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COMPLETING LORENTZ VIOLATING MASSIVE GRAVITY AT HIGH ENERGIES

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Theories with massive gravitons are interesting for a variety of physical applications, ranging from cosmological phenomena to holographic modeling of condensed matter systems. To date, they have been formulated as effective field theories with a cutoff proportional to a positive power of the graviton mass m_g and much smaller than that of the massless theory ($M_P \approx 10^{19}$ GeV in the case of general relativity). In this paper, we present an ultraviolet completion for massive gravity valid up to a high energy scale independent of the graviton mass. The construction is based on the existence of a preferred time foliation combined with spontaneous condensation of vector fields. The perturbations of these fields are massive and below their mass, the theory reduces to a model of Lorentz violating massive gravity. The latter theory possesses instantaneous modes whose consistent quantization we discuss in detail. We briefly study some modifications to gravitational phenomenology at low-energies. The homogeneous cosmological solutions are the same as in the standard cosmology. The gravitational potential of point sources agrees with the Newtonian one at distances small with respect to m_g^{-1}. Interestingly, it becomes repulsive at larger distances.

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To Valery Rubakov, Teacher and Colleague

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1. INTRODUCTION

Can gravity be mediated by a massive tensor field? This straightforward question has generated a lot of controversy since it was first formulated by Fierz and Pauli [1]. The situation is remarkably different from the case of gauge interactions mediated by vector fields, where the Higgs mechanism provides a clear-cut way to give mass to the vector bosons within a weakly coupled theory. The differences fall into two categories. First, a generic Lorentz invariant theory with massive spin-2 fields (gravitons) presents instabilities in the sector of additional polarizations appearing in the massive, as opposed to massless, case — the "Goldstone" sector. These instabilities arise around realistic backgrounds and endanger the consistency of the theory even at low energies [2, 3]. It was first realized by Rubakov [4] that these problems can be avoided by breaking the Lorentz invariance. This approach has lead to the formulation of a class of Lorentz violating (LV) massive gravities as consistent effective field theories (EFTs) [5] (see [6] for review). An alternative way to improve the behavior of the Goldstone sector while preserving the Lorentz invariance has been found in [7] and consists in a judicious choice of the couplings for the interactions of the massive gravitons (see, e. g., [8, 9]) for reviews). It has been argued [10, 11] that this tuning might be stable under quantum corrections, but it is not clear at the moment if there is any symmetry behind.

The second difference between massive spin-2 and spin-1 theories lies in their different behavior at high energies. The interactions in the Goldstone sector of massive gravity become strong and the perturbation

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theory breaks down at a certain cutoff scale[1] Λ_{low} depending on the graviton mass m_g. In the limit of vanishing mass, this scale goes down to zero and, instead of recovering the massless case, the theory ceases to exist. In fact, the same is also true for a pure theory of massive vector fields with non-abelian interaction. However, in the latter case the ultraviolet (UV) completion is known in the form of the Higgs mechanism which makes the theory renormalizable by adding a handful of new degrees of freedom (Higgs bosons). Importantly, in the resulting theory, the massless limit is perfectly smooth and corresponds to the restoration of the spontaneously broken gauge symmetry[2].

No such mechanism has been found so far for massive gravity[3]. Of course, in this case one cannot insist on an embedding into a fully UV complete theory — the massless theory being non-renormalizable anyway with the cutoff at the Planck mass $M_P \approx 10^{19}$ GeV (see, however, Sec. 3.1). Still, it makes sense to search for a setup, whose cutoff would be independent of the graviton mass and as close to the Planck scale as possible. To preserve the analogy with the Higgs mechanism, the embedding theory must contain only a finite number of new degrees of freedom compared to massive gravity itself[4], these degrees of freedom must be weakly coupled and the limit $m_g \to 0$ must be regular. The goal of this work is to present a setup fulfilling the above requirements.

The reasons for pursuing this endeavor are not merely academic. First, massive gravity is a very natural candidate for an infrared (IR) modification of general relativity (GR) [14]. Such modifications are an interesting playground to look for alternatives to the cosmological constant as the source of cosmic acceleration (see, e. g., [15]). The acceleration may be generated at the level of the background (new contribution to the Friedmann equation) or of perturbations (weaker or repulsive gravitational potential at distances larger than m_g^{-1}). It is fair to say that none of these possibilities have been satisfactory implemented so far in concrete models. In any event, any candidate to explain cosmic acceleration should have a completion at high energies, which is important for predictions related to the early universe or very dense objects. This can also allow to understand the relevance of certain tunings of the IR parameters. We will see how a concrete ultraviolet completion can have nontrivial consequences at very large distances.

Massive gravity has also been discussed in the context of the gauge/gravity correspondence [16, 17]. Various phases of massive gravity may be useful to describe different phenomenology at strong coupling. In particular, it was recently realized that LV massive gravity is relevant for the description of systems with broken translational invariance [18, 19]. The completion of the theory to smaller distances on the gravity side yields access to the operators of higher dimensions in the strongly coupled field theory[5].

Finally, the theory of massive gravity is related to the spontaneous breaking of space-time symmetries [5, 20]. This is an appropriate language to describe different states of matter within the EFT framework [21, 22]. One can speculate that the Higgs mechanism for massive gravity will be relevant to describe the phase transitions in such systems.

In this paper, we will focus on the LV massive gravity of [4, 5]. Our main motivation for this choice is the already mentioned validity of this theory as a low-energy EFT, whose structure is protected by symmetries. Besides, the fundamental role of Lorentz invariance in quantum gravity has been questioned recently [23]. It is interesting to explore if massive gravity can be naturally embedded in this framework[6]. We will assume that at the fundamental level the violation of Lorentz invariance is minimal and amounts to the existence of a preferred foliation of space time [23, 25]. It is worth noting that the presence of superluminal propagation [26–28] in the seemingly Lorentz invariant massive gravity of [7] makes a Lorentz invariant Wilsonian UV completion of this theory problematic [29]. Thus, even in this case, the UV completion (if any) is likely to be Lorentz violating.

The paper is organized as follows. In Sec. 2, we briefly summarize the formalism of LV massive gravity

[1] Throughout the paper, we identify the cutoff with the strong coupling scale of the perturbation theory around the Minkowski background. In the setup of [7], the scale of strong coupling may be raised in curved space-time due to the Vainshtein mechanism (see the discussion in [9]). However, the validity of the corresponding backgrounds is under debate [12].

[2] From a purist's viewpoint, no symmetry is broken in the Higgs mechanism, gauge invariance being just a redundancy in the description. However, we allow ourserves this abuse of terminology because of its clear intuitive meaning.

[3] Exceptions are theories in AdS where the mass of the graviton can appear due to non-trivial conditions at the time-like boundary [13–15]. However, these constructions rely heavily on the peculiar properties of the AdS geometry. In particular, the resulting graviton mass is always parametrically smaller than the inverse AdS radius.

[4] This excludes the known theories with massive spin-2 fields, such as Kaluza–Klein models or string theory: both imply the presence of an infinite tower of new degrees of freedom with masses of order Λ_{low}.

[5] We thank Riccardo Rattazzi for the discussion of this point.

[6] See [24] for an early attempt in this direction.

and define the phase that we will consider. In Sec. 3, we introduce the UV completion that allows pushing the cutoff of the theory to values close to M_P. We analyze the background solutions of the theory in Sec. 4 and show that the LV massive gravity of Sec. 2 appears in the IR limit. Section 5 is devoted to the analysis of the degrees of freedom in the theory at different scales. We also discuss in some detail the quantization of instantaneous modes present in the LV massive gravity and their relation to a certain type of non-locality along the spatial directions. First results in phenomenology are presented in Sec. 6. We conclude with the summary and discussion in Sec. 7.

2. LORENTZ VIOLATING MASSIVE GRAVITY

We will now briefly review the construction of LV massive gravity. We will formulate these theories in a language closer to the symmetry breaking mechanism by introducing Stückelberg fields [5, 20]. This formulation is useful to understand many features of the theories, in particular the strong coupling scale.

