

SCALARON PRODUCTION IN CONTRACTING ASTROPHYSICAL OBJECTS

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Received October 24, 2014

We study the creation of high-energy SM particles in the Starobinsky model of dark energy (a variant of $F(R)$-gravity) inside the regions contracting due to the Jeans instability. In this modification of gravity, the additional degree of freedom — a scalaron — behaves as a particle with the mass depending on matter density. Therefore, when the mass changes, light scalarons could be created at a nonadiabatic stage. Later, the scalaron mass grows and can reach large values, even the value 10^{13} GeV, favored by early time inflation. Heavy scalarons decay contributing to the cosmic ray flux. We analytically calculate the number density of created particles for the exponential (Jeans) contraction and find it negligibly small for the phenomenologically viable and cosmologically interesting range of model parameters. We expect similar results for a generic model of $F(R)$-gravity mimicking the cosmological constant.

Contribution for the JETP special issue in honor of V. A. Rubakov's 60th birthday

DOI: 10.7868/S0044451015030209

1. INTRODUCTION

Numerous observational data require a new component in the right-hand side of the Einstein equations, which is called dark energy and which causes the accelerated expansion of the Universe. The simplest and still viable candidate for dark energy is obviously a cosmological constant. But its unnaturally small value engenders investigation of other ways to explain the observational data.

$F(R)$-gravity provides a framework for constructing models of dark energy with the time dependent equation of state $p/\rho \equiv w = w(t)$; moreover, at some stage we have $w < -1$ (see [1] for a review). Such models of modified gravity may also explain the inflationary stage of the early Universe, providing a unified mechanism to describe both stages of accelerated expansion.

The choice of the function $F(R)$ is still phenomenological to a large extent. It must be self-consistent from the theoretical standpoint, explain the cosmological data, and pass all Solar System and astrophysical tests. The natural question is: How to distinguish $F(R)$-gravity from other models of dark energy? The most straightforward way is to improve the sensitivity of the overall cosmological analysis to the dark energy equation of state. However, besides serious systematic uncertainties, there are physically motivated degeneracies in the cosmological observables with respect to physical parameters. In particular, specific effects of $F(R)$-gravity at small spatial scales can be canceled by massive (sterile) neutrinos, whose dynamics works against modified gravity [2].

An attractive idea of probing $F(R)$-gravity [5] is associated with possible production of high-energy particles (scalarons) in space regions where the matter density changes. It was claimed in Ref. [5] that the growing curvature oscillations that decay into high-energy particles may lead to a significant impact on the flux of ultra-high energy cosmic rays. This result is rather

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unexpected, because high-frequency oscillations (i. e., heavy particles) are produced by a slow process (the structure formation) very inefficiently. Moreover, cosmological evolution naturally gives zero initial amplitude for such oscillations, which can be associated only with the scalar mode (heavy scalaron).

In this paper, we study the processes of quantum particle production in $F(R)$-gravity using the method of Bogoliubov transformations. We consider $F(R)$-gravity being equivalent to normal gravity with an additional scalar field (scalaron) having a complicated potential in the Einstein frame. The form of the potential depends on the surrounding density, providing the scalaron to be a chameleon field. Scalaron is very light at densities close to the present energy density of the Universe and heavy at larger densities. It is known that the particle with a time-dependent mass may be created in quantum theory if the typical process rate is close to the mass value. A scalaron may be born being light and then its mass may grow while the object contracts. When the scalaron becomes heavy, it decays into high-energy Standard Model (SM) particles. We consider the same $F(R)$-model as in [3], where such processes can be investigated analytically, and obtain that in contrast to [3], the number density of created particles is unfortunately too small to be observed in all realistic contracting regions (astorphysical objects) in the Universe.

This paper is organized as follows. In Sec. 2, we describe the construction of the function $F(R)$ that is appropriate for both the inflation and dark energy. In Sec. 3, we introduce the Einstein frame approach to the $F(R)$-gravity in which the additional scalar field has a mass depending on the background energy density. In Sec. 4, we calculate the number density of produced particles an object in contracting due to the Jeans instability and discuss the particle production rate in different contracting regions of the Universe (astorphysical objects). We conclude in Sec. 5.

2. DESCRIPTION OF THE MODEL

The present-day acceleration of the expansion of the Universe can be described in terms of $F(R)$-gravity by the action [4][1)]

$$S = \frac{M_P^2}{2}\int d^4x\sqrt{-g}\,F(R), \tag{1}$$

with $F(0) = 0$ reflecting the disappearance of the cosmological constant in the Minkowski flat space limit.

