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DYNAMICS OF CHARGED PLANAR GEOMETRY IN TILTED AND NONTILTED FRAMES

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We investigate the dynamics of charged planar symmetry with an anisotropic matter field subject to a radially moving observer called a tilted observer. The Einstein–Maxwell field equations are used to obtain a relation between nontilted and tilted frames and between kinematical and dynamical quantities. Using the Taub mass formalism and conservation laws, two evolution equations are developed to analyze the inhomogeneities in the tilted congruence. It is found that the radial velocity (due to the tilted observer) and the electric charge have a crucial effect on the inhomogeneity factor. Finally, we discuss the stability in the nontilted frame in the pure diffusion case and examine the effects of the electromagnetic field.

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1. INTRODUCTION

In any physical phenomenon, the significance of observers cannot be ignored. During the last few years, there has been a renewed interest in the study based on relative motion of observers. One of the reasons for this interest is to study any realistic picture of the evolution of the early universe. Physical quantities like the Hubble parameter depend on the choice of congruence and are consequently referred to as congruence-dependent quantities. Cosmological models have two timelike vector fields (congruences): the unit vector field orthogonal to the surface (geometric congruence) and the four-velocity of the matter distribution (fluid congruence). If the four-velocity is not aligned with the unit vector field, then it is called a tilted, and otherwise nontilted congruence. There has been an extensive study of homogeneous and anisotropic cosmological models describing evolution of the early universe.

Many theoretical and observational reasons have motivated the researchers to study anisotropic and inhomogeneous models including the Tolman, Szekeres, Gowdy, and some plane symmetric solutions. At small scales, the observed galaxy distribution is found to be inhomogeneous, while it is expected to become spatially homogeneous on theoretical grounds. An inhomogeneous matter distribution may lead to the formation of a naked singularity, compared to the homogeneous fluid configuration, where a black hole is more likely to be formed [1]. Some inhomogeneous solutions of a plane symmetric spacetime for a viscous fluid distribution were found in [2]. In [3], the Lemaître–Tolman–Bondi metric was studied in spherical coordinates and the effect of anisotropy and inhomogeneity on the collapse of a dust cloud was analyzed. The impact of inhomogeneity on different parameters of a spherically symmetric collapsing star radiating away its energy in the form of radial heat flux was explored in [4]. A cosmological model for isotropic expansion of an inhomogeneous universe was proposed in [5], where some exact inhomogeneous solutions for spherical and axial symmetries were also obtained.

In most of the cases, the fluid distribution is considered to be isotropic in pressure. However, pressure anisotropy and heat dissipation are also expected to play a crucial role in an expanding and collapsing universe. Many researchers [6] have taken keen interest in investigating tilted cosmological models in the presence of a heat flux. Tilted models having a disordered radiating isotropic fluid with heat flux were explored in [7] for a Bianchi type I model. Hydrodynamical and thermodynamical properties of a tilted Lemaître–Tolman–Bondi spacetime with an anisotropic matter configuration were studied in [8]. Some dynamical properties of tilted planar geometry with a radiating anisotropic

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matter distribution were explored in [9]. In [10], homogeneous tilted models with a radiating source in plane symmetry were found and the behavior of some physical parameters was examined. Recently, we have discussed the dynamics of a charged spherical star with tilted and nontilted frames [11].

The occurrence of a magnetic field in the present galactic as well as intergalactic spaces is a well-established fact and its significance is acknowledged in many astrophysical phenomena. According to [12], tilted Bianchi types I, II, and III are possible in the presence of an electromagnetic field. In [13], electromagnetic effects on cylindrically symmetric inhomogeneous cosmological models with a perfect fluid were explored and different physical and geometrical properties were discussed. The consequences of charge and dissipation (heat flux, shear viscosity, and radiation density) in the dynamics of spherically symmetric collapse were worked out in [14]. Sharif and his collaborators [15, 16] have studied the effects of electric charge on self-gravitating collapsing models with different physical backgrounds.