We focus on the setup where Lorentz invariance is broken down to the subgroup of spatial rotations [4]. To describe this situation, let us consider four scalar Stückelberg fields, ϕ^0, ϕ^a, $a = 1,2,3$, with internal $SO(3)$ symmetry acting on the indices a, coupled to the metric in a covariant way. Additional symmetries must be imposed on this sector to protect it from pathologies [5]. We start by requiring invariance under the shifts[7] of ϕ^0,

$$\phi^0 \mapsto \phi^0 + \text{const}, \tag{1a}$$

and the ϕ^0-dependent shifts of ϕ^a,

$$\phi^a \mapsto \phi^a + f^a(\phi_0), \tag{1b}$$

where f^a are arbitrary functions. We assume that in the stationary state the Stückelberg fields acquire coordinate-dependent vacuum expectation values (VEVs),

$$\phi^0 = \mu_0^2\, t, \quad \phi^a = \mu^2 x^a. \tag{2}$$

These VEVs break the product of 4-dimensional diffeomorphisms and the internal symmetries of Stückelberg fields down to the diagonal subgroup consisting of the time shifts,

$$t \mapsto t + \text{const}, \tag{3a}$$

time-dependent shifts of the spatial coordinates,

$$x^a \mapsto x^a + f^a(t), \tag{3b}$$

and $SO(3)$ spatial rotations. A simple Lagrangian that obeys the imposed symmetries and admits the VEVs (2) has the form[8]

$$\mathcal{L}_S = \mathcal{L}_{S1} + \mathcal{L}_{S2},$$

$$\mathcal{L}_{S1} = \frac{1}{8\mu_0^4}\big((\partial_\mu\phi^0)^2 - \mu_0^4\big)^2 - \frac{\kappa_0}{4\mu_0^4}\big((\partial_\mu\phi^0)^2 - \mu_0^4\big)\times \\ \times\left(\sum_a P^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^a + 3\mu^4\right), \tag{4a}$$

$$\mathcal{L}_{S2} = -\frac{1}{8\mu^4}\sum_{a,b}\big(P^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^b + \mu^4\delta^{ab}\big)^2 + \\ + \frac{\kappa}{8\mu^4}\left(\sum_a P^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^a + 3\mu^4\right)^2, \tag{4b}$$

where κ_0, κ are dimensionless constants and

$$P_{\mu\nu} = g_{\mu\nu} - \frac{\partial_\mu\phi^0\partial_\nu\phi^0}{g^{\lambda\rho}\partial_\lambda\phi^0\partial_\rho\phi^0}, \tag{5}$$

is the projector on the subspace orthogonal to the gradient of ϕ_0 which ensures invariance under (1b).

To understand the physical content of the theory, let us consider small fluctuations of the metric and expand to the quadratic order in

$$h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}.$$

Using general covariance, we can identify the coordinates with the Stückelberg fields, as in (2). In other words, we work in the gauge, where the fields ϕ^0, ϕ^a do not fluctuate; we call it "unitary gauge". Then, the quadratic Lagrangian takes the form

$$\mathcal{L}_S^{(2)} = \frac{\mu_0^4}{8}h_{00}^2 - \frac{\mu^4\kappa_0}{4}h_{00}h_{aa} - \\ - \frac{\mu^4}{8}h_{ab}h_{ab} + \frac{\mu^4\kappa}{8}h_{aa}^2, \tag{6}$$

where the summation over repeated indices is understood. This is precisely a mass term for the metric perturbation. In particular, the graviton (the helicity-2 component) acquires the mass

$$m_g = \frac{\mu^2}{M_P}, \tag{7}$$

where M_P is the Planck mass. Note that the term $h_{0a}h_{0a}$ which is missing in (6) compared to the most general expression [4] is forbidden by the residual symmetry (3b). The quadratic Lagrangian of the form (6) appears also in bimetric theories [30, 31].

[7] We start from the simple ϕ^0-shifts to make contact with [5]. Later on, we will promote them to a larger symmetry, see Eq. (14).

[8] This Lagrangian is not the most general one, but it is sufficient for our purposes, as it reproduces all possible mass terms for the metric which are allowed by the symmetries (1).

Let us return from the unitary gauge to the covariant Lagrangian (4) which is more suitable to study the non-linear properties. Importantly, in the case when μ_0, μ are much smaller than M_P, we can decouple the metric fluctuations and concentrate on the Stückelberg fields, as if they were living in flat space-time. We write

$$\phi^0 = \mu_0^2 t + \psi^0, \quad \phi^a = \mu^2 x^a + \psi^a \tag{8}$$

and obtain

$$\mathcal{L}_S = \frac{(\dot{\psi}^0)^2}{2} + \frac{\kappa_0 \mu^2}{\mu_0^2}\dot{\psi}^0 \partial_a \psi^a - \frac{\partial_a \psi^b \partial_a \psi^b}{4} - \\ - \frac{1-\kappa}{4}(\partial_a \psi^a)^2 + \mathcal{L}_{int}\left(\frac{\partial \psi^0}{\mu_0^2}, \frac{\partial \psi^a}{\mu^2}\right), \tag{9}$$

where the last term stands for the derivative interactions of cubic and higher orders. By power-counting, the strength of these interactions grows with the increase of energy or momentum and the theory breaks down at the scale

$$\Lambda = \min\{\mu_0, \mu\}. \tag{10}$$

Comparing with (7), we conclude that the cutoff is bounded from above,

$$\Lambda < \Lambda_2 \equiv \sqrt{m_g M_P}. \tag{11}$$

Actually, this conclusion is not related to the specific form of the Lagrangian (4). As discussed in [5], Λ_2 provides an absolute upper bound on the cutoff in a general massive gravity theory formulated in terms of the metric and Stückelberg fields only[9]. Thus, the theory does not admit a smooth limit $m_g \to 0$.

From the quadratic part of (9), we read off that a linear combination of ψ^0 and the longitudinal part of ψ^a has degenerate dispersion relation

$$\omega^2 = 0. \tag{12}$$

This presents a potential danger, as in non-trivial backgrounds the r.h.s. of the dispersion relation can become negative leading to an instability. In Ref. [5], it was suggested to lift the degeneracy by adding to the Lagrangian quadratic terms with higher derivatives as in the ghost condensate model [33]. In the next section, we will take a different route and embed the field ϕ^0 into the khronometric model [25].

The rest of the modes in (9) — the transverse part of ψ^a and the longitudinal component of ψ^a linearly independent from ψ^0 — obey the equations of the form

$$\bar{k}^2 \psi = 0, \tag{13}$$

where $\bar{k}$ is the absolute value of the three-momentum $\bar{k}_i$. Thus, for any non-zero spatial momentum these modes must vanish implying that there are no propagating degrees of freedom associated with ψ^a. The symmetry (1b) ensures that this property is preserved upon inclusion of higher-order operators [5] and in curved backgrounds [34]. Therefore the theories based on the symmetries (1) present a class of well-defined EFTs[10].

3. GOING BEYOND Λ_2: INGREDIENTS

3.1. The field ϕ^0

There are two natural ways to deal with the field ϕ^0 to complete the previous actions to energies higher than (10). First, as we mentioned above, the degeneracy of the dispersion relation (12) can be lifted by adding higher derivative terms as in the ghost condensate [33]. This theory is still an effective theory with a cutoff of order μ_0, but since this is independent of the mass of the graviton, this scale can be quite high. Phenomenological bounds set the constrain $\mu_0 \lesssim 10$ MeV. It was argued in Ref. [35] that these bounds can be relaxed by the non-linear dynamics which may push the upper limit on μ_0 to 100 GeV. This still remains much below the Planck scale. The way to raise the cutoff of the theory to (almost) Planckian was proposed in Ref. [36]. It uses the embedding of the ghost condensate into the khronometric model [25] and requires the introduction of a new degree of freedom — khronon — at a scale below μ_0. One could use this strategy here.

However, it is more economical to use the second option and identify the Stückelberg field ϕ^0 directly with the khronon, thus keeping only a single degree of freedom in this sector. In this case, the symmetry (1a) is extended to the full reparameterization invariance,

$$\phi^0 \mapsto f^0(\phi^0), \tag{14}$$

for an arbitrary monotonous function f^0. This larger symmetry forbids the terms present in (4a). Instead, the Lagrangian must be constructed using the unit vector

$$u_\mu \equiv \frac{\partial_\mu \phi^0}{\sqrt{g^{\lambda\rho}\partial_\lambda \phi^0 \partial_\rho \phi^0}} \tag{15}$$

invariant under the symmetry (14). The most general Lagrangian with the lowest number of derivatives, and thus dominant at low energies, reads [25]

[9] The cutoff is even lower, $\Lambda < \Lambda_3 \equiv (m_g^2 M_P)^{1/3}$, if one restricts to the Lorentz invariant theories [20, 32].

[10] A subtle issue of the proper treatment of the non-propagating modes at the quantum level will be discussed in Sec. 5.3.

$$\mathcal{L}_{kh} = -\frac{M_P^2}{2}\big(R + \beta\nabla_\mu u^\nu \nabla_\nu u^\mu + \\ + \lambda(\nabla_\mu u^\mu)^2 + \alpha u^\mu u^\nu \nabla_\mu u^\rho \nabla_\nu u_\rho\big), \quad (16)$$

where we have also included the standard GR action; α, β, λ are dimensionless coupling constants. This can be combined with (4b) to give an action of LV massive gravity. We note that in the unitary gauge[11)] (2) the Lagrangian (16) gives rise only to terms with derivatives of the metric perturbations and thus does not contribute to the mass term for $h_{\mu\nu}$. The latter reduces to

$$\mathcal{L}_{\rm mass} = -\frac{\mu^4}{8}h_{ab}h_{ab} + \frac{\mu^4\kappa}{8}h_{aa}^2. \quad (17)$$

This can be understood as the consequence of the time-reparameterization invariance

$$t \mapsto f^0(t), \quad (18)$$

which, being now a residual symmetry in the unitary gauge, forbids any contributions containing h_{00} without derivatives. This version of massive gravity was considered in Ref. [37].