[1)] The metric signature is $(-+++)$.

Any viable $F(R)$ function must obey the classical and quantum stability conditions: $F'(R) > 0$ and $F''(R) > 0$. It was introduced to mimic the ΛCDM model in the late-time Universe, and hence in the limit of small curvatures, $F(R) \approx R - 2\Lambda$ with the dark energy density $\rho_\Lambda = \Lambda M_P^2$. Moreover, the second derivative of $F(R)$ must also be bounded from above, $F''(R) <$ const (see [5] for the details) to avoid early time singularities at $R \to \infty$. The last requirement is easily satisfied for any $F(R)$ after adding an R^2-term. This term with a specially selected coefficient may also provide the Starobinsky inflation in the early Universe [6].

An example of a function appropriate for the dark energy proposed by Starobinsky is [4]

$$F(R) = R + \lambda R_0\left(\left(1+\frac{R^2}{R_0^2}\right)^{-n} - 1\right). \tag{2}$$

In the regime $R \gg R_0$, we have $F(R) \approx R - \lambda R_0$, providing a cosmological constant. The parameter R_0 fixes a scale that corresponds to the dark energy density ρ_Λ (this is valid for $\lambda \gtrsim 1$):

$$R_0 \equiv \frac{2}{\lambda}\frac{\rho_\Lambda}{M_P^2}. \tag{3}$$

To avoid the early time singularity, we hereafter use the function $F(R)$ with the R^2-term added:

$$F(R) = R + \lambda R_0\left(\left(1+\frac{R^2}{R_0^2}\right)^{-n} - 1\right) + \frac{R^2}{6M^2}. \tag{4}$$

As discussed above, the last term in (4) is also appropriate for the usual Starobinsky inflation [6] in the early Universe if we choose $M = 3\cdot 10^{13}$ GeV; an impact of the second term in (4) is negligible for the corresponding large values of curvatures.

There is a problem (described in [5]) with the subsequent stage of scalaron oscillations. At this stage, zero and even negative values of R may be obtained. Then it is easy to see that later the Universe unavoidably arrives at $F''(R) < 0$, leading to quantum instability of the theory. However it is possible to construct a function $F(R)$ that provides a similar late-time cosmology but does not suffer from instabilities at the post-inflationary epoch. For example (where we neglect the presence of the R^2-term),

$$F''(R) \propto \frac{1}{1+(R/R_0)^{2n+2}} \tag{5}$$

leads to the results similar to those following from (4) at large curvatures. In what follows, we use function (4) for simplicity, being interested only in the late time evolution of the Universe. But we need to ensure that the evolution of R does not put its value to the region of forbidden curvatures.

Starobinsky model (4) has two free parameters, λ and n (R_0 is fixed by (3), while M is fixed to explain the early time inflation). A stringent restriction on the value of n follows from local gravity constraints on the chameleon gravity [7]. It gives $n \gtrsim 1$. The parameter λ is bounded only from the stability condition for the de Sitter minimum (see [8] for a review). This bound mildly varies with n as

$$\lambda > f(n) \quad \text{and} \quad f(n) \approx n/(2n - 4/3). \tag{6}$$

3. EINSTEIN FRAME PICTURE: THE SCALARON AS A CHAMELEON

$F(R)$-gravity can be considered in the Einstein frame, where it describes the usual Einstein gravity with an additional scalar field (scalaron) coupled to the matter fields as a chameleon field [9]. The scalaron potential is

$$V(\phi) = \frac{M_P^2}{2F'(R)^2}\left(RF'(R) - F(R)\right), \tag{7}$$

where $R = R(\phi)$ solves the equation

$$F'(R) = \exp\frac{\sqrt{2}\phi}{\sqrt{3}M_P}. \tag{8}$$

Through the gravity interaction, the scalaron couples to all the matter fields effectively described in the cosmological context as the ideal fluid with an energy density ρ and a pressure p. This coupling modifies the scalaron potential [9]:

$$V_{eff}(\phi) = V(\phi) + \frac{\rho - 3p}{4}e^{-4\bar{\phi}}. \tag{9}$$

The minimum ϕ_{min} of V_{eff} can be obtained substituting the solution R_{min} of the equation

$$2F(R) - RF'(R) = \frac{\rho - 3p}{M_P^2} \tag{10}$$

in (8), with ϕ_{min} written in terms of R_{min}.