This paper is devoted to exploring the dynamics of plane symmetric spacetime with the congruence of a tilted observer consisting of radiating anisotropic matter in the presence of the electromagnetic field. The paper has the following format. In Sec. 2, we present the Einstein–Maxwell field equations for both tilted and nontilted observers and find some relations between them. In the latter case, the matter content is no longer charged dissipative but charged dust cloud. Section 3 deals with some kinematical and dynamical quantities that are used to investigate the Ellis evolution equation for the tilted congruence as well as the inhomogeneity factor. We also discuss stability analysis in the nontilted frame with the effects of the electromagnetic field. Section 4 concludes our results.

2. FLUID CONFIGURATION AND BASIC FORMALISM

To investigate inhomogeneities in the present accelerated expansion phase of the universe, we take nonstatic plane symmetric geometry in the form [17]

$$ds^2 = -A^2(t,z)\,dt^2 + B^2(t,z)\left(dx^2+dy^2\right) + C^2(t,z)\,dz^2. \quad (1)$$

The energy–momentum tensor for a charged dust cloud in a nontilted frame is

$$T_{\alpha\beta} = \bar{\rho} u_\alpha u_\beta + \frac{1}{4\pi}\left(F_\alpha^\gamma F_{\beta\gamma} - \frac{1}{4}F^{\gamma\delta}F_{\gamma\delta}g_{\alpha\beta}\right), \quad (2)$$

where u_α, $\bar{\rho}$, and $F_{\alpha\beta}$ are the four-velocity, the energy density, and the Maxwell field tensor. In comoving coordinates, it takes the form

$$u_\alpha = (A,0,0,0), \quad F_{\alpha\beta} = -\phi_{\alpha,\beta} + \phi_{\beta,\alpha},$$

where ϕ_α is the four-potential. The Maxwell field equations are

$$F^{\alpha\beta}_{\ \ ;\beta} = \mu_0 J^\alpha, \quad F_{[\alpha\beta;\gamma]} = 0, \quad (3)$$

where J_α and $\mu_0 = 4\pi$ is the four-current and the magnetic permeability. In comoving coordinates, the four-current and four-potential become

$$J^\alpha = \xi u^\alpha, \quad \phi^\alpha = \phi\delta_0^\alpha,$$

where ϕ and ξ respectively denote the scalar potential and charge density; both are functions of t and z. With these used in Eq. (3), the Maxwell-field equations yield the independent components

$$\frac{\partial^2\phi}{\partial z^2} - \left(\frac{A'}{A} + \frac{C'}{C} - \frac{2B'}{B}\right)\frac{\partial\phi}{\partial z} = \xi\mu_0 AC^2, \quad (4)$$

$$\frac{\partial^2\phi}{\partial t\partial z} - \left(\frac{\dot{A}}{A} + \frac{\dot{C}}{C} - \frac{2\dot{B}}{B}\right)\frac{\partial\phi}{\partial z} = 0. \quad (5)$$

Here, the prime and the dot respectively stand for z and t differentiation. Integrating Eq. (4) with respect to z leads to

$$\phi' = \frac{s(z)\mu_0 AC}{B^2}, \quad \text{where} \quad s(z) = \int\limits_0^z \xi CB^2\,dz, \quad (6)$$

which equivalently satisfies Eq. (5).

The corresponding Einstein–Maxwell field equations in the nontilted frame yield

$$\left(8\pi\bar{\rho} + \frac{s^2\mu_0^2}{B^4}\right)A^2 = \left(\frac{2\dot{C}}{C} + \frac{\dot{B}}{B}\right)\frac{\dot{B}}{B} - \left(\frac{A}{C}\right)^2 \times \left[\frac{2B''}{B} + \left(\frac{B'}{B} - \frac{2C'}{C}\right)\frac{B'}{B}\right], \quad (7)$$

$$0 = -2\left(\frac{\dot{B}'}{B} - \frac{\dot{B}A'}{BA} - \frac{\dot{C}B'}{CB}\right), \quad (8)$$

$$-\frac{s^2\mu_0^2C^2}{B^4} = -\left(\frac{C}{A}\right)^2\left[\left(\frac{\dot{B}}{B}\right)^2 + \frac{2\ddot{B}}{B} - \frac{2\dot{A}\dot{B}}{AB}\right] + \left(\frac{B'}{B}\right)^2 + \frac{2A'B'}{AB}, \quad (9)$$

$$\frac{s^2\mu_0^2}{B^2} = \left(\frac{B}{C}\right)^2\left[\frac{B''}{B} - \frac{A'}{A}\left(\frac{C'}{C} - \frac{B'}{B}\right) + \frac{A''}{A} - \frac{B'C'}{BC}\right] - \\ - \left(\frac{B}{A}\right)^2\left[\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} - \frac{\dot{A}}{A}\left(\frac{\dot{C}}{C} + \frac{\dot{B}}{B}\right) + \frac{\dot{C}\dot{B}}{CB}\right]. \quad (10)$$