The Lagrangian (16) contains higher derivatives of the field ϕ^0 and one may be worried that this can lead to pathologies (ghosts or gradient instabilities). In fact, this does not happen, as the extra derivatives act in the spatial directions. This property becomes explicit in the gauge where the time coordinate is identified with ϕ^0 as in the first equation of (2). We note, that this identification still leaves the free choice of the spatial coordinates, so it should not be confused with the unitary gauge where all coordinates are fixed. We will call this partial gauge fixing ADM gauge. The action of the khronometric model takes the form

$$S_{kh} = \frac{M_P^2}{2}\int dt\, d^3x\, \sqrt{\gamma}N\bigg[(1-\beta)K_{ij}K^{ij} - \\ - (1+\lambda)K^2 + {}^{(3)}R + \alpha\left(\frac{\partial_i N}{N}\right)^2\bigg], \quad (19)$$

where we used the Arnowitt–Deser–Misner (ADM) decomposition for the metric,

$$ds^2 = dt^2 - \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \quad (20)$$

the extrinsic curvature of the constant-time slices

$$K_{ij} = \frac{1}{2N}(\dot\gamma_{ij} - \nabla_i N_j - \nabla_j N_i), \quad K = \gamma^{ij}K_{ij}, \quad (21)$$

and denoted ${}^{(3)}R$ the three-dimensional curvature constructed from the metric γ_{ij}. Apart from the symmetry (18), this action is invariant under time-dependent spatial diffeomorphisms

$$x^i \mapsto \tilde x^i(\mathbf{x}, t). \quad (22)$$

We will refer to the group consisting of the transformations (18) and (22) as foliation-preserving diffeomorphisms (FDiff). Clearly, the action (19) leads to equations of motion which are second order in time derivatives. We will work in the ADM gauge from now on.

The choice

$$\alpha = \beta = \lambda = 0$$

corresponds to GR and the restoration of the full diffeomorphisms-invariance. However, the limit α, β, $\lambda \to 0$ is not smooth. At any non-zero values of α, β, λ the theory propagates in addition to the helicity-2 gravitons a single helicity-0 mode (khronon). The latter has linear dispersion relation; in the case α, β, $\lambda \ll 1$ (which is the relevant one for phenomenology), it reads[12)] [25]

$$\omega^2 = \frac{\beta+\lambda}{\alpha}\,k^2. \quad (23)$$

Due to the non-linear interactions of the khronon present in (19), the model has a cutoff

$$\Lambda_{kh} \sim M_P \min\left\{\sqrt{\alpha}, \sqrt{\beta}, \sqrt{\lambda}\right\}. \quad (24)$$

Phenomenological considerations put upper bounds on α, β, λ [25, 38, 39] and hence constrain the cutoff to be somewhat smaller than the Planck scale,

$$\Lambda_{kh} \lesssim 10^{15}\,\text{GeV}. \quad (25)$$

Still, this is well above virtually any scale that can appear in the astrophysical or cosmological context[13)]. Furthermore, it is known how to complete the action (19) beyond Λ_{kh} by embedding it into the Hořava gravity [23, 40]. The latter presents a power-counting renormalizable theory. However, due to the technical complexity, the question about its renormalizability in the strict sense and UV behavior still remains open (see Refs. [41, 42] addressing this issue in restricted settings). In these circumstances, a cautious reader may prefer to take modest attitude and view the khronometric model as an EFT with the cutoff (24), which is sufficient for the purposes of this work.

Finally, let us mention the following peculiarity of the action (19). As described in Ref. [25], it leads to a certain type of instantaneous interactions mediated by

11) Because of the invariance (14), the choice of the constant μ_0^2 in the first formula of (2) is now arbitrary and unrelated to the parameters of the theory.

12) This relation gets modified — in particular, the khronon acquires a mass gap — when the action (19) is coupled to the other sectors needed to reproduce the massive gravity, see Sec. 6 below.

13) In the applications unrelated to astrophysics, such as non-relativistic holography or description of solids, the parameters α, β, λ are *a priori* constrained only by the stability requirements, that are mild, and the scale Λ_{kh} can be as high as M_P.

a non-propagating mode. The latter is similar to the non-propagating modes of massive gravity discussed in Sec. 2. We will study the instantaneous modes in more detail in Sec. 5.3.

3.2. The fields ϕ^a and their coupling to Higgs vectors

Next we consider the triplet of Stückelberg fields invariant under

$$\phi^a \mapsto \phi^a + f^a(t), \tag{26}$$

which is nothing but the symmetry (1b) in the ADM gauge. We want a Lagrangian that admits the coordinate-dependent VEVs (2), but at the same time is UV complete past the scale μ. This precludes from introducing any self-interaction of the Stückelberg fields involving the scale μ. Then the simplest option is to choose the Lagrangian to be quadratic in ϕ^a. To respect the symmetry (26), it must depend only on the spatial derivatives of these fields,

$$S_\phi = \int dt\, d^3x\, \sqrt{\gamma} N \left[-\frac{1}{2}\gamma^{ij}\partial_i\phi^a\partial_j\phi^a \right]. \tag{27}$$

This does not introduce any new strong coupling scale. However, without any further interactions, this Lagrangian is not enough to provide non-zero graviton mass. Though the configuration

$$\phi^a = \Phi x^a$$

is a solution of the equations following from (27) for any constant Φ, it introduces non-vanishing energy density and pressure which make the universe expand[14]. As will become clear in the Sec. 4, in this case the generated mass will decrease with time and will asymptotically vanish. Time varying masses can be interesting (see, e. g., [34]) but are not the aim of this paper. To give constant graviton mass, the VEVs in an expanding universe must grow proportionally to the scale factor,

$$\phi^a \propto a(t) x^a,$$

which is not a solution of the field equations implied by (19) and (27). We have to add more ingredients.

Consider a triplet of vector fields with purely spatial components V_a^i. These transform as vectors under the diffeomorphisms preserving the foliation structure of the ADM gauge, which act on the i-index. Besides, they form the fundamental representation of a global internal $SO(3)$ acting on the index a. We do not assume any gauge invariance associated to these vectors. To avoid strong coupling, we focus on Lagrangians which are renormalizable in flat space-time. By the standard power-counting, they can contain the derivatives of V_a^i only quadratically and up to quartic terms in the potential. The generic Lagrangian satisfying these properties and invariant under FDiff $\times$ $SO(3)$ reads

$$\begin{aligned} S_V = \int dt\, d^3x \sqrt{\gamma}\, \times \\ \times N \Bigg[\frac{1}{2N^2}(\dot{V}_a^i - N^j\nabla_j V_a^i + V_a^j \nabla_j N^i)^2 - \\ - \frac{c_1^2}{2}(\nabla_i V_a^j)^2 - \frac{c_2^2}{2}(\nabla_i V_a^i)^2 - \\ - \frac{\varkappa_1}{4}(V_a^i V_b^j \gamma_{ij} - M_V^2 \delta_{ab})^2 - \\ - \frac{\varkappa_2}{4}(V_a^i V_a^j \gamma_{ij} - 3M_V^2)^2 \Bigg], \end{aligned} \tag{28}$$

where c_1, c_2, $\varkappa_1$, and $\varkappa_2$ are dimensionless couplings and we have chosen the overall constant in the potential to have vanishing vacuum energy (we will shortly introduce a cosmological constant term in a different part of the action). For clarity, we have omitted non-minimal interactions with the metric, such as ${}^{(3)}R_{ij} V_a^i V_a^j$ and ${}^{(3)}R V_a^i V_a^j \gamma_{ij}$, which vanish in Minkowski space-time. These terms would make the analysis more cumbersome without changing it qualitatively.

When $M_V^2 > 0$, the vectors develop VEVs,

$$V_a^i = M_V \delta_a^i, \tag{29}$$

which break the product $SO(3) \times SO(3)$ of spatial and internal rotations down to the diagonal subgroup, cf. [43–46]. Below the scale $\sim \sqrt{\varkappa} M_V$, the dynamics is described by the σ-model corresponding to this pattern of symmetry breaking with the coset space defined by

$$V_a^i V_b^j \gamma_{ij} = M_V^2 \delta_{ab}. \tag{30}$$

As the vector VEVs introduces an additional source of Lorentz symmetry breaking, it is natural to expect that the phenomenological constraint on the scale M_V will be similar to that of Λ_{kh}, Eq. (25). Notice, however, that M_V is not related to the cutoff and can be much lower than Λ_{kh} without jeopardizing the validity of the theory.

Finally, we complete our Lagrangian with a term mixing the vectors and the Stückelberg fields,

$$S_{V\phi} = \int dt\, d^3x \sqrt{\gamma} N \left[m_A V_a^i \partial_i \phi^a - \mathcal{V}_0 \right]. \tag{31}$$

This mixing operator has dimension 3 and thus is just a relevant deformation of the previous action. It does not affect the UV properties of the theory, in particular, it does not introduce any new UV cutoff, and the parameter m_A can be arbitrarily small without encountering any singularity. We are going to see that in IR this

[14] These density and pressure cannot be canceled by any bare cosmological constant.

coupling leads to the generation of the VEVs (2) with $\mu^2 = m_A M_V$ and the graviton mass (7). The last term in (31) represents a cosmological constant and can be tuned to cancel the negative vacuum energy that would be generated otherwise (see below). We note that it is technically natural to take the parameter m_A to be much smaller than the other scales of the theory as it is protected from large quantum corrections by the discrete symmetry[15] $\phi^a \mapsto -\phi^a$. In what follows we will assume the hierarchy of scales,

$$M_P \gtrsim \Lambda_{kh} \gtrsim M_V \gg m_A. \tag{32}$$

It is worth stressing that this hierarchy is not required by the internal consistency of the theory. For example, one could consider instead $m_A \sim M_V$. However, assuming (32) makes the physical picture particularly transparent.