For (4), we can approximate the solution as (the greater λ is, the better the accuracy)

$$R_{min} \approx (4+\tau)\frac{\rho_\Lambda}{M_P^2}, \qquad \bar{\phi}_{min} = \frac{\sqrt{3}M_P}{\sqrt{2}}\log(F'(R_{min})), \tag{11}$$

where

$$\tau \equiv (\rho - 3p)/\rho_\Lambda\,.$$

The effective scalaron mass at this minimum is

$$m_{eff}^2 = \frac{1}{3F''(R_{min})}\left(1 - \frac{R_{min}F''(R_{min})}{F'(R_{min})}\right) \approx \approx \frac{1}{3F''(R_{min})}. \tag{12}$$

Hence, for model (4), we obtain the scalaron mass that depends on the surrounding energy density and pressure:

$$m_{eff}^2 = \frac{M^2 m^2}{M^2 + m^2}, \tag{13}$$

where

$$m^2 = \frac{1}{12n(2n+1)}\left(\frac{\lambda}{2}\right)^{2n}\frac{\rho_\Lambda}{M_P^2}(4+\tau)^{2n+2}. \tag{14}$$

In a particular range of densities m_{eff} strongly depends on τ. Obviously (see Eqs. (14) and (13)), m is small there, $m < M$, leading to

$$4 < \tau < \left(\frac{MM_P}{\sqrt{\rho_\Lambda}}\right)^{1/(n+1)}\left(\frac{2}{\lambda}\right)^{n/(n+1)} \times \times (12n(2n+1))^{1/(n+1)} = \left(1.4\cdot 10^{55}\right)^{1/(n+1)} \times \times \left(\frac{2}{\lambda}\right)^{n/(n+1)}(12n(2n+1))^{1/(n+1)}. \tag{15}$$

The situation where the scalaron behaves as chameleon (i. e., inequality (15) is fulfilled) can be realized in a large variety of astrophysical objects at different scales depending mostly on a choice of n. When the mass of a particle varies with changing the surrounding density, that particle can be created if the adiabaticity condition is violated. In the next section, we calculate the corresponding number of created particles.

The scalaron is unstable because it is coupled to all nonconformal fields that are present in the matter Lagrangian. If we consider the Standard Model of particle physics to describe all the matter content, then the scalaron presumably decays to Higgs bosons (if kinematically allowed) [10] with the decay rate

$$\Gamma = \frac{m_{eff}^3}{48\pi M_P^2}. \tag{16}$$

Higgs bosons decay immediately, producing a flux of protons, electrons, and neutrinos, which provides a possibility to find such events in the case of significant scalaron production.

4. PARTICLE PRODUCTION IN CONTRACTING OBJECTS

4.1. Mathematical problem

A particle with its mass depending on the surrounding density can be produced in contracting clouds when the adiabaticity condition is violated. We consider an object contracting due to the Jeans instability with $\rho(t) = \rho_0 \exp(t/t_J)$ ($t > 0$, with $t_J = M_P/\sqrt{\rho_0}$). To calculate the number of produced particles, we use the standard approach of Bogoliubov transformations described, e.g., in textbook [11]. According to this method, we write an equation for the scalaron mode function φ with a momentum k:

$$\ddot{\varphi} + (k^2 + m_0^2 e^{2\beta t})\varphi = 0, \qquad (17)$$

where

$$m_0 \equiv \frac{\alpha^{n+1}}{\sqrt{12n(2n+1)}} \left(\frac{\lambda}{2}\right)^n \frac{\sqrt{\rho_\Lambda}}{M_P}, \qquad \alpha \equiv \frac{\rho_0}{\rho_\Lambda}, \quad \beta \equiv \frac{(n+1)}{t_J}. \qquad (18)$$

For the chosen $\rho(t)$, the contraction starts at $t = 0$, and we therefore postulate vacuum initial conditions $\varphi = 1/\sqrt{2\omega}$, $\dot{\varphi} = -i\omega\varphi$ (where $\omega = \sqrt{k^2 + m_0^2 e^{2\beta t}}$) at $t = -\infty$. Because $\rho = \text{const}$ for $t < 0$, such conditions can be set for any instant $t < 0$, e.g., at $t \to (-0)$, just before the contraction starts. The adiabaticity condition [11][2)]

$$\left|\frac{\ddot{\omega}}{\omega^3} - \frac{3}{2}\left(\frac{\dot{\omega}}{\omega^2}\right)^2\right| \ll 1 \qquad (19)$$

is violated only for $t < 1/\beta$, with no particles produced after this time.