We perform a Lorentz transformation on the nontilted congruence of the observer to obtain a tilted frame in which the matter configuration has a radial velocity ω. The unit four-vectors then take the form

$$U^\alpha = \left(\frac{1}{A\sqrt{1-\omega^2}}, 0, 0, \frac{\omega}{C\sqrt{1-\omega^2}}\right),$$
$$S^\alpha = \left(\frac{\omega}{A\sqrt{1-\omega^2}}, 0, 0, \frac{1}{C\sqrt{1-\omega^2}}\right), \quad (11)$$
$$l^\alpha = \left(\frac{\omega+1}{A\sqrt{1-\omega^2}}, 0, 0, \frac{\omega+1}{C\sqrt{1-\omega^2}}\right).$$

The energy–momentum tensor coincides with that of an imperfect matter distribution when we deal with the tilted congruence. We assume the matter content in our systematic analysis to be locally anisotropic in pressure dissipating in both streaming out and diffusion approximation in the presence of an electromagnetic field. Such a fluid distribution is represented by the second-rank stress–energy tensor

$$T_{\alpha\beta} = (P_\perp + \rho)U_\alpha U_\beta - (P_\perp - P_z)S_\alpha S_\beta + \epsilon l_\alpha l_\beta + \\ + q_\alpha U_\beta + P_\perp g_{\alpha\beta} + q_\beta U_\alpha + \\ + \frac{1}{4\pi}\left(F^\gamma_\alpha F_{\beta\gamma} - \frac{1}{4}F^{\gamma\delta}F_{\gamma\delta}g_{\alpha\beta}\right), \quad (12)$$

where ρ, P_z, $P_\perp$, q^α, and ϵ are respectively the energy density, pressure anisotropy, heat flux, and radiation density. The heat flux $q^\alpha = qS^\alpha$ satisfies the equation $U_\alpha q^\alpha = 0$, while

$$U^\alpha U_\alpha = -S^\alpha l_\alpha = -1 = -S^\alpha S_\alpha = U^\alpha l_\alpha, \quad S^\alpha U_\alpha = 0.$$

In comoving coordinates, the four-current in the tilted frame takes the form $J^\alpha = \xi U^\alpha$. With this used in Eq. (3), the Maxwell field equations lead to the independent components

$$\frac{\partial^2\phi}{\partial z^2} - \left(\frac{C'}{C} + \frac{A'}{A} - \frac{2B'}{B}\right)\frac{\partial\phi}{\partial z} = \frac{\xi\mu_0 C^2 A}{\sqrt{1-\omega^2}}, \quad (13)$$

$$\frac{\partial^2\phi}{\partial t\partial z} - \left(\frac{\dot{C}}{C} + \frac{\dot{A}}{A} - \frac{2\dot{B}}{B}\right)\frac{\partial\phi}{\partial z} = -\frac{\xi\mu_0 C A^2\omega}{\sqrt{1-\omega^2}}. \quad (14)$$

Integrating Eq. (13) with respect to z leads to

$$\phi' = \frac{\tilde{s}(z)\mu_0 CA}{B^2}, \\ \text{where} \quad \tilde{s}(z) = \int\limits_0^z \frac{\xi CB^2}{\sqrt{1-\omega^2}}\,dz, \quad (15)$$

which identically satisfies Eq. (14). For the tilted congruence, the Einstein–Maxwell field equations yield

$$\frac{8\pi A^2}{1-\omega^2}\left(\tilde{\rho} + \omega^2\tilde{P}_z + 2\omega\tilde{q}\right) + \frac{\tilde{s}^2\mu_0^2 A^2}{B^4} = \\ = \left(\frac{2\dot{C}}{C} + \frac{\dot{B}}{B}\right)\frac{\dot{B}}{B} - \left(\frac{A}{C}\right)^2 \times \\ \times \left[\frac{2B''}{B} - \left(\frac{2C'}{C} - \frac{B'}{B}\right)\frac{B'}{B}\right], \quad (16)$$