4. GENERATION OF VEVs IN EXPANDING BACKGROUNDS

Let us now show that the construction of the previous section gives rise to the VEVs for the fields ϕ^a of the desired form. We assume a homogeneous and isotropic ansatz with spatially flat metric allowing for a general cosmological evolution,

$$\begin{gathered} N(t), \quad \gamma_{ij} = a^2(t)\delta_{ij}, \quad V_a^i = \frac{M_V}{a(t)}\delta_a^i, \\ \phi^a = \Phi(t)x^a. \end{gathered} \tag{33}$$

Substituting this ansatz into the equations of motion obtained from putting together the actions (19), (27), (28) and (31), we obtain[16]

$$3M_c^2 H^2 - \frac{3\Phi^2}{2a(t)^2} + \frac{3\mu^2\Phi}{a(t)} - \mathcal{V}_0 = \rho_{mat}, \tag{34a}$$

$$\begin{aligned} 2M_c^2\dot{H} + 3M_c^2H^2 - \frac{\Phi^2}{2a(t)^2} &+ \\ + \frac{2\mu^2\Phi}{a(t)} - \mathcal{V}_0 &= -p_{mat}, \end{aligned} \tag{34b}$$

where

$$\mu^2 = m_A M_V \tag{35}$$

and

$$M_c^2 \equiv M_P^2\left(1 + \frac{\beta + 3\lambda}{2}\right) - \frac{M_V^2}{2} \tag{36}$$

is the "cosmological Planck mass". In Eqs. (34) we fixed the gauge $N = 1$ and assumed that the universe is filled with matter with energy density ρ_{mat} and pressure p_{mat}. Taking the derivative of (34a) and using the energy conservation in the matter sector,

$$\dot{\rho}_{mat} + 3H(\rho_{mat} + p_{mat}) = 0, \tag{37}$$

we obtain the following equation for Φ,

$$\dot{\Phi}(\Phi - \mu^2 a(t)) = 0. \tag{38}$$

This has two branches of solutions. On the branch $\Phi = \text{const}$ the VEVs of the Stückelberg fields actually disappear with time. Indeed, the invariant quantity

$$\gamma^{ij}\partial_i\phi^a\partial_j\phi^a = \frac{3\Phi^2}{a(t)^2}$$

decreases as the universe expands. Besides, we will see shortly that this branch is unstable at late times whenever $\mu \neq 0$. The other branch is

$$\Phi = \mu^2 a(t). \tag{39}$$

It corresponds to constant strength of the Stückelberg fields' gradients and is stable. In this latter case the cosmological equations (34) reduce to the form,

$$3M_c^2H^2 = \rho_{mat} + \mathcal{V}_0 - \frac{3\mu^4}{2}, \tag{40a}$$

$$2M_c^2\dot{H} + 3M_c^2H^2 = -p_{mat} + \mathcal{V}_0 - \frac{3\mu^4}{2}. \tag{40b}$$

We see that in this phase μ produces a negative shift of the cosmological constant to a smaller value. In what follows we will assume that this contribution is canceled by the bare cosmological constant,

$$\mathcal{V}_0 = \frac{3\mu^4}{2}, \tag{41}$$

so that the Minkowski space-time is a solution in the absence of matter. This is just the usual fine-tuning of the cosmological constant.

5. HIERARCHY OF EFTs AND THE GRAVITON MASS

5.1. Phases with massive gravitons

To understand the effect of the mixing term (31) on the spectrum of the theory let us study small perturbations. As before, we work in the ADM gauge and first

[15] A similar argument is used to protect the small coupling between a time-like vector field and an ordinary massless scalar in the technically natural dark energy model of [47].

[16] The simplest way to derive these equations is to substitute ansatz (33) into the action and perform variation with respect to the free functions $N(t)$ and $a(t)$ afterwards. Note, however, that it would be incorrect to vary this action with respect to Φ as the corresponding variation $\delta\phi^a = \delta\Phi x^a$ would not be bounded at spatial infinity.

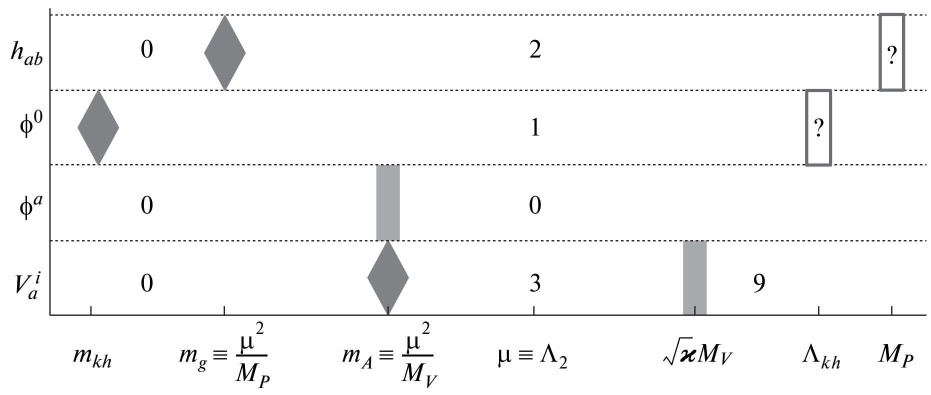


Fig. 1. Relevant energy scales in the theory and the number of propagating degrees of freedom in each sector at different scales. Rectangles represent the energy scales at which a sector gets UV completed (we do not make any assumptions about the UV completion of the khronon and spin-2 sectors, but Hořava gravity [23] would be a natural option). Rhomboids mark the scales below which a sector loses all its propagating degrees of freedom. The khronon mass m_{kh} will be derived in Sec. 6 (see Eq. (83)). Note that nothing happens at the scale $\mu \equiv \Lambda_2$ which sets the cutoff in the original EFT formulation of massive gravity

focus on the phase with a vacuum from the branch of solutions (39). To simplify the analysis, we freeze out the perturbations in the khronometric sector by sending M_P and Λ_{kh} to infinity while keeping M_V and m_A finite. For the perturbations of the vectors and the Stückelberg fields we write,

$$V_a^i = M_V \delta_a^i + v_a^i, \quad \phi^a = \mu^2 x^a + \psi^a. \qquad (42)$$

If we are interested in energies below $\sqrt{\varkappa} M_V$, we can adopt the σ-model description. Inserting (42) in the constraint equation (30) yields

$$v_a^i = A_a^i - \frac{A_i^j A_a^j}{2M_V} + O(A^3), \qquad (43)$$

where A_a^i is an antisymmetric matrix,

$$A_a^i + A_i^a = 0.$$

Substituting this into the Lagrangian, we obtain

$$\mathcal{L}_\phi + \mathcal{L}_{V\phi} = -\frac{(\partial_i \psi^a)^2}{2} - \frac{m_A^2}{2} A_a^j A_a^j + m_A A_a^i \partial_i \psi^a. \qquad (44)$$

The second term gives mass of order m_A to the antisymmetric perturbations A_a^i. Below this scale the perturbations of the vectors can be integrated out completely. From (44), we find

$$A_a^i = \frac{1}{2m_A}(\partial_i \psi^a - \partial_a \psi^i), \qquad (45)$$

which substituted back into (44) gives (up to a total derivative)

$$\mathcal{L}_\phi + \mathcal{L}_{V\phi} = -\frac{\partial_i \psi^a \partial_i \psi^a}{4} - \frac{(\partial_a \psi^a)^2}{4}. \qquad (46)$$

This coincides with the third and fourth terms (with $\kappa = 0$) of the quadratic Stückelberg Lagrangian (9) arising in massive gravity. The “massless” fields ψ^a can be interpreted as the Goldstone bosons for the broken symmetries

$$\text{FDiff} \times SO(3) \to SO(3)_{diag}.$$

Recall that since we are dealing with LV theories, the counting and properties of such fields are different from the Lorentz invariant case, see, e.g., [48–50]. In the same spirit, the vector fields A_a^i that have been integrated out can be interpreted as the “Higgs” fields regularising the bad behaviour of the Goldstone sector at energies above m_A. Given the previous results, we expect that the graviton in this model will acquire the mass (7). For the case $M_P \gg M_V$, the vector and graviton masses are well separated and at energies $m_A \gg E \gg m_g$ the dynamics is well described by the EFT for the Stückelberg fields. The hierarchy of various scales in the theory and the corresponding EFT descriptions are summarized in Fig. 1.

