle_{CFT} \equiv \\ \equiv \left\langle \exp(\varphi\hat{J})\right\rangle^f_{CFT}, \quad (2.1)$$

$$\hat{J}\mathbf{f}^{-1}\hat{J} = \int dx\,dy\,\hat{J}(x)\mathbf{f}^{-1}(x,y)\hat{J}(y), \\ \varphi\hat{J} = \int dx\,\varphi(x)\,\hat{J}(x). \quad (2.2)$$

For the sake of generality of our formalism we write the operator $\mathbf{f} = \mathbf{f}(x, y)$ in what follows in a rather general form even though it is ultralocal in CFT models, $\mathbf{f}(x,y) = f\delta(x,y)$, and we also use the condensed notation omitting the sign of integration over d-dimensional coordinates. A functional dependence in the d-dimensional space is denoted by round brackets, like $S_{CFT}(\Phi) \equiv S_{CFT}(\Phi(x))$, and the operators acting in this space, like $\mathbf{f}$, are boldfaced.

Representing the part of the exponential in (2.1) quadratic in J as a Gaussian integral over an auxiliary field ϕ (the Habbard–Stratonovich transform), we have

$$\Big\langle \exp(\varphi \hat{J})\Big\rangle^f_{CFT} = (\det \mathbf{f})^{1/2} \int d\phi\, \times \\ \times \left\langle \exp\left(-\frac{1}{2}\phi\mathbf{f}\phi + (\phi+\varphi)\hat{J}\right)\right\rangle_{CFT}, \qquad (2.3)$$

where $\det \mathbf{f}$ denotes the functional determinant of the operator $\mathbf{f}(x,y)$ on the space of functions of d-dimensional coordinates.

As usual in large-N CFT, we assume the vanishing expectation value of $\hat{J}$, $\langle\hat{J}\rangle = 0$, and the smallness of higher-order correlators $\langle\hat{J}\hat{J}\ldots\hat{J}\rangle$ as $N\to\infty$,

$$\Big\langle \exp(\varphi\, \hat{J})\Big\rangle_{CFT} \approx \exp\left(\frac{1}{2}\,\varphi\langle\hat{J}\,\hat{J}\rangle\,\varphi\right) \equiv \\ \equiv \exp\left(-\frac{1}{2}\,\varphi\,\mathbf{F}\varphi\right), \qquad (2.4)$$

$$\big\langle\hat{J}(x)\,\hat{J}(y)\big\rangle = -\mathbf{F}(x,y), \qquad (2.5)$$

where $-\mathbf{F}$ is the notation for the undeformed two-point correlator of J. From the new Gaussian integration in (2.3), we then have

$$\Big\langle \exp(\varphi \hat{J})\Big\rangle^f_{CFT} = (\det \mathbf{f})^{1/2}\,(\det \mathbf{F}_f)^{-1/2} \times \\ \times \exp\left(-\frac{1}{2}\,\varphi\,\frac{1}{\mathbf{F}^{-1}+\mathbf{f}^{-1}}\varphi\right), \qquad (2.6)$$

where

$$\mathbf{F}_f \equiv \mathbf{F} + \mathbf{f}. \qquad (2.7)$$

Therefore, the correlator $\langle\hat{J}\hat{J}\rangle^f_{CFT}$ in the double-trace deformed CFT interpolates between the UV and IR fixed points of the theory:

$$\langle\hat{J}\hat{J}\rangle^f_{CFT} = -\frac{1}{\mathbf{F}^{-1}+\mathbf{f}^{-1}} \to \\ \to \begin{cases} -\mathbf{F} + \ldots\,, & \mathbf{f}^{-1}\mathbf{F} \ll 1, \\ -\mathbf{f} + \mathbf{f}(\mathbf{f}^{-1}\mathbf{F})^{-1} + \ldots\,, & \mathbf{f}^{-1}\mathbf{F} \gg 1. \end{cases} \qquad (2.8)$$

For an ultralocal $\mathbf{f} = f\delta(x,y)$ in the CFT with a single-trace scalar operator $\hat{J}$ of dimension Δ, the correlator $\langle\hat{J}\hat{J}\rangle_{CFT} = -\mathbf{F}$ in the coordinate and momentum representations behaves as

$$-\mathbf{F} \sim \frac{1}{|x-y|^{2\Delta}} \sim \frac{1}{k^{d-2\Delta}}.$$

Thus, the above two limits indeed correspond to the respective UV,

$$f^{-1}\mathbf{F} \sim -\frac{1}{fk^{d-2\Delta}} \ll 1,$$

and IR,

$$f^{-1}\mathbf{F} \sim -\frac{1}{fk^{d-2\Delta}} \gg 1,$$

fixed points. In the IR limit, the correlator (modulo the contact term $\mathbf{f} = f\delta(x,y)$) is dominated by the second term

$$\mathbf{f}(\mathbf{f}^{-1}\mathbf{F})^{-1} \sim \frac{1}{|x-y|^{1/(d-2\Delta)}}$$

in the long-distance regime $|x-y| \gg |f|^{1/(d-2\Delta)}$ [15]. The renormalization group flow interpolates between two phases in which the operator $J(x)$ has different dimensions, $\Delta = \Delta_+$ in IR and $d/2 - \Delta = \Delta_-$ in UV.