$$\frac{4\pi CA}{1-\omega^2}\left\{\omega(\tilde{\rho} + \tilde{P}_z) + (1+\omega^2)\tilde{q}\right\} = \\ = \frac{\dot{B}'}{B} - \frac{\dot{B}A'}{BA} - \frac{\dot{C}B'}{CB}, \quad (17)$$

$$\frac{8\pi C^2}{1-\omega^2}\left(\omega^2\tilde{\rho} + \tilde{P}_z + 2\omega\tilde{q}\right) - \frac{\tilde{s}^2\mu_0^2 C^2}{B^4} = \\ = -\left(\frac{C}{A}\right)^2\left[\left(\frac{\dot{B}}{B}\right)^2 + \frac{2\ddot{B}}{B} - \frac{2\dot{B}\dot{A}}{BA}\right] + \\ + \left(\frac{B'}{B}\right)^2 + \frac{2B'A'}{BA}, \quad (18)$$

$$8\pi B^2 P_\perp + \frac{\tilde{s}^2\mu_0^2}{B^2} = \left(\frac{B}{C}\right)^2 \times \\ \times \left[\frac{B''}{B} - \frac{A'}{A}\left(\frac{C'}{C} - \frac{B'}{B}\right) + \frac{A''}{A} - \frac{B'C'}{BC}\right] - \\ - \left[\frac{\ddot{C}}{C} + \frac{\ddot{B}}{B} - \frac{\dot{A}}{A}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{\dot{B}\dot{C}}{BC}\right]\left(\frac{B}{A}\right)^2, \quad (19)$$

where $\tilde{\rho} = \epsilon + \rho$, $\tilde{P}_z = \epsilon + P_z$, and $\tilde{q} = \epsilon + q$. By comparing Eqs. (7)–(10) and (16)–(19), we have some relations between the physical variables of nontilted and tilted frames:

$$\left(\tilde{\rho} + \omega^2\tilde{P}_z + 2\omega\tilde{q}\right) + \frac{\mu_0^2}{8\pi B^4}(\tilde{s}^2 - s^2)(1-\omega^2) = \\ = \bar{\rho}(1-\omega^2),$$

$$\omega(\tilde{\rho} + \tilde{P}_z) + (1+\omega^2)\tilde{q} = 0, \quad 8\pi B^2 P_\perp + \frac{\mu_0^2}{B^2}(\tilde{s}^2 - s^2) = 0,$$

$$\frac{1}{1-\omega^2}\left(\omega^2\tilde{\mu} + \tilde{P}_z + 2\omega\tilde{q}\right) = \frac{\mu_0^2}{8\pi B^4}(\tilde{s}^2 - s^2).$$

3. STRUCTURE SCALARS AND DYNAMICAL EQUATIONS

In this section, we investigate some scalars associated with the kinematical quantities, like acceleration and expansion scalars in a tilted frame. We also explore the conservation of the energy–momentum tensor and express it in terms of these kinematical quantities. The scalar associated with the Weyl tensor is known as the Weyl scalar, while the scalars associated with the Riemann tensor are known as structure scalars [18]. The Weyl scalar is important in describing the inhomogeneities in the universe due to the tidal wave nature, while structure scalars are used to discuss the structure and evolution of the universe. Moreover, all solutions of the field equation can be written in terms of these scalar functions for the static case. Using the Taub mass function for plane symmetry [19] with structure scalars and the Weyl scalar, two evolution equations have been formulated that play a key role in investigating the inhomogeneity in the matter distribution. In the tilted frame, the acceleration and expansion scalars take the form [8]

$$a = \frac{A^2}{(1-\omega^2)^{3/2}}\left(\dot{\omega} + \frac{A\omega\omega'}{B} + \frac{\omega\dot{B}}{B}(1-\omega^2) + \right. \\ \left. + \frac{\omega^2 A'}{B} - \frac{\omega^2 AB'}{B^2}\right), \qquad (20)$$

$$\Theta = \frac{1}{\sqrt{1-\omega^2}}\left[\frac{2\dot{B}}{B} + \frac{\dot{C}}{C} + \frac{\dot{A}}{A} + \frac{\omega\dot{\omega}}{1-\omega^2} + \right. \\ \left. + \frac{\omega' A}{B(1-\omega^2)} + \frac{\omega A'}{B} - \frac{\omega AB'}{B^2}\right]. \qquad (21)$$