Equation (17) with the vacuum initial conditions can be solved analytically in terms of Bessel functions. An exact form of the relevant Bogoliubov coefficient is (up to an irrelevant complex phase)

$$B = \exp\left(\frac{\pi k}{2\beta}\right) \frac{\sqrt{\pi}}{2\sqrt{2\beta}} \sqrt[4]{m_0^2 + k^2} \times \left(\frac{i m_0 \left(H^{(2)}_{ik/\beta+1}\left(\frac{m_0}{\beta}\right) - H^{(2)}_{ik/\beta-1}\left(\frac{m_0}{\beta}\right)\right)}{2\sqrt{m_0^2+k^2}} + H^{(2)}_{ik/\beta}\left(\frac{m_0}{\beta}\right)\right), \qquad (20)$$

where $H^{(2)}_a(x)$ is the Hankel function.

We first discuss the case $m_0 < \beta$. It corresponds to the situation where the scalaron is created being light at production ($m \sim \beta$) and slightly later, when adiabaticity condition (19) is restored. After that, its mass grows until the instant when the scalaron decays to SM particles as discussed in the preceding section. The number of created scalarons can be numerically obtained in this case as

$$n_p = \frac{4\pi}{(2\pi)^3} \int k^2 |B|^2 \, dk \approx C\beta^3, \qquad (21)$$

where $C = 4.9 \cdot 10^{-4}$.

In the opposite case $m_0 > \beta$, one expects that particle production is suppressed because massive particles cannot be created in a slow process. But numerically we obtain

$$n_p = \frac{4\pi}{(2\pi)^3} \int k^2 |B|^2 \, dk = C m_0 \beta^2 \qquad (22)$$

with $C = 6.23 \cdot 10^{-3}$. It looks strange that the larger the mass is, the more particles are produced. The reason is connected with the fact that the mass depends on time in a nonsmooth way in the simple mathematical model that we considered. Particles are produced mostly at the time close to $t = 0$, where the mass dependence on time is not smooth. The divergent second-order time derivative of ω leads to a violation of adiabaticity condition (19) at $t = 0$. But it is actually natural to expect that the contraction starts in a smooth way with a typical time $t_0 > L > t_J$ (for causality reasons), where L is the size of the object. If we use such kind of smoothing, we obtain the expected suppression of particle production because in this case modes evolve adiabatically.

To illustrate such a suppression, we consider a smooth dependence $m^2(t) = m_0^2(1 + e^{2\beta t})$ with $m_0 > \beta$ and set vacuum initial conditions at $t = -\infty$. We can then analytically solve an equation like (17) and obtain the Bogoliubov coefficient to be (up to an irrelevant phase)

2) Usually, it is equivalent to the condition $\dot{\omega}/\omega^2 \ll 1$, which means that the mass of a particle must exceed the characteristic rate of the corresponding process.

$$B = \frac{1}{\sqrt{2\pi q}} |\Gamma(1 - iq)| \exp\left(-\frac{\pi q}{2}\right), \quad q = \frac{\sqrt{m_0^2 + k^2}}{\beta}. \tag{23}$$

In the limit of large q, we find $B \approx e^{-\pi q}$. For $m_0 > \beta$, we can write the produced number density as

$$n_p = \frac{4\pi}{(2\pi)^3} \int k^2 |B|^2 dk = \frac{4\pi\beta^3}{(2\pi)^3} \int\limits_{m_0/\beta}^{\infty} |B(q)|^2 \sqrt{q^2 - (m_0/\beta)^2}\, q\, dq \approx \tag{24}$$

$$\approx \frac{4\pi\beta^3}{(2\pi)^3} \int\limits_{m_0/\beta}^{\infty} e^{-2\pi q} \sqrt{q^2 - (m_0/\beta)^2}\, q\, dq \sim \beta^3 \times \exp\left(-\frac{2\pi m_0}{\beta}\right). \tag{25}$$

We see that in a realistic model, particle production is exponentially suppressed in the case $m_0 > \beta$. This is in accordance with the Rubakov theorem: If you do everything correctly, the result is correct. We cannot calculate the exact number of produced particles in a model-independent way, because it depends on the details of the Jeans instability development.