This double-trace deformation picture also applies in the context of the dual description of higher-spin conformal gauge fields [12, 15]. Since the $O(N)$ or $U(N)$ scalar or fermion CFT has a tower of nearly conserved higher-spin currents $J_{\mu_1\ldots\mu_s}(x)$, their gauging results in the corresponding tower of higher-spin gauge fields $\varphi^{\mu_1\cdots\mu_s}(x)$:

$$J = J_{\mu_1\ldots\mu_s}(x) \sim \Phi^i(x)\,\partial_{\mu_1}\ldots\partial_{\mu_s}\Phi^i(x), \\ \varphi = \varphi^{\mu_1\cdots\mu_s}(x). \qquad (2.9)$$

This class of theories was conjectured to be dual to Vasiliev theories of higher-spin gauge fields in AdS (a very incomplete list of references is contained in [17–22]). The description of these dualities can be summarized as follows.

In AdS_{d+1} with the coordinates $X \equiv X^A = X^1,\ldots,X^{d+1}$, there exist totally symmetric transverse gauge fields $\varPhi = \varPhi^{A_1\cdots A_s}(X)$ with the quadratic action

$$S_{d+1}[\varPhi] = \int\limits_{\mathrm{AdS}} d^{d+1}X\,\mathcal{L}\big(\varPhi(X), \nabla\varPhi(X)\big) \qquad (2.10)$$

that generates linearized equations for massless spin-s tensor fields. The covariant form of this quadratic action is known [23], but its concrete expression is not needed in what follows. At the boundary of AdS_{d+1}, which is either R^d or S^d (or $S^1\times S^{d-1}$ in the thermal case) and is parameterized by coordinates $x \equiv x^\mu = x^1,\ldots,x^d$ via the embedding functions $X = e(x)$, the boundary values of the tangential components of $\varPhi$,

$$\varPhi| \equiv \varPhi^{\mu_1\cdots\mu_s}(e(x)) = \varphi^{\mu_1\cdots\mu_s}(x), \qquad (2.11)$$

represent the gauge fields of the d-dimensional CFT, coupled to its conserved higher-spin currents. Then the $\mathrm{AdS}_{d+1}/\mathrm{CFT}_d$ conjecture means that the generating functional of the correlators of conserved currents of the undeformed CFT living on the boundary $\partial(\mathrm{AdS}_{d+1})$ can be obtained from the path integral of the dual theory of gauge fields in the AdS_{d+1} spacetime subject to Dirichlet boundary conditions at this boundary:

$$\Big\langle \exp(\varphi\hat{J})\Big\rangle_{CFT} = \frac{\displaystyle\int\limits_{\Phi|=\varphi} D\Phi \exp(-S_{d+1}[\Phi])}{\displaystyle\int\limits_{\Phi|=0} D\Phi \exp(-S_{d+1}[\Phi])}. \tag{2.12}$$

In what follows, we always use a vertical bar to denote the restriction of a bulk quantity to the boundary.

Using this relation in the right hand side of (2.3), we obtain

$$\Big\langle \exp(\varphi\hat{J})\Big\rangle^f_{CFT} = (\det\mathbf{f})^{1/2}\frac{\displaystyle\int d\phi \exp\left(-\frac{1}{2}\phi\mathbf{f}\phi\right)\int\limits_{\Phi|=\phi+\varphi} D\Phi \exp(-S_{d+1}[\Phi])}{\displaystyle\int\limits_{\Phi|=0} D\Phi \exp(-S_{d+1}[\Phi])} =$$
$$= (\det\mathbf{f})^{1/2}\times \frac{\displaystyle\int\limits_{\text{all }\Phi} D\Phi \exp\left(-S_{d+1}[\Phi] - \frac{1}{2}(\Phi|-\varphi)\mathbf{f}(\Phi|-\varphi)\right)}{\displaystyle\int\limits_{\Phi|=0} D\Phi \exp(-S_{d+1}[\Phi])}, \tag{2.13}$$

where the total action in the functional integrand contains both the bulk part and the boundary part located at $\partial(\mathrm{AdS}_{d+1}) = M_d$

$$S[\Phi] \equiv S_{d+1}[\Phi] + \frac{1}{2}(\Phi| - \varphi)\mathbf{f}(\Phi| - \varphi) = \\ = \frac{1}{2}\int\limits_{\mathrm{AdS}} d^{d+1}X\, \Phi(X)\overleftrightarrow{F}(\nabla)\,\Phi(X) + \\ + \frac{1}{2}\int\limits_{M_d} d^dx\big(\Phi|(x) - \varphi(x)\big)\mathbf{f}\big(\Phi|(x) - \varphi(x)\big), \tag{2.14}$$

and the integration in the denominator runs over the fields Φ both in the bulk and on the boundary. This means that the boundary conditions on a saddle-point configuration Φ_f are affected by the boundary part of the action (that is, by $\mathbf{f}$, which is a kernel of the quadratic boundary action in (2.14)), and that is why we label it by the subscript f.

The kernel of the bulk Lagrangian in given by the second-order operator $F(\nabla)$, whose derivatives $\nabla \equiv \partial_X$ are integrated by parts in such a way that they form bilinear combinations of first-order derivatives acting on two different fields (this is denoted by the left-right arrow over $F(\nabla)$). Integration by parts gives nontrivial surface terms on the boundary. In particular, this operation results in the Wronskian relations for generic test functions $\Phi_{1,2}(X)$ on any spacetime domain M_{d+1} with the boundary ∂M_{d+1}:

$$\int\limits_{M_{d+1}} d^{d+1}X\left(\Phi_1\overrightarrow{F}(\nabla)\Phi_2 - \Phi_1\overleftarrow{F}(\nabla)\,\Phi_2\right) = \\ = -\int\limits_{\partial M_{d+1}} d^dx\left(\Phi_1\,\overrightarrow{W}\Phi_2 - \Phi_1\,\overleftarrow{W}\Phi_2\right), \tag{2.15}$$

$$\int\limits_{M_{d+1}} d^{d+1}X\,\Phi_1\overleftrightarrow{F}\Phi_2 = \int\limits_{M_{d+1}} d^{d+1}X\,\Phi_1(\overrightarrow{F}\Phi_2) + \\ + \int\limits_{\partial M_{d+1}} d^dx\,\Phi_1\overrightarrow{W}\Phi_2\,\Big|. \tag{2.16}$$

The arrows here indicate the direction of the action of derivatives on either Φ_1 or Φ_2. These relations can be regarded as a definition of the first-order Wronskian operator $W = W(\nabla)$ for $F(\nabla)$. In simple models on the AdS background and its conformal boundary, parameterized by coordinates $X^A = y, x^\mu$, it basically reduces to the normal to the boundary derivative, $W(\nabla) \sim \partial_y$.