The Weyl tensor can be decomposed into electric and magnetic parts as

$$H_{\alpha\beta} = \frac{1}{2}\epsilon_{\alpha\mu\nu\gamma}C^{\nu\gamma}{}_{\beta\rho}U^\mu U^\rho, \quad E_{\alpha\beta} = C_{\alpha\mu\beta\nu}U^\mu U^\nu,$$

where $\epsilon_{\alpha\mu\nu\gamma} = \sqrt{-g}\eta_{\alpha\mu\nu\gamma}$, and $\eta_{\alpha\mu\nu\gamma}$ is the Levi-Civita tensor. Its magnetic part turns out to be zero due to planar symmetry, while the electric part has the independent components

$$E_{00} = \frac{\omega^2}{3(1-\omega^2)}\mathcal{E}, \quad E_{01} = \frac{A^3 B\omega}{3(1-\omega^2)}\mathcal{E},$$

$$E_{11} = \frac{\omega^2 A^4}{3(1-\omega^2)}\mathcal{E},$$

$$E_{22} = \frac{A^2 B^2}{3(1-\omega^2)}(1+2\omega^2 A^2)\mathcal{E},$$

$$E_{33} = \frac{A^2 C^2}{3(1-\omega^2)}(\omega^2 - 2)\mathcal{E},$$

where

$$\mathcal{E} = \frac{1}{2A^2}\left[\frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\left(\frac{\dot{C}}{C} - \frac{\dot{B}}{B}\right) - \frac{\ddot{B}}{B} + \right. \\ \left. + \frac{\dot{A}}{A}\left(\frac{\dot{C}}{C} - \frac{\dot{B}}{B}\right)\right] + \frac{1}{2C^2} \times \\ \times \left[\frac{B''}{B} - \frac{A''}{A} + \frac{B'}{B}\left(\frac{C'}{C} + \frac{B'}{B}\right) - \frac{A'}{A}\left(\frac{C'}{C} + \frac{B'}{B}\right)\right].$$

Similarly, the Riemann tensor can be decomposed into its electric and magnetic parts. Here, we take the electric part and the second dual of the magnetic part of the Riemann tensor and decompose them into trace and trace-free scalar parts as [20]

$$Y_{\alpha\beta} = \frac{1}{3}Y_T h_{\alpha\beta} + Y_{TF}\left(S_\alpha S_\beta - \frac{1}{3}h_{\alpha\beta}\right), \qquad (22)$$

$$X_{\alpha\beta} = \frac{1}{3}X_T h_{\alpha\beta} + X_{TF}\left(S_\alpha S_\beta - \frac{1}{3}h_{\alpha\beta}\right), \\ \text{with} \quad h_{\alpha\beta} = g_{\alpha\beta} + U_\alpha U_\beta. \qquad (23)$$

Using the field equations together with unit four-vectors of plane symmetry in the tilted frame, we obtain

$$Y_T = 4\pi(\tilde{\rho} + 3\hat{P}) + \frac{\mu_0^2 \tilde{s}^2}{B^4}, \\ Y_{TF} = \mathcal{E} - 4\pi\Pi + \frac{\mu_0^2 \tilde{s}^2}{B^4}, \qquad (24)$$

$$X_T = 8\pi\tilde{\rho} + \frac{\mu_0^2 \tilde{s}^2}{B^4}, \quad X_{TF} = -\mathcal{E} - 4\pi\Pi + \frac{\mu_0^2 \tilde{s}^2}{B^4}, \qquad (25)$$

with $\hat{P} = (2P_\perp + \tilde{P}_z)/3$ and $\Pi = \tilde{P}_z - P_\perp$.