Alternatively, we can work in the unitary gauge and fix $\psi^a = 0$ at the expense of allowing for the fluctuations of the metric

$$N = 1 + n, \quad N^i, \quad \gamma_{ij} = \delta_{ij} + h_{ij}. \qquad (47)$$

The relevant part of the Lagrangian takes the form

$$\mathcal{L}_\phi + \mathcal{L}_{V\phi} = \mu^4 \left[-\frac{\gamma^{aa}}{2} + \frac{V_a^a}{M_V} - \frac{3}{2} \right]. \qquad (48)$$

The solution of the constraint (30) now reads

$$v_a^i = A_a^i - \frac{M_V}{2}h_{ai} - \frac{A_i^j A_a^j}{2M_V} - \frac{A_i^j h_{ja}}{4} - \frac{A_a^j h_{ji}}{4} + \\ + \frac{3M_V}{8}h_{ji}h_{ja} + O(A^3, h^3). \quad (49)$$

Substituting this formula and the expression

$$\gamma^{ij} = \delta_{ij} - h_{ij} + h_{ik}h_{kj} + O(h^3) \quad (50)$$

into (48), we obtain at the quadratic level

$$\mathcal{L}_\phi + \mathcal{L}_{V\phi} = -\frac{m_A^2}{2}A_a^i A_a^i - \frac{\mu^4}{8}h_{ia}h_{ia}. \quad (51)$$

The first term again gives mass to the antisymmetric perturbations, while the second explicitly provides the mass term for helicity-2 graviton. As we are going to see in Sec. 6, it also gives mass to the khronon (see Eq. (83)). Note that we obtain only one of the two structures for the metric mass term allowed by the symmetries, cf. (17). This a consequence of the assumption $M_V \gg m_A$ which implies that the symmetric part of the vector fluctuations is much heavier (with the mass of order $\sqrt{\varkappa}M_V$) than the antisymmetric part. This renders the parameter κ in (17) suppressed by the ratio $m_A^2/\varkappa M_V^2$ which we neglected in the above analysis. Were we to make a different assumption $M_V \sim m_A$, we would obtain both terms of (17) with comparable coefficients. Finally, if instead of the khronometric setting one used the ghost condensate for the ϕ^0-sector, which in the ADM gauge amounts to promoting all couplings in the action to functions of the lapse N [25], one would be able to reproduce also the other terms in the general Lagrangian (6) of the massive gravity discussed in Sec. 2.

5.2. Other phases?

In the previous section, we focused on branch (39) of the background solutions. However, as noticed before, equation (38) also admits a second branch

$$\dot\Phi = 0. \quad (52)$$

On this branch, the effect of the Stückelberg gradients (if non-zero initially) decays with time in an expanding universe. For completeness we now analyze the small perturbations around this branch. We write

$$\phi^a = \Phi x^a + \psi^a, \quad (53)$$

with $\Phi = \text{const}$ and take the Friedmann–Robertson–Walker (FRW) form for the metric. We again work in the decoupling limit $M_P, \Lambda_{kh} \to \infty$, so that the metric fluctuations are frozen. Below the scale $\sqrt{\varkappa}M_V$ the fluctuations of the vectors are restricted to the antisymmetric part A_a^i. Expanding the relevant part of the action to quadratic order we obtain

$$S_V + S_\phi + S_{V\phi} = \int dt\, d^3x \Big[\frac{a^3}{2}(\dot A_a^i)^2 - \frac{a}{2}\Big(c_1^2(\partial_i A_a^j)^2 + \\ + c_2^2(\partial_i A_a^i)^2 + (\partial_i\psi^a)^2\Big) - \frac{a^2 m_A \Phi}{2M_V}(A_a^i)^2 + \\ + a^2 m_A A_a^i \partial_i \psi^a\Big]. \quad (54)$$

Restricting to the modes with frequencies much higher than the Hubble rate, we can neglect the terms with derivatives of the scale factor in the equations of motion. This yields

$$-\ddot A_a^i + \frac{c_1^2}{a^2}\partial_j^2 A_a^i + \frac{c_2^2}{a^2}\partial_j\partial_{[i}A_{a]}^j - \frac{m_A\Phi}{M_V a}A_a^i + \\ + \frac{m_A}{a}\partial_{[i}\psi^{a]} = 0, \quad (55a)$$

$$\partial_i^2\psi^a - a m_A \partial_i A_a^i = 0, \quad (55b)$$

where the square brackets stand for the antisymmetrization of indices. Let us perform the Fourier transform and concentrate on the transverse modes

$$\psi^a = e_a^{(\alpha)}\psi_{(\alpha)}, \quad A_a^i = \frac{\bar k_i e_a^{(\alpha)} - \bar k_a e_i^{(\alpha)}}{\bar k}A_{(\alpha)}, \quad (56)$$

where $e_i^{(\alpha)}$, $\alpha = 1, 2$, are unit polarization vectors orthogonal to the three-momentum $\bar k_i$. Substituting this into Eqs. (55) and eliminating $\psi_{(\alpha)}$ we obtain

$$\left[\omega^2 - \left(c_1^2 + \frac{c_2^2}{2}\right)\frac{\bar k^2}{a^2} - \frac{m_A}{M_V}\left(\frac{\Phi}{a} - \frac{\mu^2}{2}\right)\right]A_{(\alpha)} = 0, \quad (57)$$

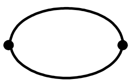
where we used μ^2 defined in (35). We see that whenever $\Phi < \mu^2 a/2$ the mode is tachyonic. In particular, the trivial configuration of the Stückelberg fields $\phi^a = 0$ is unstable. Furthermore, in an expanding universe $\mu^2 a/2$ will exceed any constant value of Φ and the instability will set in at late times. Thus, we conclude that in an expanding universe this branch is unstable and we do not consider it any more in this paper.

5.3. Quantum treatment of instantaneous modes

We have argued above that the constructed model is a valid quantum theory up to the scale (24). We have based this claim on the scaling argument borrowed from relativistic theories, so it is worth taking a closer look at it to check if it is not spoiled by Lorentz violation. To get a flavor of the potential problems, we consider the instantaneous modes ϕ^a. Let us first switch off their mixing with the vectors by setting $m_A = 0$ and perform their perturbative quantization using the path integral formalism. From (27) one reads off the propagator,

$$\phi^a \ \overset{p}{\text{———}}\ \phi^b = -\frac{i}{\bar{p}^2}\delta^{ab}, \quad (58)$$

where we denoted the spatial part of a four-vector $p_\mu = (p_0, \bar{p}_i)$ by the bar. This propagator does not depend on the frequency p_0. The fields ϕ^a couple to the metric and contribute into the effective action for the perturbations h_{ij}. For example, the one-loop contribution into the quadratic part is

h^{ij}  h^{kl}

$$h^{ij} \bigcirc h^{kl} = \frac{1}{4}h^{ij}(p)h^{kl}(-p)\int\frac{dq_0}{2\pi}\times \\ \times\int\frac{d^3\bar{q}}{(2\pi)^3}\frac{\bar{q}_i\bar{q}_k(\bar{q}+\bar{p})_j(\bar{q}+\bar{p})_l}{\bar{q}^2(\bar{q}+\bar{p})^2}. \quad (59)$$

This expression contains two types of divergences. The integral over the spatial momentum can be regulated by subtracting a finite number of local counterterms. However, the whole contribution will still be infinite because of the overall divergent integral over q_0. We note that this divergence has non-polynomial dependence on the external spatial momentum $\bar{p}_i$ and therefore is non-local in space. On the other hand, it does not depend on p_0 and hence is local in time. Thus it can be regulated by introducing a spatially non-local counterterm in the bare action. Though unusual, such counterterms do not spoil the validity of the theory. In particular, the diagram (59) does not contain any imaginary part and thus the corresponding counterterm does not violate unitarity.

One may object that allowing for non-locality, even restricted to only spatial dimensions, introduces an infinite freedom in the choice of the bare action. However, we now argue that there is a natural choice of counterterms for the diagrams where, like in Eq. (59), a divergent integral over the loop frequency completely factorizes out of a frequency-independent part. This consists in canceling these diagrams altogether. In the present case this would mean that all loop diagrams containing the instantaneous fields ϕ^a must be put to zero. Two arguments support this prescription. First, the integrals over frequency diverge linearly and thus vanish identically in dimensional regularization. Second, we can appeal to the canonical quantization. In this formalism, the fields ϕ^a are subject to second-class constraints which force them to vanish. Indeed, as the action does not depend on the time-derivative of these fields, the canonical momenta conjugate to them vanish trivially, while the fields themselves obey the equation

$$\nabla_i(N\nabla^i\phi^a) = 0. \quad (60)$$

Supplemented by the vanishing boundary conditions at spatial infinity it forces[17] $\phi^a = 0$. In the canonical approach such constrained degrees of freedom must be eliminated from the start, even prior to quantization, implying that they completely drop off from the quantum theory[18].

There is a way to implement the above prescription within the path integral approach without introducing non-local counterterms from the start. One notices that the overall result of integration over ϕ^a is a factor

$$\left[\det(i\gamma^{ij}\partial_i\partial_j)\right]^{-3/2} \quad (61)$$

in the partition function. This can be canceled by adding to the system three real bosonic fields $\tilde{\phi}^a$ and three complex fermionic fields η^a with the action,

$$S_{\tilde{\phi}\eta} = \int dt\, d^3x\sqrt{\gamma}N\times \\ \times\left[-\frac{1}{2}\gamma^{ij}\partial_i\tilde{\phi}^a\partial_j\tilde{\phi}^a - \gamma^{ij}\partial_i\eta^a\partial_j\bar{\eta}^a\right]. \quad (62)$$

Integrating out these "remover" fields multiplies the partition function by,

$$\frac{\left[\det(i\gamma^{ij}\partial_i\partial_j)\right]^3}{\left[\det(i\gamma^{ij}\partial_i\partial_j)\right]^{3/2}} = \left[\det(i\gamma^{ij}\partial_i\partial_j)\right]^{3/2}, \quad (63)$$

which precisely compensates (61). The expression (63) corresponds to the spatially non-local counterterms discussed above.