The saddle-point approximation for the path integral in the numerator of (2.13) is dominated by the contribution of a stationary point of the total action (2.14). In view of (2.16), the requirement of the vanishing first-order variation contains bulk and surface terms,

$$\int\limits_{\mathrm{AdS}_{d+1}} d^{d+1}X\,\delta\Phi\,(\overrightarrow{F}\Phi)+ \\ +\int\limits_{M_d} d^dx\,\delta\Phi\left(\big(\overrightarrow{W}+\mathbf{f}\big)\,\Phi\,|-\mathbf{f}\varphi\right)=0, \quad (2.17)$$

which must vanish independently because $\delta\Phi(X)$ is nonvanishing both in the bulk and at the boundary since Φ is being integrated over all spacetime points. Thus we obtain equations of motion for a stationary configuration $\Phi_f(X)$ in the bulk and the boundary condition at M_d,

$$F(\nabla)\,\Phi_f(X)=0, \quad \big(\overrightarrow{W}\,(\nabla)+\mathbf{f}\big)\,\Phi_f\big|=\mathbf{f}\varphi. \quad (2.18)$$

The latter is the generalized Neumann (or Robin) boundary condition involving the normal-to-the-boundary derivative of $\Phi(X)$ contained in $W(\nabla)$. On the contrary, $\mathbf{f}$ is an entirely d-dimensional operator, which is ultralocal in the CFT theory of double-trace deformations, but we keep it as a more general (differential or even nonlocal pseudo-differential) operator in M_d in what follows. The solution to this boundary value problem can be given in terms of the Green's function $G_{N_f}(X,Y)$ of $F(\nabla)$ subject to this (homogeneous) Neumann boundary condition:

$$\begin{aligned} &F(\nabla)\,G_{N_f}(X,Y)=\delta(X,Y),\\ &(\overrightarrow{W}+\mathbf{f})\,G_{N_f}(X,Y)\Big|_{X\in\partial M_{d+1}}=0, \end{aligned} \quad (2.19)$$

and is given by

$$\Phi_f(X)=\int\limits_b dy\,G_{N_f}(X,y)\,\mathbf{f}\varphi(y)\equiv G_{N_f}|\,\mathbf{f}\varphi. \quad (2.20)$$

Here,

$$G_{N_f}(X,y)\equiv G_f(X,Y)\,|_{Y=e(y)}$$

is the notation for the boundary-to-bulk propagator — the Green's function with its second argument put on the boundary via the embedding function $Y^A=e^A(y^\mu)$. Its subscript indicates that this Green's function is determined by the generalized Neumann boundary conditions with a particular function $\mathbf{f}$.

Then the stationary (on-shell) value of the action (2.14) is

$$S[\Phi_f]=\frac{1}{2}\int\limits_b dx\,dy\,\varphi(x)\mathbf{f}\,\times \\ \times\Big(\mathbf{f}^{-1}(x,y)-G_{N_f}(x,y)\Big)\mathbf{f}\varphi(y)\equiv \\ \equiv\frac{1}{2}\,\varphi\big[\mathbf{f}-\mathbf{f}\,G_{N_f}\big|\big|\,\mathbf{f}\big]\varphi, \quad (2.21)$$

where

$$G_{N_f}(x,y)\equiv G_{N_f}(X,Y)\,|_{X=e(x),\,Y=e(y)}\equiv \\ \equiv G_{N_f}|| \quad (2.22)$$

is the notation for the boundary-to-boundary propagator — the restriction of both Green's function arguments to the boundary, denoted by two vertical bars for brevity. Again using the condensed notation on the boundary, we omitted the sign of integration over the boundary coordinates in (2.20), (2.21)[1]. Thus, finally, we have

$$\int D\Phi\exp\left(-S_{d+1}[\Phi]-\frac{1}{2}(\Phi|-\varphi)\mathbf{f}(\Phi|-\varphi)\right)= \\ =\big(\mathrm{Det}_{N_f}F\big)^{-1/2}\times \\ \times\exp\left(-\frac{1}{2}\,\varphi\big[\mathbf{f}-\mathbf{f}\,G_{N_f}\big|\big|\,\mathbf{f}\big]\varphi\right), \quad (2.23)$$

where

$$\mathrm{Det}_{N_f}F=(\mathrm{Det}\,G_{N_f})^{-1}$$

is the bulk functional determinant of F on the space of functions subject to the generalized Neumann boundary conditions (2.19). The denominator of (2.13) is given of course by the functional determinant with the Dirichlet boundary conditions corresponding to $\Phi|=0$,

$$\int\limits_{\Phi|=0} DA\exp(-S_{d+1}[\Phi])=\big(\mathrm{Det}_DF\big)^{-1/2}. \quad (2.24)$$

Functional determinants of operators acting in the $(d+1)$-dimensional bulk here and in what follows are denoted by Det with a subscript indicating the type of boundary conditions for the class of functions on which the determinant is calculated (in contrast to det for operators acting on the boundary).