The conservation of the stress–energy tensor leads to the equations

$$\tilde{\rho}^* + \tilde{q}^\dagger + \Theta\tilde{\rho} + \left[\sqrt{1-\omega^2}\frac{A'}{B} + \Theta\omega + \frac{2\dot{\omega}}{\sqrt{1-\omega^2}} - \right. \\ - \frac{AB'}{B^2\sqrt{1-\omega^2}} + \frac{\dot{A}\sqrt{1-\omega^2}}{A}(2\omega - 1) + \\ \left. + \frac{\omega^3\omega' A}{B\sqrt{1-\omega^2}} - \omega\left(\frac{2\dot{B}}{B} + \frac{\dot{C}}{C}\right)\left(\frac{1+\omega^2}{1-\omega^2}\right)\right]\tilde{q} = 0, \qquad (26)$$

$$\tilde{q}^* + \tilde{P_z}^\dagger + (\tilde{P}_z + \tilde{\rho})\frac{a}{A^2} + \frac{\tilde{q}}{\sqrt{1-\omega^2}} \times \\ \times \left\{\omega\dot{\omega} + \frac{\omega\dot{\omega}}{1-\omega^2} + \frac{A\omega'}{B}\right\} - \frac{\tilde{s}\tilde{s}'\mu_0^2}{4\pi B^4 C^2} = 0, \qquad (27)$$

where $w^* = w_{,\alpha}U^\alpha$ and $w^\dagger = w_{,\alpha}S^\alpha$. The corresponding Taub mass function is [19]

$$m(t,z) = \frac{B}{2}\left(\frac{\dot{B}^2}{A^2} - \frac{B'^2}{C^2}\right) + \frac{\tilde{s}^2}{2B}. \tag{28}$$

Differentiating with respect to t and z leads to

$$\dot{m} = -\frac{4\pi B^2}{1-\omega^2}\left[\frac{AB'}{C}\left\{\omega(\tilde{P}_z + \tilde{\rho}) + (\omega^2+1)\tilde{q}\right\} + \right. \\ \left. + \dot{B}\left\{\omega^2\tilde{\rho} + 2\omega\tilde{q} + \tilde{P}_z\right\}\right], \tag{29}$$

$$m' = \frac{4\pi B^2}{1-\omega^2}\left[\frac{C\dot{B}}{A}\left\{\omega(\tilde{P}_z + \tilde{\rho}) + (\omega^2+1)\tilde{q}\right\} + \right. \\ \left. + B'\left\{\tilde{\rho} + 2\omega\tilde{q} + \omega^2\tilde{P}_z\right\}\right] + \frac{\mu_0^2\tilde{s}\tilde{s}'}{B}. \tag{30}$$

Now, we explore two evolution equations for the Weyl tensor, which are important in identifying the inhomogeneities in the matter distribution. Using the procedure in [8], these equations are obtained from Eqs. (16)–(18) and (28)–(30) and written in terms of X_{TF} as

$$12\pi\tilde{q}\left(A^2(\ln B)^\dagger\omega + \frac{\dot{B}}{B}\sqrt{1-\omega^2}\right) = \\ = 3(\ln B)^\dagger X_{TF} + (X_{TF} + 4\pi A^2\tilde{\rho})^\dagger - \\ - \frac{2\mu_0^2\tilde{s}^2}{B^4}(\ln B)^\dagger + \frac{\mu_0^2\tilde{s}\tilde{s}'}{CB^4}\frac{1}{\sqrt{1-\omega^2}}, \tag{31}$$

$$4\pi\tilde{q}\left(\frac{2\dot{\omega}}{\sqrt{1-\omega^2}} + \omega\Theta - \frac{B'}{BC}\sqrt{1-\omega^2}\right) + 4\pi\tilde{\rho}\Theta = \\ = 3(\ln B)^* X_{TF} + X_{TF}^* - 4\pi A^2\tilde{q}^\dagger + \\ + \frac{2\mu_0^2\tilde{s}^2}{B^4}(\ln B)^* - \frac{\mu_0^2\tilde{s}\tilde{s}'}{CB^4}\frac{1}{\sqrt{1-\omega^2}}. \tag{32}$$

It can be easily seen that these equations depend on the congruence of a tilted observer and have a contribution of the electromagnetic field as well.