The result in (21) for $m_0 < \beta$ is still correct because particles are mostly produced not at $t \approx 0$ but at the instant when $m \sim \beta$, the mass dependence on time is smooth, and our approximation works. Also, it can be proved that the number of produced particles does not depend on the details of how the Jeans instability starts to evolve.

4.2. Applications

4.2.1. Cosmic structure formation

With inequality (6), the condition $m_0 < \beta$ is satisfied only for initial densities $\alpha = \rho/\rho_\Lambda < 8$ for the viable region of model parameters that corresponds to recent and ongoing structure formation processes. Using (21), we can estimate the number of created particles inside the region of a typical size $L \approx 1$ Mpc:

$$N = n_p L^3 = 5 \cdot 10^{-4} \beta^3 (1\ \text{Mpc})^3 \lesssim 10^{-12} (n+1)^3. \tag{26}$$

We can see that only a negligible number of scalarons can be created in the structure formation process. Only an enormous value of n may lead to noticable production, which we disregard. As discussed above, particle production is strongly suppressed when $m_0 > \beta$, and hence the earlier structure formation (star and galaxy formation) gives a much smaller impact.

4.2.2. Star formation in our Galaxy

The density range of contracting clouds that now form stars in our Galaxy corresponds to the case $m_0 \gg \beta$. As discussed above, we cannot correctly describe the particle creation process because we need to know in detail how the Jeans instability evolves. But we can obtain an upper bound on the number density of created particles as $n_p \sim \beta^3$ (in reality, n_p is much smaller because of exponential suppression (25)) and calculate the corresponding flux of high-energy particles:

$$F = \frac{n_p L^3}{r^2 t_J} N \sim 3\,(n+1)^3 \cdot 10^{-78}\ \text{cm}^{-2} \cdot \text{s}^{-1}. \tag{27}$$

Here, $L = c_s\, t_J$ is the cloud size, $c_s \sim \sqrt{T/m_p}$ is the sound speed in the gas (we here take the temperature $t = 10$ K and m_p to be the hydrogen molecular mass), $r = 10$ kpc is a characteristic distance in our Galaxy, and N is the total number of objects that may be obtained from the known star formation rate of 3 $M_\odot$ per year [12].

The measured flux of ultra-high-energy cosmic rays even at the energy 10^{20} eV, $F \sim 10^{-21}$ cm$^{-2} \cdot$ s^{-1} [13], is many orders greater than the number obtained in (27). Hence, in any case, the scalaron creation has a negligible effect in astrophysics.

4.2.3. Expanding Universe

Expansion of the Universe obviously implies changing the energy density. Scalarons are expected to be created at the instant when $m_{eff} \sim H$ (H is the Hubble parameter), and hence the number of created particles can be roughly estimated from dimensional analysis as $n_p \sim H^3$ (see also [14]). There were two instants in the past of the Universe when m_{eff} was close to H. The first instant corresponds to the period just after inflation. Scalarons created at that time have decayed in the very early Universe and do not affect the present Universe. The second instant, if it exists (for large λ, the scalaron mass m_{eff} is always greater than H) is very close to the present moment corresponding to the redshift $z < 0.2$. We therefore expect the number density of created scalarons to be $n_p \sim H_0^3$, which means that there is only one particle inside the present horizon or even less.

5. CONCLUSIONS AND DISCUSSION

We studied particle production in media with changing density, occurring in $F(R)$-gravity or other chameleon models of dark energy. We performed a calculation of scalaron creation in the Einstein frame for the Starobinsky dark energy model [4]. In the case of exponential contraction due to the Jeans instability, the corresponding equation has an analytic solution. Imposing vacuum initial conditions at the instant when contraction starts, we calculated the Bogoliubov coefficient and the number density of created particles. In all realistic situations, the scalaron production is very inefficient. In all cases, less than one particle inside the corresponding Jeans volume is produced. The subsequent scalaron decay contirbution to the cosmic ray flux is found to be infinitesimal. In the Starobinsky variant of $F(R)$-gravity considered here, formally, the production rate increases with the parameter n. But the latter must be enormously huge to produce a noticable effect. This implies a drastic change in the $F(R)$ function at the particular value of curvature ($R = R_0$), which is a very unnatural and ungrounded choice.

We thank F. Bezrukov, A. Dolgov, K. Postnov, and S. Sibiryakov for the discussions. This paper was supported by the Russian Science Foundation grant 14-12-01430.

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