Substituting these results in (2.13), we obtain

$$\Big\langle\exp(\varphi\hat{J})\Big\rangle^f_{CFT}=(\det\mathbf{f})^{1/2}\left(\frac{\mathrm{Det}_{N_f}F}{\mathrm{Det}_DF}\right)^{-1/2}\times \\ \times\exp\left(-\frac{1}{2}\,\varphi\big[\mathbf{f}-\mathbf{f}\,G_{N_f}\big|\big|\,\mathbf{f}\big]\varphi\right), \quad (2.25)$$

and the comparison of the exponentials and preexponential factors here and in (2.6) then yields the tree-level and one-loop relations

$$G_{N_f}\big|\big|=\mathbf{F}_f^{-1}, \quad (2.26)$$

$$\mathrm{Det}_{N_f}F=\det\mathbf{F}_f\;\mathrm{Det}_DF. \quad (2.27)$$

[1] It is useful to apply this DeWitt condensed notation for integral operations on the boundary, because these operations have properties of formal matrix contraction and multiplication.

These relations are direct consequences of AdS/CFT correspondence (2.12) in the lowest two orders of the $1/N$ expansion, but the logic of this derivation can be reversed. If we start with these relations, then the holographic duality is enforced in this approximation. As we see shortly, a simple exercise on linear algebra and Gaussian integration provides a proof that these relations are very general and hold for a generic second-order differential operator $F(\nabla)$ acting on an arbitrary spin–tensor field for a generic manifold with a boundary. By a special rule, it induces a (generically nonlocal pseudodifferential) operator $\mathbf{F}_f$ acting on the boundary that can be regarded as the inverse (boundary-to-boundary) propagator of the surface theory induced from the bulk theory. No particular geometry of the bulk spacetime or its boundary is assumed in this construction. All this means that the holographic duality between d- and $(d+1)$-dimensional theories can be extended beyond AdS isometries and conformal invariance under the single assumption that the d-dimensional theory is induced from the bulk theory by integrating out its bulk degrees of freedom.

3. HOLOGRAPHIC DUALITY AND THE INDUCED BOUNDARY THEORY

For Eqs. (2.26)–(2.27) to hold, the boundary operator $\mathbf{F}_f$ should be related to the operator $F(\nabla)$ acting in the bulk and to relevant boundary conditions N_f on ∂M_{d+1}. To establish this, we address the duality relation (2.12) at the tree level. For a quadratic $(d+1)$-dimensional action of the form

$$S_{d+1}[\Phi] = \frac{1}{2}\int\limits_{\mathrm{AdS}} dX\, \Phi(X)\, \overleftrightarrow{F}(\nabla)\, \Phi(X), \tag{3.1}$$

the tree-level holographic duality (2.12) implies that

$$\Big\langle \exp(\varphi\hat{J})\Big\rangle_{CFT} = \frac{\exp(-S_{d+1}[\Phi_D(\varphi)])}{\exp(-S_{d+1}[\Phi_D(0)])}, \tag{3.2}$$

where $\Phi_D(\varphi)$ is a solution of the problem, $F(\nabla)\phi_D(X) = 0$, $\phi_D\,| = \varphi(x)$, with inhomogeneous Dirichlet boundary conditions. In view of relations (2.15) and (2.16), this solution and its on-shell value of the action can be represented in terms of the Dirichlet Green's function $G_D(X,Y)$,

$$\begin{aligned} &F(\nabla)\, G_D(X,Y) = \delta(X,Y),\\ &G_D(X,Y)\Big|_{X=e(x)} = 0, \end{aligned} \tag{3.3}$$

as

$$\Phi_D(X) = -\int\limits_{M_d} dy\; G_D(X,Y)\,\overleftarrow{W}\Big|_{Y=e(y)}\varphi(y) \equiv \\ \equiv -G_D\,\overleftarrow{W}\,|\;\varphi, \tag{3.4}$$

$$S[\Phi_D] = \frac{1}{2}\int\limits_{M_d} dx\,dy\,\varphi(x)\Big[-\overrightarrow{W}\,G_D\,\overleftarrow{W}\,(x,y)\Big]\varphi(y) \equiv \\ \equiv \frac{1}{2}\,\varphi\Big[-\overrightarrow{W}\,G_D\,\overleftarrow{W}\,||\Big]\varphi. \tag{3.5}$$

The expression $\overrightarrow{W}\,G_D\,\overleftarrow{W}\,||$ implies that Wronskian operators act on both arguments of the kernel of the Dirichlet Green's function, and the result is restricted to the boundary,

$$\overrightarrow{W}\,G_D\,\overleftarrow{W}\,||\,(x,y) \equiv \\ \equiv \overrightarrow{W}(\nabla_X)\, G_D(X,Y)\, \overleftarrow{W}(\nabla_Y)\Big|_{X=e(x),\,Y=e(y)}. \tag{3.6}$$