3.1. Inhomogeneity factor

In this section, we explore the factor describing the inhomogeneity caused by different factors of the fluid configuration and called the inhomogeneity factor. The concept of the inhomogeneity factor was introduced in [8] in discussing the inhomogeneity of a tilted Lemaître–Tolman–Bondi metric and examining its thermodynamical and hydrodynamical properties. Since then, many researchers have investigated the inhomogeneity factors with different physical backgrounds of cylindrical, spherical, and planar geometries. Here, we construct the inhomogeneity factor of planar geometry with charged anisotropic dissipative matter in the framework of a tilted congruence. When the fluid is regular everywhere and $\tilde{\rho}^\dagger = 0 = \tilde{s}$, the inhomogeneity factor in Eq. (31) becomes

$$\chi = X_{TF} - \frac{1}{B^3}\int_0^k FB^3\,dk \tag{33}$$

with

$$F = 12\pi\tilde{q}\left(\frac{\dot{B}}{B}\sqrt{1-\omega^2} + \omega A^2(\ln B)^\dagger\right),$$

where k is an integration parameter on the curves of the congruence defined by S^α. It can be easily seen that the inhomogeneity factor is affected by dissipating quantities and the congruence of the observer in the absence of an electromagnetic field. Now, evaluating X_{TF} from Eq. (32) and then substituting it in (33), we ontain the evolution of the inhomogeneity factor as

$$\chi = \frac{1}{B^3}\int_0^u\left(\frac{\mu_0^2\tilde{s}\tilde{s}'}{BC}\frac{\omega}{\sqrt{1-\omega^2}} - \frac{2\mu_0^2\tilde{s}^2}{B}(\ln B)^*\right)du - \\ - \frac{1}{B^3}\int_0^k FB^3dk + \frac{1}{B^3}\int_0^u\left[4\pi A^2\tilde{q}\left(\frac{2\dot{\omega}}{\sqrt{1-\omega^2}} + \right.\right. \\ \left.\left. + \Theta\omega - \frac{B'}{BC}\sqrt{1-\omega^2}\right) + 4\Theta\pi\tilde{\rho} + 4\pi A^2\tilde{q}^\dagger\right]R^3du, \tag{34}$$

where the curves on the congruence U^α are represented by the parameter u.

3.2. Transport equation and stability of the nontilted frame

A transport equation is required to examine the behavior of fluid variables and to analyze the transportation of mass and heat. For a dissipative matter configuration, it is defined by the second-order partial differential equation given by the Müller–Israel–Stewart theory [21] as

$$\tau h^{\alpha\beta}U^\gamma q_{\beta;\gamma} = -Kh^{\alpha\beta}(Ta_\beta + T_{,\beta}) - q^\alpha - \\ - \frac{1}{2}\left(\frac{\tau U^\beta}{KT^2}\right)_{;\beta} q^\alpha KT^2. \tag{35}$$

Here T, K, and τ respectively represent the temperature, thermal conductivity and relaxation time, where the time required for a disturbed system to return to equilibrium is known as the relaxation time. For the nontilted configuration, we choose the fluid distribution to be charged dust. The only nonvanishing component of transport equation (35) becomes

$$\tau\left(\dot{q}+\frac{A\omega}{B}q'\right)+q(1-\omega^2)^{1/2}+\tau q\times$$
$$\times\left[(1-A^2)\left(\frac{\omega' A}{(1-\omega^2)B}+\frac{\dot{\omega}}{\omega(1-\omega^2)}+\frac{\dot{B}}{B}\right)+\right.$$
$$\left.+\frac{\omega A^3 B'}{B^2(1-\omega^2)}+\frac{\dot{A}}{A}\right]=$$
$$=-K\left[\frac{1-\omega^2}{\omega}\left\{\left(\frac{1}{1-\omega^2}-\frac{1}{A^2}\right)\dot{T}+\frac{A}{B}T'\right\}+\right.$$
$$+T\omega\left(\frac{\omega' A}{B(1-\omega^2)}+\frac{\dot{B}}{B}+\frac{\dot{\omega}}{\omega(1-\omega^2)}-\frac{\omega A^3 B'}{B^2(1-\omega^2)}+\right.$$
$$\left.\left.+\frac{\dot{A}A}{1-\omega^2}+\frac{\omega A^2 A'}{B(1-\omega^2)}\right)\right]-$$
$$-\frac{1}{2}KT^2 q\left[\left(\frac{\dot{\tau}}{KT^2}\right)+\frac{\omega A}{B}\left(\frac{\tau}{KT^2}\right)'\right]-\frac{1}{2}\tau q\Theta(1-\omega^2)^{1/2}+$$
$$+\frac{1}{2}\tau q\left(\frac{\omega' A}{B(1-\omega^2)}-\frac{\omega A B'}{B^2}-\frac{\omega\dot{A}}{B}+\right.$$
$$\left.+\frac{\omega A'}{B}-\frac{\dot{\omega}A}{B(1-\omega^2)}\right). \quad (36)$$