Turning on m_A makes the situation less trivial. However, given that mixing (31) is a relevant deformation it clearly cannot spoil the UV consistency of the theory. A comprehensive analysis of the quantum properties of the theory introduced in Sec. 3 is beyond the scope of this paper. Instead, we illustrate the expected behavior in a toy model containing a scalar and a vector without any VEVs in an external non-dynamical metric (we assume $N_i = 0$),

[17] Multiplying (60) by ϕ^a and integrating over the three-dimensional space we obtain

$$0 = \int d^3x\,\phi^a\nabla_i(N\nabla^i\phi^a) = -\int d^3x\,N\nabla_i\phi^a\nabla^i\phi^a.$$

As the lapse function is non-zero everywhere, one concludes that $\nabla_i\phi^a = 0$ and hence ϕ^a vanishes due to the boundary conditions.

[18] There is no modification of the canonical structure for the remaining fields as in the case at hand Dirac and Poisson brackets are identical.

$$S = \int dt\, d^3x \sqrt{\gamma} N \left[\frac{\gamma_{ij} \dot{V}^i \dot{V}^j}{2N^2} - \frac{\gamma_{jk} \nabla_i V^j \nabla^i V^k}{2} - \right.$$
$$\left. - \frac{1}{2} \gamma^{ij} \partial_i \phi \partial_j \phi + m_A V^i \partial_i \phi - \frac{M_V^2}{2} \gamma_{ij} V^i V^j \right]. \quad (64)$$

For simplicity, we have retained only one of the gradient terms for the vector putting the coefficient in front of it to $c_1^2 = 1$. As before, there are two ways to proceed. In the canonical approach, we have to solve for the field ϕ before quantization,

$$\phi = \frac{m_A (\nabla_i V^i + a_i V^i)}{\gamma^{kl} \nabla_k \nabla_l + a^l \nabla_l}, \quad (65)$$

where

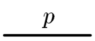
$$a_i \equiv N^{-1} \partial_i N. \quad (66)$$

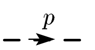
Substituting this into (64), we obtain a non-local action which depends only on V^i,

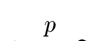
$$S = \int dt\, d^3x \sqrt{\gamma} N \left[\frac{\gamma_{ij} \dot{V}^i \dot{V}^j}{2N^2} - \frac{\gamma_{jk} \nabla_i V^j \nabla^i V^k}{2} - \right.$$
$$- \frac{M_V^2 \gamma_{ij} V^i V^j}{2} - (\nabla_i V^i + a_i V^i) \times$$
$$\left. \times \frac{m_A^2}{2(\gamma^{kl} \nabla_k \nabla_l + a^k \nabla_k)} (\nabla_j V^j + a_j V^j) \right]. \quad (67)$$

The Dirac bracket remains identical to the canonical commutator. One observes that the limit $m_A \to 0$ is smooth and corresponds to restoration of locality. As non-locality is purely spatial, it does not, in principle, present an obstruction to canonical quantization.

However, in practice it is very inconvenient to work with the non-local action (67). It is more efficient to use the path integral approach and retain ϕ as a quantum field. Assuming that the metric is close to flat, we obtain from (64) the propagators for ϕ and V^i,

ϕ  $\phi = -\dfrac{i}{\bar{p}^2} + \dfrac{i m_A^2}{\bar{p}^2 (p_0^2 - \bar{p}^2 - M_V^2 + m_A^2)}$, (68a)

ϕ  $V^i = \dfrac{-m_A \bar{p}_i}{\bar{p}^2 (p_0^2 - \bar{p}^2 - M_V^2 + m_A^2)}$, (68b)

V^i  $V^j = \left(\delta_{ij} - \dfrac{\bar{p}_i \bar{p}_j}{\bar{p}^2} \right) \dfrac{i}{p_0^2 - \bar{p}^2 - M_V^2} + \dfrac{\bar{p}_i \bar{p}_j}{\bar{p}^2} \dfrac{i}{p_0^2 - \bar{p}^2 - M_V^2 + m_A^2}$. (68c)

To avoid cluttered formulas, we will set $M_V = m_A$ in what follows. This does not affect the UV properties of the theory. We consider again diagram (59). Now it contains three contributions. The first one comes from the product of the first terms in propagator (68a) and, as before, is eliminated by adding to the path integral the fields $\tilde{\phi}$, η with action (62). Besides, there are contributions coming from the cross-product of the two terms in (68a),

$$- \frac{m_A^2}{2} h^{ij}(p) h^{kl}(-p) \int \frac{dq_0 d^3\bar{q}}{(2\pi)^4} \times$$
$$\times \frac{\bar{q}_i \bar{q}_k (q + \bar{p})_j (\bar{q} + \bar{p})_l}{\bar{q}^2 (q_0^2 - \bar{q}^2)(\bar{q} + \bar{p})^2}, \quad (69)$$

as well as from the square of the second term,

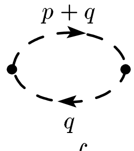
$$\frac{m_A^4}{4} h^{ij}(p) h^{kl}(-p) \int \frac{dq_0 d^3\bar{q}}{(2\pi)^4} \times$$
$$\times \frac{\bar{q}_i \bar{q}_k (\bar{q} + \bar{p})_j (\bar{q} + \bar{p})_l}{\bar{q}^2 (q_0^2 - \bar{q}^2)(\bar{q} + \bar{p})^2 ((q_0 + p_0)^2 - (\bar{q} + \bar{p})^2)}. \quad (70)$$

The divergences in these expressions can be removed by genuinely local counterterms. We consider, for example, Eq. (69). Introducing Feynman parameters we obtain

$$\int \frac{dq_0 d^3\bar{q}}{(2\pi)^4} \frac{\bar{q}_i \bar{q}_k (\bar{q}+\bar{p})_j (\bar{q} + \bar{p})_l}{\bar{q}^2 (q_0^2 - \bar{q}^2)(q+p)^2} = 2 \int_0^1 \frac{dx_1}{\sqrt{x_1}} \int_0^{1-x_1} dx_2 \times$$
$$\times \int \frac{dq_0' d^3\bar{q}}{(2\pi)^4} \frac{\bar{q}_i \bar{q}_k (\bar{q} + \bar{p})_j (\bar{q} + \bar{p})_l}{({q_0'}^2 - \bar{q}^2 - 2\bar{q}\bar{p}x_2 - \bar{p}^2 x_2)^3}, \quad (71)$$

where in the last integral we rescaled the loop frequency. The integral over the four-momentum on the r.h.s. has the standard form and its divergent part is a polynomial in momenta $\bar{p}$. It is straightforward to check that the integration over Feynman parameters does not contain any additional divergences. Thus, we conclude that the overall divergence of (71) is local both in time and space. Similar reasoning applies to (70).

One may worry that a divergence in the Feynman parameters can appear in the diagrams that contain the loop frequency in the numerator of the integrand, because then more powers of the Feynman parameters descend into the denominator. Let us show that this does not happen. We consider the diagram arising from the interactions given by the first and the third terms in (64),

h^{ij}  $h^{kl} = \dfrac{m_a^2}{2} h_{ij}(p) h_{kl}(-p) \displaystyle\int \frac{dq_0 d^3\bar{q}}{(2\pi)^4} \times$

$$\times \int \frac{q_0 (q_0 + p_0) \bar{q}_i \bar{q}_k (\bar{q} + \bar{p})_j (\bar{q} + \bar{p})_l}{\bar{q}^2 (q_0^2 - \bar{q}^2)(\bar{q} + \bar{p})^2 ((q_0 + p_0)^2 - (\bar{q} + \bar{p})^2)}. \quad (72)$$

Passing to the Feynman parameterization, we obtain

$$\int \frac{dq_0 d^3\bar{q}}{(2\pi)^4} \frac{q_0(q_0+p_0)\bar{q}_i\bar{q}_k(\bar{q}+\bar{p})_j(\bar{q}+\bar{p})_l}{\bar{q}^2(q_0^2-\bar{q}^2)(\bar{q}+\bar{p})^2((q_0+p_0)^2-(\bar{q}+\bar{p})^2)} =$$

$$= 6\int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int_0^{1-x_1-x_2} dx_3 \int \frac{dq_0 d^3\bar{q}}{(2\pi)^4} \times$$

$$\times \frac{q_0(q_0+p_0)\bar{q}_i\bar{q}_k(\bar{q}+\bar{p})_j(\bar{q}+\bar{p})_l}{\left[(x_1+x_2)q_0^2+2q_0p_0x_2+p_0^2x_2-\bar{q}^2-2\bar{q}\bar{p}(x_2+x_3)-\bar{p}^2(x_2+x_3)\right]^4}. \quad (73)$$

Upon rescaling of the loop frequency,

$$q_0 \mapsto q_0' = q_0/\sqrt{x_1+x_2},$$

the most singular contribution in the integral over Feynman parameters is proportional to $(x_1+x_2)^{-3/2}$ which is again integrable. At the heuristic level this can be understood as follows. The divergences in the integrals over Feynman parameters are usually associated to the infrared (or collinear) divergences, which are absent in our case because original expressions (69) and (72) are IR safe.