The result in (3.5) is exactly the tree-level boundary effective action obtained from the original action (4.3) by integrating out the bulk fields subject to fixed boundary values $\varphi(x)$, $\mathbf{S}(\varphi) = S\,[\phi_D(\varphi)]$. Accordingly, the kernel of the quadratic form of (3.5) in φ is the inverse propagator of the boundary theory,

$$\mathbf{F} \equiv \frac{\delta^2\mathbf{S}}{\delta\varphi\,\delta\varphi} = -\,\overrightarrow{W}\,G_D\,\overleftarrow{W}\,||, \tag{3.7}$$

which is generically a nonlocal operator in the space of boundary coordinates x. Thus, the generating functional of correlation functions in the undeformed CFT is given by

$$\begin{aligned} &\Big\langle \exp(\varphi\hat{J})\Big\rangle_{CFT} = \exp\left(-\frac{1}{2}\,\varphi\mathbf{F}\varphi\right),\\ &\langle\hat{J}\hat{J}\rangle_{CFT} = -\mathbf{F}, \end{aligned} \tag{3.8}$$

with the two-point correlator of the $\hat{J}$ (cf. Eq. (2.5)) induced from the $(d+1)$-dimensional bulk. In fact, this is a basic relation of the linearized tree-level AdS/CFT correspondence, which has been checked in numerous models starting with [3, 4]. This fixes the boundary operator $\mathbf{F}_f = \mathbf{F} + \mathbf{f}$ in the right-hand sides of our basic relations (2.26), (2.27) in terms of the bulk operator $F(\nabla)$. We now proceed with the proof of these relations.

4. FUNCTIONAL DETERMINANTS RELATIONS

The idea of the derivation of relations (2.26) and (2.27), that was first given in [16], is based on a sequence of Gaussian functional integrations. Any action $S[\Phi]$ quadratic in its field $\Phi(X)$ can give rise to two Gaussian functional integrals. One of them is of the form

$$Z = \int\limits_{\rm all} D\Phi\, \exp\left(-S[\Phi]\right), \qquad (4.1)$$

where integration runs over all fields both in the bulk and on its boundary, and the other,

$$Z(\varphi) = \int\limits_{\Phi|=\varphi} D\Phi \exp\left(-S[\Phi]\right), \qquad (4.2)$$

implies integration with fixed values of Φ at the boundary. Obviously, these path integrals are related by the equation $Z = \int d\varphi\, Z(\varphi)$, and hence independent calculations of its left- and right-hand sides yield certain tree-level and one-loop relations. As we see in what follows, under an appropriate choice of $S[\Phi]$ they turn out to be exactly the ones advocated above.

We consider the bulk–boundary action of the field $\Phi(X)$ in the $(d+1)$-dimensional (bulk) spacetime M_{d+1} and its boundary $M_d = \partial M_{d+1}$,

$$S[\Phi] = \frac{1}{2}\int\limits_{M_{d+1}} dX\, \Phi(X)\, \overset{\leftrightarrow}{F}(\nabla)\, \Phi(X) + \\ + \int\limits_{M_d} dx \left(\frac{1}{2}\varphi(x)\, \mathbf{f}(\partial)\, \varphi(x) + j(x)\, \varphi(x)\right), \qquad (4.3)$$

$$\Phi\big| \equiv \Phi(X)\big|_{\partial M_{d+1}} = \Phi(e(x)) = \varphi(x). \qquad (4.4)$$

We recall that the boundary embedding into the bulk in terms of $x = x^\mu$ is denoted by $X^A = e^A(x^\mu)$ and, as previously, the vertical bar denotes the restriction of a bulk quantity to the boundary. The field $\Phi(X)$ and the second-order differential operator $F(\nabla)$ have absolutely generic spin–tensor structure, and there are no restrictions on the geometry of the bulk M_{d+1} and its boundary M_d. Similarly to (2.16), the derivatives of $F(\nabla)$ in the bulk part are integrated by parts in such a way that they form bilinear combinations of first-order derivatives. As a kernel, the boundary part of the action contains some local or nonlocal (pseudodifferential) operator $\mathbf{f} = \mathbf{f}(\partial)$, $\partial = \partial_x$ acting in the space of x. In contrast to the bulk part, integration by parts on the boundary is irrelevant for our purposes, because M_d is assumed either to be closed compact or to have trivial vanishing boundary conditions at its infinity. Function $j(x)$ plays the role of sources conjugate to $\varphi(x)$ and located on the boundary.

The calculation of (4.2) repeats the derivation in Sec. 2, and the answer is given by

$$Z = \left(\mathrm{Det}_{N_f} F\right)^{-1/2} \exp(-S[\Phi_f]), \qquad (4.5)$$

where Φ_f is a stationary point of action (4.3) satisfying the problem with inhomogeneous generalized Neumann boundary conditions

$$F(\nabla)\, \phi_f(X) = 0, \quad (\overrightarrow{W} + \mathbf{f})\, \phi_f\, \Big| + j(x) = 0, \qquad (4.6)$$

and $\mathrm{Det}_{N_f} F$ denotes the bulk ($(d+1)$-dimensional) functional determinant of $F(\nabla)$ on the space of functions subject to these (homogeneous) boundary conditions.

Similarly to (2.17), problem (4.6) naturally follows from the action (4.3) and Wronskian relations for $F(\nabla)$, because the variation of the action is given by the sum of bulk and boundary terms, which should vanish separately since the action should be stationary also with respect to arbitrary variations of the boundary fields $\delta\varphi$. The Neumann Green's function of this problem, Eq. (2.19), gives a solution of (4.6) that in the condensed notation of Sec. 2 (cf. Eq. (2.20)) has the form $\Phi_f(X) = -G_{N_f}|\, j$ and gives rise to the on-shell value of the action as a functional of the boundary source $j(x)$:

$$S[\phi_f] = -\frac{1}{2}\int\limits_b dx\, dy\, j(x)\, G_{N_f}(x,y)\, j(y) \equiv \\ \equiv -\frac{1}{2}\, j\, G_{N_f}||\, j. \qquad (4.7)$$