We consider the perturbation time scale to be much smaller than the relaxation time and the hydrostatic time scale at $t=0$ with the pure diffusion approximation (i.e., $\epsilon=0$ for the sake of convenience) while retaining the plane symmetry. Thus, we have

$$\omega=0=q, \quad \dot{q}\approx\dot{\omega}\neq 0.$$

With the above conditions used in Eq. (36), it follows that

$$\tau\dot{q}=-TK\dot{\omega}, \quad (37)$$

and similarly Eq. (27) yields

$$\bar{\rho}\dot{\omega}+\dot{q}-\frac{ss'\mu_0^2}{4\pi CB^4}=0. \quad (38)$$

Using $\dot{q}$ from Eq. (37) in (38), we find

$$\bar{\rho}\dot{\omega}\left(1-\frac{KT}{\bar{\rho}\tau}\right)-\frac{ss'\mu_0^2}{4\pi CB^4}=0. \quad (39)$$

We see that this equation has an extra factor, $KT/\bar{\rho}\tau=\lambda$, which results in the loss of stability and causes contraction due to the electric charge. It is found that the role of the electromagnetic field in the dynamics of a nontilted planar object is the same as obtained in cylindrical and spherical systems. If the system contracts in such a way that the term $1-\lambda$ vanishes, for example, by an increment in thermal conductivity or/and temperature, then a bouncing would occur from a collapsing configuration. A systematic analysis of relaxation effects on the dynamics of collapsing spherical stars has been done in [22], strictly within the diffusion approximation. The specific effect of bouncing by assuming an increment in the factor produced by the inertial term of the transport equation has also been discussed numerically [23]. It was shown in [24] that the same factor affects the inertial mass and the gravitational force term of the coupled dynamical transport equation. We have presented the coupled dynamical transport equation for cylindrically symmetric anisotropic fluids and found a condition on the charge for which the gravitational mass increases and causes rapid collapse [25]. In that case, forces opposing the contraction (e.g., pressure gradients) may overcome gravitation. We see that if such a decrease is significant enough to reverse the balance of forces, then a bounce back from contraction might occur due to $1-\lambda<1$. When $1-\lambda>1$, the gravitational term becomes positive (repulsive), which implies a repulsive force producing further contraction.

4. SUMMARY

This paper explores the effects of anisotropic pressure, heat dissipation, and electric charge on the dynamical properties of a plane matter distribution relative to the motion of a tilted observer. In this context, a Lorentz boost is applied to a nontilted congruence to obtain a radially moving observer having a radial velocity ω. We have constructed Einstein–Maxwell field equations for both congruences and found relations between the physical variables of both frames. The kinematical quantities, namely, acceleration and expansion scalars, are explored and used to describe the conservation of the energy–momentum tensor. We have developed some dynamical quantities called structure scalars, which have physical significance in describing the evolution of self-gravitating collapsing stars. One of them (X_{TF}, the trace-free part of the second dual of the magnetic part of the Riemann tensor) is identified as the inhomogeneity factor for an anisotropic matter field in the presence/absence of a magnetic field [17, 26].

Here, we have found that the energy density inhomogeneity depends on the value of χ with the congruence of the tilted observer. We see that the charge tends to increase the inhomogeneity in fluid configurations impelled by other matter variables. We have also investigated that in nontilted planar geometry, the electromagnetic field disturbs the stability of a charged dust cloud and decreases the collapse rate, which is well consistent with [16]. It is interesting to mention here that all our solutions reduce to the charge-free case when we set $s = 0 = \tilde{s}$.

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