By extending the above reasoning to other diagrams in model (64) the reader will easily convince herself that the only class of divergences that require (spatially) non-local counterterms are those where all propagators in a given loop are equated to the first term in (68a). These divergences are independent of m_A and are exactly canceled by the remover fields $\tilde{\phi}$, η with action (62). Furthermore, this cancellation persists upon making the metric h_{ij} dynamical and allowing it to propagate in the loops. Thus, it is natural to conjecture that no matter how complicated a diagram is (see an example in Fig. 2), it will require only local counterterms after addition of similar diagrams with the fields $\tilde{\phi}$ and η.

So far we have discussed only the instantaneous modes associated with the Stückelberg fields ϕ^a of massive gravity. The dynamics of these fields is relatively simple: they enter UV action (27), (31) quadratically and do not contain any propagating degrees of freedom. This allowed us to eliminate all unusual non-local divergences appearing due to these fields by adding the "remover" sector $\tilde{\phi}^a$, η^a with simple action (62). However, as pointed out in Ref. [25], another source of instantaneous interactions is the khronon field ϕ^0. Here the situation appears to be more complicated: the khronon describes, besides the instantaneous mode, a genuine propagating degree of freedom and, furthermore, enters the action non-linearly. This produces difficulties with the quantization which are intrinsic of Hořava (or khronometric) proposal. We plan to address them elsewhere. For now, we just point out that the discussion of this section suggests that a consistent quantization of the theory exists. Indeed, in the decoupling limit the propagator of the khronon has the form similar to the second term in (68a) [25]. We have seen that the divergences associated with such propagators can be removed by local counterterms.

6. MODIFICATION OF THE NEWTON's LAW

Having addressed the theoretical consistency of the model, we now study its immediate phenomenological consequences. Let us consider the gravitational field of a point mass $M_\odot$ at a fixed position $x^i = 0$. We will focus on the weak field (linear) regime and assume the minimal coupling of the metric to the matter sector; the latter is justified by the phenomenological constraints on deviations from the Lorentz invariance [51]. There are two important changes with respect to the massive gravity phase (3) described in [5, 52]. First, at any energy, the role of the Stückelberg field ϕ^0 is played by the khronon. Second, the theory is defined also above the energy Λ_2, which can have experimental consequences at relevant short distances, e. g., in the early universe or in very dense stars. We will only consider the large distance modification in this section.

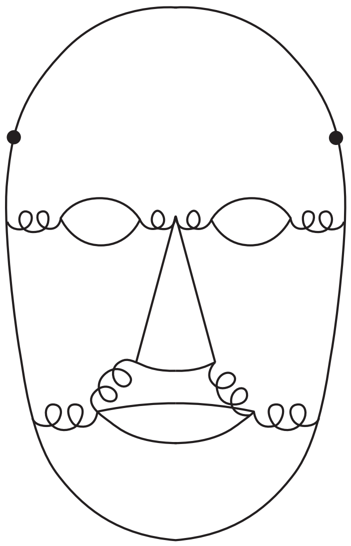


Fig. 2. Generic diagram with instantaneous modes and gravitons propagating in the loops. Summing it with the diagrams of the same topology where the different subsets of the ϕ-loops are replaced by those of $\tilde{\phi}$ and η will remove all non-local divergences

We work in the unitary gauge[19)], and restrict the vector fields to coset space (30). We consider the scalar part of the perturbations. The expansion around the Minkowski background to linear order reads

$$N = 1 + \varphi, \tag{74a}$$

$$N_i = \partial_i B, \tag{74b}$$

$$\gamma_{ij} = \delta_{ij} - 2\left(\delta_{ij} - \frac{\partial_i\partial_j}{\Delta}\right)\Psi - 2\frac{\partial_i\partial_j}{\Delta}E, \tag{74c}$$

$$V_a^i = M_V\delta_a^i + \epsilon_{iaj}\partial_j C + M_V\left(\delta_{ia} - \frac{\partial_i\partial_a}{\Delta}\right)\Psi + M_V\frac{\partial_i\partial_a}{\Delta}E, \tag{74d}$$

where we have used the linear part of Eq. (49). Using expression (51) and expanding khronometric and vector Lagrangians (19) and (28) to quadratic order, we obtain[20)]

$$\mathcal{L}^{(2)}_{scal} = \frac{M_P^2}{2}\Big[(1-\beta)(-2\dot\Psi^2 + 4\Psi\ddot E + 4\Psi\Delta\dot B) - (\lambda+\beta)(2\dot\Psi + \dot E + \Delta B)^2 - 2\Psi\Delta\Psi + 4\varphi\Delta\Psi + \alpha(\partial_i\varphi)^2\Big] + \frac{M_V^2}{2}\Big(2\dot\Psi^2 + (\dot E + \Delta B)^2 - 4(c_1^2 + c_2^2)(\partial_i\Psi)^2\Big) + (\partial_i\dot C)^2 - c_1^2(\partial_i\partial_k C)^2 - \mu^4\Psi^2 - \frac{\mu^4}{2}E^2 - m_A^2(\partial_i C)^2 - \varphi M_\odot\delta(\mathbf{x}). \tag{75}$$

We see that the pseudoscalar mode C completely decouples and has the dispersion relation

$$\omega^2 = c_1^2\bar k^2 + m_A^2. \tag{76}$$

For the other components, we obtain the set of equations,

$$2M_P^2\Delta\Psi - \alpha M_P^2\Delta\varphi - M_\odot\delta(\mathbf{x}) = 0, \tag{77a}$$

$$2M_P^2(1+\lambda)\dot\Psi + \big[M_P^2(\lambda+\beta) - M_V^2\big](\dot E + \Delta B) = 0, \tag{77b}$$

$$\big[M_P^2(1+\beta+2\lambda) - M_V^2\big]\ddot\Psi + M_P^2(1+\lambda)(\ddot E + \Delta\dot B) - \big[M_P^2 - 2M_V^2(c_1^2 + c_2^2)\big]\Delta\Psi - \mu^4\Psi + M_P^2\Delta\varphi = 0, \tag{77c}$$

$$2M_P^2(1+\lambda)\ddot\Psi + \big[M_P^2(\lambda+\beta) - M_V^2\big](\ddot E + \Delta\dot B) - \mu^4 E = 0. \tag{77d}$$

Combining the second and fourth equations, we find

$$E = 0, \quad \Delta B = -\frac{2M_P^2(1+\lambda)}{M_P^2(\lambda+\beta) - M_V^2}\dot\Psi. \tag{78}$$

Substituting this into Eq. (77c) and using (77a) to express φ, we find the equation for the single variable Ψ,

$$-\alpha M_P^2\frac{M_P^2(2+3\lambda-\beta) - M_V^2}{M_P^2(\lambda+\beta) - M_V^2}\ddot\Psi + 2M_P^2\left(1 - \frac{\alpha}{2}\right)\Delta\Psi - \mu^4\alpha\Psi = M_\odot\delta^{(3)}(\mathbf{x}), \tag{79}$$

where we have assumed

$$\alpha, \beta, \lambda, M_V/M_P \ll 1$$

and kept only up to the first subleading order in these parameters.

Let us momentarily put the source to zero, $M_\odot = 0$. Then (79) reduces to the wave equation for the helicity-0 graviton mode — the khronon. To the leading order, its dispersion relation reads

$$\omega^2 = \bar k^2\left(\frac{\lambda+\beta}{\alpha} - \frac{M_V^2}{\alpha M_P^2}\right) + \frac{\mu^4}{2M_P^2}\left(\lambda + \beta - \frac{M_V^2}{M_P^2}\right). \tag{80}$$

19) Recall that it is consistent to first fix the unitary gauge and take the variation of the action afterwards [53].
20) We remind that μ is defined in (35).

One makes two observations. First, the velocity of the khronon,

$$c_{kh} = \sqrt{\frac{\lambda+\beta}{\alpha} - \frac{M_V^2}{\alpha M_P^2}}, \tag{81}$$

gets renormalized compared to the pure khronometric theory (see Eq. (23)) due to the VEVs of the vector fields. The requirement that the velocity square remains positive puts an upper bound,

$$M_V < M_P\sqrt{\lambda+\beta}. \tag{82}$$

This condition is automatically satisfied within our assumptions (32). Second, the khronon acquires a mass gap,

$$m_{kh} = \frac{\mu^2}{M_P}\sqrt{\frac{\lambda+\beta}{2} - \frac{M_V^2}{2M_P^2}}, \tag{83}$$

which is parametrically smaller than the mass of graviton (7). This is in striking contrast to the case of [5] where the Stückelberg field ϕ^0 remains massless. It is worth stressing that the appearance of gap (83) is an IR phenomenon and depends only on the properties of the Stückelberg sector ϕ^0, ϕ^a at energies below m_A. Thus one expects it to be a universal property of massive gravities where this sector obeys the symmetries (1b), (14).