Here, again,

$$G_{N_f}(x,y) \equiv G_{N_f}(X,Y)\, |_{X=e(x),\, Y=e(y)} \equiv G_{N_f}||$$

is the notation for the boundary-to-boundary propagator, with the restriction of both Green's function arguments to the boundary denoted by two vertical bars for brevity. To simplify the formalism, we omitted the sign of integration over the boundary coordinates in (4.7)[2]. Thus, we finally have

$$Z = \left(\mathrm{Det}_{N_f} F\right)^{-1/2} \exp\left(\frac{1}{2}\, j\, G_{N_f}||\, j\right). \qquad (4.8)$$

Alternatively, we can calculate the same integral by splitting the integration procedure into two steps: first integrating over bulk fields with fixed boundary values and then integrating over the latter. This allows

[2] It is useful to apply this DeWitt condensed notation for integral operations on the brane, because these operations have properties of formal matrix contraction and multiplication.

rewriting the same result in the form $Z = \int d\varphi\, Z(\varphi)$, where the inner integral (4.2),

$$Z(\varphi) \equiv \int\limits_{\Phi|=\varphi} D\Phi \exp\left(-S[\Phi]\right) = \\ = \left(\mathrm{Det}_D\, F\right)^{-1/2} \exp(-S[\Phi_D]), \quad (4.9)$$

is given by the contribution of the solution of Dirichlet problem (3.4) with the Dirichlet Green's function $G_D(X,Y)$ (cf. Eq. (3.3)). The corresponding on-shell action equals

$$S[\Phi_D] = \frac{1}{2}\varphi\Big[-\overrightarrow{W}G_D\overleftarrow{W}\,||+\mathbf{f}\Big]\varphi + j\,\varphi \equiv \\ \equiv \frac{1}{2}\varphi\,\mathbf{F}_f\,\varphi + j\,\varphi. \quad (4.10)$$

The part quadratic in φ here coincides with the induced action (3.5) in Sec. 3 modulo the additional $\mathbf{f}$-term.

Substituting (4.9) with (4.10) in $Z = \int d\varphi\, Z(\varphi)$, we again obtain the Gaussian integral over φ that is saturated by the saddle point φ_0 of the above boundary action (4.10), $\varphi_0 = -\mathbf{F}_f^{-1} j$, and the final result is

$$Z = \left(\mathrm{Det}_D\, F\right)^{-1/2}\left(\det \mathbf{F}_f\right)^{-1/2} \times \\ \times \exp\left(\frac{1}{2}\, j\,\mathbf{F}_f^{-1} j\right), \quad (4.11)$$

where we recall that det denotes functional determinants in the d-dimensional boundary theory.

In view of the arbitrariness of the boundary source j, comparing the tree-level and one-loop (preexponential) parts with those of (4.8) immediately yields two relations

$$G_{N_f}\,|| = \mathbf{F}_f^{-1} \equiv \Big[-\overrightarrow{W}G_D\overleftarrow{W}\,||+\mathbf{f}\Big]^{-1}, \quad (4.12)$$

$$\mathrm{Det}_{N_f}\, F = \det \mathbf{F}_f\, \mathrm{Det}_D F. \quad (4.13)$$

These are exactly the relations (2.26), (2.27) that underly the dual AdS description of the double-trace deformation of CFT models. The one-loop-order equation (4.13) here relates functional determinants of the bulk operator on different functional spaces defined by Neumann and Dirichlet boundary conditions and intertwines them via the determinant of the boundary operator[3)].

When applied to a large-N CFT, these relations describe a deformation of the boundary CFT that induces a renormalization group flow from the infrared ($f = \infty$) to the ultraviolet ($f = 0$) fixed points of this theory and generates the corresponding increase in the central charge [24] (or the conformal anomaly a-coefficient in the 4D case [25]). From (4.13), the change of the f parameter is determined by the ratio

$$\frac{\mathrm{Det}_{N_{f_1}} F}{\mathrm{Det}_{N_{f_2}} F} = \frac{\det \mathbf{F}_{f_1}}{\det \mathbf{F}_{f_2}} \equiv \frac{\det\big(\mathbf{1} + f_1^{-1}\,\mathbf{F}\big)}{\det\big(\mathbf{1} + f_2^{-1}\,\mathbf{F}\big)}, \quad (4.14)$$

where in the second equality we took into account that for an ultralocal kernel $\mathbf{f} = f\delta(x,y)$, its determinant $\det \mathbf{f} = 1$ (e. g., in dimensional regularization) does not give any contribution. This is the relation that was formulated in [26, 27] as the ratio of the bulk theory partition functions with different values of the f coefficient in terms of the $\langle \hat{J}\hat{J}\rangle_{CFT}$ correlator of the unperturbed boundary CFT, $-\mathbf{F} = \overrightarrow{W}G_D\overleftarrow{W}\,||$[4)].

While the right-hand side of this equation was derived on the CFT side by using the Hubbard–Stratonovich transform [15], the left-hand side equality was proved in [26] by using an expression for the functional determinant of the Sturm–Lioville operator in terms of its basis functions [28, 29] or by the explicit use of the operator spectra on the AdS background. On the contrary, the power of our result (4.13) is that it holds for generic bulk–boundary backgrounds for operators $F(\nabla)$ and $\mathbf{f}$ of the most general type and admits any type of covariant regularization for UV divergences [16].

4.1. The case of gauge theories

An important remark is that the functional determinant duality relation (4.13) also applies to gauge theories, which is the case of major interest for us because our goal is the holographic duality for towers of higher-spin fields in the bulk and its boundary. A potential difficulty here might be the fact that in the bulk, the totally symmetric spin-s fields $\Phi^{A_1\ldots A_s}(X)$ have bulk indices ranging over $d+1$ values, while the boundary fields $\varphi^{\mu_1\ldots\mu_s}(x)$ have only d-dimensional tensor components, and hence the bulk F and boundary $\mathbf{F}$ operators have essentially different spin structures. This controversy is reconciled, however, by noting that spin $s > 0$ theories are gauge invariant under transformations of the form

$$\Phi \to \Phi^{\Xi} = \Phi + \Delta^{\Xi}\Phi, \quad \varphi \to \varphi^{\xi} = \varphi + \Delta^{\xi}\varphi, \\ \Xi^{||}\,| = \xi, \quad (4.15)$$

[3)] This might perhaps be a field-theoretic analogue of Vasiliev's determinant relation in the operator algebra of conformal currents [22] based on different star products — a counterpart of different functional spaces on the field theory side.

[4)] To compare (4.14) with the formalism in [26] we should bear in mind that our $\mathbf{f}$ is the negative inverse of f in [26], and our $\mathbf{F}$ is the negative of the $\langle \hat{J}\hat{J}\rangle_{CFT}$ correlator denoted by G in [26].

$$\Delta^{\Xi}\Phi^{A_1\ldots A_s}(X) = \nabla^{(A_1}\Xi^{A_2\ldots A_s)}(X), \qquad (4.16)$$

$$\Delta^{\xi}\varphi^{\mu_1\ldots\mu_s}(x) = D^{(\mu_1}\xi^{\mu_2\ldots\mu_s)}, \qquad (4.17)$$

generated by a spin-$(s-1)$ field $\Xi(X)$ with the tangential components $\Xi^{\|} = \xi(x)$ (D_μ denotes the covariant derivative on the boundary). The balance of physical degrees of freedom in the bulk and on the boundary is then maintained by imposing gauge conditions fixing these transformations. Background covariant gauges of the form

$$H(\Phi) = H^{A_1\ldots A_{s-1}}(X) \sim \nabla_B\Phi^{BA_1\ldots A_{s-1}}(X) = 0$$

fix them incompletely: there remain residual gauge transformations that are the zero modes of the second-order bulk Faddeev–Popov operator $Q = Q^{A_1\ldots A_{s-1}}_{B_1\ldots B_{s-1}}$ defined by

$$\Delta^{\Xi}H(\Phi) = Q\Xi. \qquad (4.18)$$

These modes are parameterized by the boundary values $\Xi| = \xi(x)$, which perform gauge shift (4.15) of the boundary fields φ. Therefore, these residual gauge transformations can be gauged out by imposing the boundary gauge conditions on φ of the form

$$h(\varphi) = h^{\mu_1\ldots\mu_{s-1}}(x) \sim D_\nu\varphi^{\nu\mu_1\ldots\mu_{s-1}}(x).$$

In their turn, these generate a nondegenerate boundary Faddeev–Popov operator $\mathbf{Q} = \mathbf{Q}^{\mu_1\ldots\mu_{s-1}}_{\nu_1\ldots\nu_{s-1}}$ defined by

$$\Delta^{\xi}h(\varphi) = \mathbf{Q}\xi. \qquad (4.19)$$

Altogether, this is equivalent to introducing the Faddeev–Popov gauge-breaking factor $\delta\,[H(\Phi)]\,\times$ $\times\,\delta(h(\varphi))M_{H,h}[\Phi]$ under the path integral sign with[5)]

$$(M_{H,h}[\Phi])^{-1} = \int D\Xi\,\delta\left[H(\Phi^{\Xi})\right]\,\delta(h(\varphi^{\xi})) \equiv \equiv (\mathrm{Det}_N Q)^{-1}. \qquad (4.20)$$

Again, using the obvious relation

$$\int D\Xi\,(\ldots) = \int d\xi \int\limits_{\Xi|=\xi} D\Xi\,(\ldots)$$

(meaning that the integral over the full algebra of gauge transformations decomposes into the integration over the algebra in the bulk with fixed transformations on the boundary and the subsequent integration over these boundary transformations), we evaluate the Faddeev–Popov gauge fixing factor as

$$\int D\Xi\,\delta\left[H(\Phi^{\Xi})\right]\,\delta\big(h(\varphi^{\xi})\big) = \int d\xi\,\delta(\mathbf{Q}\xi)\int\limits_{\Xi|=\xi} D\Xi\,\delta[Q\Xi] = (\det\mathbf{Q})^{-1}\int\limits_{\Xi|=0} D\Xi\,\delta[\,Q\Xi\,] = (\det\mathbf{Q})^{-1}(\mathrm{Det}_D Q)^{-1}, \qquad (4.21)$$

which similarly to (4.13) factores into the product of the bulk Dirichlet and boundary counterparts. We can use the 't Hooft trick to convert delta-function type gauges into the bulk and boundary gauge breaking terms

$$\delta\,[H(\Phi)]\,\delta(h(\varphi)) \to \to \exp\left(-\frac{1}{2}\int d^{d+1}X\,H^2(\Phi(X)) - -\frac{1}{2}\int d^dx\,h^2(\varphi(x))\right). \qquad (4.22)$$

They contribute their respective gauge-breaking parts to the operators F and $\mathbf{F}_f$ and make both of them nondegenerate. Then, ultimately in higher-spin gauge theories, relation (4.13) for the dual one-loop prefactors takes the form

$$\frac{\mathrm{Det}_N Q}{\left(\mathrm{Det}_{N_f}F\right)^{1/2}} = \frac{\det\mathbf{Q}}{\left(\det\mathbf{F}_f\right)^{1/2}}\,\frac{\mathrm{Det}_D Q}{\left(\mathrm{Det}_D F\right)^{1/2}}, \qquad (4.23)$$

and it can again be laid in the basis of holographic duality. Details of this bulk–boundary factorization, including the Ward identities, which guarantee gauge independence of both boundary and bulk factors in the right-hand side of this relation (of the choice of $h(\varphi(x))$ and $H(\Phi(X))$ respectively), can be found in [31]. The analysis in [31] was done in the spin-two case, but it can easily be extended to all s.