Next, we restore the source in (79) and focus on static configurations. We find

$$\Psi = -\frac{G_N M_\odot}{r}\exp(-m_{kh}r/c_{kh}), \tag{84}$$

where we have introduced the Newton's constant,

$$G_N \equiv \frac{1}{8\pi M_P^2(1-\alpha/2)}. \tag{85}$$

Clearly, the gravitational field has a Yukawa-type behavior. Finally, from (77a) we obtain the Newton's potential

$$\varphi = -\frac{G_N M_\odot}{r}\left[1 - \frac{2}{\alpha}\Big(1 - \exp(-m_{kh}r/c_{kh})\Big)\right]. \tag{86}$$

This potential is plotted in Fig. 3. One observes that it markedly deviates from the Newtonian potential of general relativity. The most striking feature is that the gravitational force becomes repulsive at distances $r > 1/m_g$. At large distances, the potential goes to zero. This is different from the case of massive gravities with gapless field ϕ^0 [5], where the gravitational potential generically presents linear growth with distance[21)]

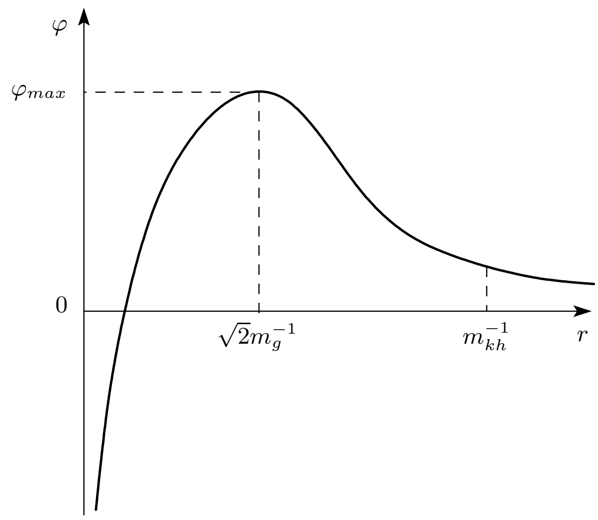


Fig. 3. The shape of the Newton potential in the massive gravity model in this paper. The gravitational force becomes repulsive at distances larger than the inverse graviton mass

[34, 52]. Note also that there is no van Dam–Veltman–Zakharov (vDVZ) discontinuity [56, 57]: in the limit $\mu \to 0$ the potentials φ, Ψ reduce to their GR expressions.

To understand the behavior of the Newton potential in more detail, we expand the exponent at

$$r \ll c_{kh}m_{kh}^{-1} = \left(\sqrt{\alpha}\, m_g\right)^{-1}.$$

At these distances the khronon mass is irrelevant and one expects the potential to coincide with the results existing in the literature. We obtain

$$\varphi = G_N M_\odot\left[-\frac{1}{r} + \sqrt{\frac{2}{\alpha}}m_g - \frac{m_g^2 r}{2} + \ldots\right], \tag{87}$$

where dots stand for the terms that are suppressed by the powers of the combination $\sqrt{\alpha}m_g r$. The second term in brackets gives a constant shift of the Newton potential which drops off from the observables involving only distances $r \lesssim 1/m_g$. The third term gives precisely the linear contribution discussed in [34, 52]. Note that for our model this contribution is repulsive. The potential reaches a maximum at $r = \sqrt{2}/m_g$ where

$$\varphi_{max} = \sqrt{2/\alpha}\, G_N M_\odot m_g.$$

For the validity of the linearized approximation φ_{max} must be much smaller than one. This translates into the condition that the graviton mass must be smaller than the inverse Schwarzschild radius of the source multiplied by $\sqrt{\alpha}$. Unless α is extremely small, this condition is not very restrictive.

21) This growth may be cut by non-linearities of the model [5, 54, 55] or by non-stationary evolution of the background [34]. Also, it is absent if the coefficients in the mass term (6) satisfy certain relations [5, 52].

Stronger phenomenological constraints come from the requirement that the gravitational field of localized sources should not significantly deviate from the standard form at astrophysical scales. The Solar System tests put a limit on the difference between the two gravitational potentials φ and Ψ. In the post-Newtonian framework this is traditionally parameterized by the ratio $\gamma \equiv \Psi/\varphi$ and the current constraint (measured at the orbit of Saturn by the Cassini satellite) reads [58]

$$\gamma - 1 = (2.1 \pm 2.3) \cdot 10^{-5}. \tag{88}$$

From expressions[22] (84), (87), we obtain the formula for γ in our model at distances shorter than inverse khronon mass,

$$\gamma = 1 - \frac{(m_g r)^2}{2}. \tag{89}$$

This gives an upper bound $m_g < 4 \cdot 10^{-17}$ cm^{-1} $\sim$ $\sim$ 120 pc^{-1}. A tighter limit comes from the gravitational field of galaxies. The requirement that it matches the standard expression implies

$$m_g \lesssim (1\ \text{Mpc})^{-1}. \tag{90}$$

It is likely that yet stronger bounds can be obtained from the large scale structure and the cosmic microwave background (CMB). We leave this analysis for future.

It would be also interesting to explore if the gravitational repulsion found above can be active at the cosmological scales and lead to accelerated expansion of the universe. Note that this mechanism of acceleration would rely crucially on the presence of inhomogeneities, as the homogeneous FRW ansatz does not exhibit any self-accelerated behavior (see Sec. 4).

Before closing this section, let us mention that a complementary way to constrain the graviton mass is by looking directly at the modifications in the helicity-2 sector. These have consequences for radiation and propagation of gravity waves [52, 59, 60]. Having a more complete theory allows to put these studies on the firm ground in the situations with characteristic scales smaller than Λ_2^{-1}, such as inflation and reheating.

7. SUMMARY AND DISCUSSION

In this paper, we have proposed an embedding of Lorentz violating massive gravity above the scale $\Lambda_2 \equiv \equiv \sqrt{m_g M_P}$. The proposed theory has a high cutoff scale only a few orders of magnitude below the Planck mass and independent of the mass of the graviton. At high energies the theory possesses a large symmetry FDiff $\times$ $SO(3)$ which is spontaneously broken at lower energy to a diagonal global $SO(3)$ subgroup[23]. This pattern of symmetry breaking is realized by a triplet of space-like vector fields which develop non-zero VEVs and play the role of the "Higgs" fields. A crucial technical role is played by a quadratic mixing between the vectors and the Stückelberg fields ϕ^a of massive gravity. Once the vectors acquire VEVs, this mixing forces the Stückelbergs to develop coordinate-dependent profiles, which eventually translates into the graviton mass. This means that no non-linear interactions in the Stückelberg sector are required to do this job and one can restrict to purely quadratic action for the fields ϕ^a, thus eliminating any strong coupling from this sector. This mechanism is reminiscent of the proposal for the (partial) UV completion of the ghost condensate model [36, 47] where a mixing between a time-like vector acquiring a VEV and a massless scalar forces the latter to evolve in time.

The graviton mass in the model is proportional to the product of the vector VEVs and the coefficient in front of the vector-Stückelberg mixing. Thus, it vanishes both if the vector VEVs disappear (in the unbroken phase) or if the mixing is switched off. The action stays regular in the limit of vanishing mass and therefore one expects all observable quantities, with the quantum corrections included, to behave smoothly in this limit. In this sense, our mechanism is analogous to the Higgs mechanism of gauge theories. It is worth stressing that in our model the mixing between the vector and Stückelberg fields is protected by a discrete symmetry $\phi^a \mapsto -\phi^a$ and thus a small coefficient in front of it is technically natural. This implies that the graviton mass is stable under quantum corrections.

We analyzed the structure of the theory at different energies and explicitly verified the expectation that new degrees of freedom, besides those of pure massive gravity, must exist below the scale Λ_2. Indeed, we found that certain components of the vector fields propagate at these energies. These degrees of freedom have a mass gap which is parametrically smaller than Λ_2, but still bigger than m_g. It would be interesting to work out the consequences of these new light degrees of freedom for phenomenology.

We also found that the helicity-0 component of the graviton, which in our model is identified with the khronon of the khronometric model, acquires a mass parametrically lower than m_g. This has important im-

[22] We subtract the constant piece from (87).

[23] Notice that Lorentz invariance is broken explicitly all the way up to the cutoff.

plications for the gravitational potentials of localized sources: unlike previous models of LV massive gravity, in our case the potentials fall of exponentially at large distances. Remarkably, the shape of the Newton potential is not monotonous. It grows from negative values at short distances, changes sign, reaches a positive maximum at $r = \sqrt{2}\, m_g^{-1}$ and then decreases towards $r \to \infty$. This implies that the gravitational force becomes repulsive at $r > \sqrt{2} m_g^{-1}$. This property may lead to a rich phenomenology which we leave for future studies. An interesting question is whether the gravitational repulsion between the inhomogeneities present in the universe can provide the accelerated expansion at recent epoch, despite the fact that for the strictly homogeneous ansatz our model does not exhibit any self-acceleration.

A subtle theoretical aspect of our model, inherited from the effective theory of LV massive gravity, is the presence of instantaneous interactions. We have addressed the issue of quantization of the instantaneous modes and argued that it can be performed consistently. We also pointed out that in the canonical formalism the instantaneous modes must be interpreted as a certain type of non-locality along the spatial dimensions. To make the discussion concise, we focused on simplified toy models. A more comprehensive study of this topic is definitely required owing to its importance for LV proposals for quantum gravity [23, 25].

Another open question left for future research is to understand how the strong coupling of LV massive gravity manifests itself at the level of Feynman diagrams and how it is canceled by the new degrees of freedom appearing in our model (see [32, 61] for related works in the Lorentz invariant context). This may shed light on possible generalizations of the mechanism proposed in this paper to other IR modifications of gravity, such as multi-metric theories and the Lorentz invariant setup of [7]. In particular, it would be interesting to prove at the diagrammatic level the (im)possibility of a Lorentz invariant Wilsonian UV completion of the latter setup.

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