5. CONCLUSIONS AND DISCUSSION

Thus, we have a strong evidence that the holography principle extends beyond conformal symmetry and AdS isometry of the underlying theories. In the class of AdS/CFT dualities associated with the double-trace

5) The ghost factor we use here involves a generic gauge, whereas the works on higher spin gauge fields on AdS background [30] usually use a particular (DeWitt background covariant) gauge defined by the generator of the gauge transformation. Moreover, in [30] the power of the Faddeev–Popov determinant in the ghost factor is different, because all the determinants are defined on functional spaces of symmetric tensor fields constrained by conditions of transversality and tracelessness.

deformation of CFT, holography is dutifully enforced at the one-loop level wherever the holographic duality holds at the tree level in the form of the boundary theory induced from the bulk via the Dirichlet boundary value problem. This opens up prospects for the further progress in the holographic concept. First, the arbitrariness of the background gives a firm ground for the tree-level duality beyond the quadratic approximation for the action of bulk and boundary theories. Second, the obvious identity

$$\int\limits_{\text{all}} D\Phi\, e^{-NS[\Phi]} = \int d\varphi \int\limits_{\Phi|=\varphi} D\Phi\, e^{-NS[\Phi]} \qquad (5.1)$$

applied to a nonlinear bulk–boundary action with $1/N \to 0$ playing the role of $\hbar$,

$$S[\Phi] = \int\limits_{M_{d+1}} d^{d+1}X \left(\frac{1}{2}\, S_{(2)}\,\Phi^2 + \frac{1}{3!} S_{(3)} \Phi^3 + \ldots\right) + \\ + \int\limits_{M_d} d^d x \left(\frac{1}{2}\,\mathbf{f}_{(2)}\varphi^2 + \frac{1}{3!}\,\mathbf{f}_{(3)}\varphi^3 + \ldots\right), \qquad (5.2)$$

suggests sequence of new higher-loop identities starting with (4.12), (4.13) and involving tree-level vertices of the action. This might help extending the known results on the AdS/CFT correspondence beyond the one-loop approximation.

Of course, there are certain limitations in the applicability of the suggested method. It seems to be working in only one direction: from a local theory in the bulk to a potentially nonlocal theory on the boundary (we recall that the critical point of our derivation is a local bulk operator $F(\nabla)$ of the second order in derivatives, the corresponding definition of its Wronskian operator $W(\nabla)$, and the related Dirichlet and Neumann boundary value problems). At the same time, known numerous checks of the AdS/CFT correspondence [11] start from a free local CFT at the boundary and match with partition functions of local, although apparently nonlinear, dual theories in the AdS bulk. In order to invert the setting in our holography derivation, perhaps one might start with the attempt to solve a mathematical problem as follows. Given a generic boundary action functional $\mathbf{S}(\varphi)$ of the field $\varphi(x)$, find the functional of the bulk action $S[\Phi]$ on M_{d+1} whose on-shell value (subject to Dirichlet data on ∂M_{d+1}) matches $\mathbf{S}(\varphi)$,

$$\frac{\delta S[\Phi_0]}{\delta \Phi_0} = 0, \quad \Phi_0\big| = \varphi \to S[\Phi_0(\varphi)] = \mathbf{S}(\varphi). \qquad (5.3)$$

Apparently, this problem does not have a unique solution, but the requirement of locality of $S[\Phi]$ might restrict the class of possible solutions (if any), and then, given the boundary theory with an action $\mathbf{S}(\varphi)$, one may apply the above derivation by first recovering the local $S[\Phi]$.

The practical importance of functional determinant relations (4.12), (4.13) is that they can be used in concrete physical problems. In [16], these relations were demonstrated to be useful for the derivation of surface terms of the Schwinger–DeWitt (Gilkey–Seely) coefficients in the heat kernel trace expansion — a method important for the calculation of the Casimir energy, the boundary UV divergences, etc. The bulk–boundary/brane action (4.3) finds application in the Randall–Sundrum brane-world model [32], where the operator $\mathbf{f}$ is generated by the tension term on the brane. In the Dvali–Gabadadze–Porrati (DGP) model [33], $\mathbf{f}$ is a second-order operator induced by the brane Einstein term, $\mathbf{f}(\partial) \sim \Box/\mu$, where μ is the DGP scale responsible for the cosmological acceleration [34]. In the context of the Born–Infeld action in D-brane string theory with vector gauge fields, $\mathbf{f}(\partial)$ is a first-order operator [35].

Very interesting is the class of models in which the holographic duality is not associated with the conformal infinity of the AdS spacetime but is realized for dynamically evolving (cosmological) branes that are nontrivially embedded into the spacetime with extra dimensions [1, 32–34]. One such model is the large-N CFT-driven 4D cosmology whose partition function serves as a source of quasi-thermal initial conditions for the Universe [36]. It is dual to the 5D Schwarzschild–de Sitter spacetime with an embedded spherical shell carrying the 4D Einstein action [37] — a realization of the dS/CFT correspondence [38] rather than the AdS/CFT one. It is important that this 4D shell surrounding the Euclidean bulk black hole is not static, but rather its radius is periodically oscillating. This oscillatory dynamics in the bulk incorporates a dual description of the self-consistent 4D cosmological evolution driven by the large-N CFT in a quasi-thermal state, the amount of its quasi-equilibrium radiation being related to the bulk black hole mass. Without a doubt, there are many more potential revelations and applications within this approach in perturbative and nonperturbative quantum gravity.

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