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DRAG OF BALLISTIC ELECTRONS BY AN ION BEAM

V. L. Gurevich, M. I. Muradov *

Ioffe Institute, Russian Academy of Sciences
194021, Saint Petersburg, Russia

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Drag of electrons of a one-dimensional ballistic nanowire by a nearby one-dimensional beam of ions is considered. We assume that the ion beam is represented by an ensemble of heavy ions of the same velocity $\mathbf{V}$. The ratio of the drag current to the primary current carried by the ion beam is calculated. The drag current turns out to be a nonmonotonic function of velocity V. It has a sharp maximum for V near $v_{nF}/2$, where n is the number of the uppermost electron miniband (channel) taking part in conductance and v_{nF} is the corresponding Fermi velocity. This means that the phenomenon of ion beam drag can be used for investigation of the electron spectra of ballistic nanostructures. We note that whereas observation of the Coulomb drag between two parallel quantum wires may be in general complicated by phenomena such as tunneling and phonon drag, the Coulomb drag of electrons of a one-dimensional ballistic nanowire by an ion beam is free from such spurious effects.

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1. FORMULATION OF THE PROBLEM

Drag as a physical phenomenon in solids can be described as follows. We consider a solid with two types of quasiparticles (type 1 and type 2) and create a flux of the quasiparticles of type 2, the so-called active, or driving current. As a result of the interaction between particles, a current of quasiparticles of type 1, the so-called passive, or drag current is excited. An example of this phenomenon is the Coulomb drag, where due to Coulomb interaction between the electrons, a current in a conductor creates a current in an adjacent conductor. This phenomenon was predicted in seminal papers by Pogrebinskii [1] and Price [2].

In this paper, we consider a physically entirely different situation where the driving current is created by real heavy particles outside the conductor (rather than by Fermi degenerate quasiparticles within another conductor).

Two formulations of the problem are feasible.

1. The dragging flux consists of heavy ions of almost the same velocity $\mathbf{V}$.

2. A flux of weakly ionized gas is in thermal equilibrium, having some temperature T and hydrodynamical velocity $\mathbf{V}$ (cf. with Ref. [3]).

In this paper, we treat the first possibility. In other words, we consider an ion beam, i. e., a flux of ions having the same velocity $\mathbf{V}$. For the simplest situation, the value of velocity $\mathbf{V}$ is determined by the accelerating voltage $\mathfrak{V}$ and the ion mass M as

$$\frac{MV^2}{2} = e_I \mathfrak{V}, \quad (1)$$

where e_I is the charge of an ion.

It is interesting to compare in advance the situation we discuss in this paper with the drag in the case where both conductors are one-dimensional (1D) structures with the electrons performing ballistic (collisionless) motion. Such nanoscale systems may have rather low electron densities, which can be varied by means of the gate voltage. The e–e (electron–electron) interaction can be treated as e–e collisions between the electrons belonging to the drive (active) and drag (passive) wires (see Refs. [4–11]).

Experimentally, in our opinion, the situation with two 1D quantum wires cannot be considered settled. Two 1D quantum wires interacting via a Coulomb potential are usually created in solids artificially (e. g., by split gates), and therefore special care should be taken in order that there be no tunneling between the wires, because tunneling can hamper observation of the drag. On the other hand, a change in the split gate voltage may result not only in the shift of the chemical potentials of individual nanowires but also in variation of the barrier width (or a spatial distance) between the

*E-mail: mag.muradov@mail.ioffe.ru

wires. In some cases, even a change in the Coulomb drag current direction is observed [9]. Furthermore, a phonon-mediated contribution [12, 13] to the drag is in general inevitable for two nanowires formed by split gates.

It is usually assumed that the electrons of the quantum wires are degenerate and the temperature is low compared to the electron Fermi energy. The collisionless quantum wires act as waveguides for the electron de Broglie waves. For a strong Fermi degeneracy

$$T \ll \mu, \tag{2}$$

where μ is the Fermi energy (we use the energy units for the temperature T assuming $k_B \equiv 1$), each miniband of transverse quantization (channel) makes the contribution to the conductance given by [14]

$$G_0 = \frac{e^2}{\pi\hbar} \tag{3}$$

(e being the electron charge), and hence the total conductance of a quantum wire is

$$G = \mathcal{N} G_0,$$

where $\mathcal{N}$ is the number of active channels, i. e., minibands with bottoms $\epsilon_n(0)$ below the Fermi level μ. It is assumed that each quantum wire is connected to ideal electronic reservoirs attached to its ends. The relaxation processes in the reservoirs are considered to be so fast that each of them is in thermal equilibrium. The e–e interaction within a single quantum wire does not result in a current variation because of the quasimomentum conservation in e–e collisions in a semiconductor. However, if two such wires, 1 and 2, are near one another and are parallel, the Coulomb interaction of electrons belonging to different wires can transfer quasimomentum between the wires, which eventually gives rise to a drag effect. The drag force due to the ballistic current in wire 2 creates a sort of permanent acceleration on the electrons of wire 1. As wire 1 has a finite length L, a steady drag current J_d is established. Within the Fermi-liquid approach, we should restrict ourselves to direct e–e collisions mediated by the Coulomb interaction.

For such e–e collision to be possible, the absolute values of the four electron energies should be within the stripes of width approximately equal to T near the corresponding Fermi levels, μ_d and μ_a. This means that the relation

$$|\mu_a - \mu_d| \lesssim T \tag{4}$$

should hold. In other words, because of the conservation of the electron energy and quasimomentum in combination with the Fermi degeneracy, the drag current exists only if the Fermi levels of the electrons of both wires coincide within the accuracy of thermal broadening. A 1D quantum wire can have several minibands of transverse quantization (channels), and there is a Fermi level associated with each such channel. The coincidence of any pair of Fermi levels of the active (drive) and drag wires should result in a sharp spike of the drag current [4].

The primary aim of this paper is to consider the situation where many (or some) of the above-mentioned experimental difficulties do not arise and the picture is as clear as possible, such that the Coulomb drag could be investigated exactly, retaining the principal features of the 1D drag situation as closely as possible. As regards the drag by an ion beam, quite unlike the situation with two Fermi-degenerate conductors, the velocity of ions V can be varied in experiment. As we see in what follows, this possibility provides a tool to investigate the electron spectrum of a ballistic 1D conductor. We see below that varying the velocity V allows observing a maximum of the drag current $J_d(V)$. The position of the maximum corresponds to the condition

$$V = v_F/2, \tag{5}$$

where v_F is the Fermi velocity corresponding not to any miniband (as in the case of two quantum wires outlined above) but to the uppermost miniband taking part in conduction of the drag current. The point is in a different physics behind these two types of oscillatory behavior.

Our purpose is to investigate the main features of this drag phenomenon. We assume that the distance d between the ion beam and the wire is much larger than the width of the wire, and hence the Coulomb interaction of ions and electrons is a smooth function on the scale of this width. Then the selection rules for the corresponding matrix elements require that the electrons involved in the transitions change their quasimomenta and remain in the first approximation within the initial transverse quantized channel n. We can vary the velocity $\mathbf{V}$ of the ions with the accelerating voltage $\mathfrak{V}$ and measure the resulting variation of the drag current (or drag voltage). We let $\mathcal{V}_\mathbf{r}$ denote the volume occupied by the nanowire and $\mathcal{V}_\mathbf{R}$ denote the volume where the flux of ions propagates and interacts with the electrons of the nanowire. We assume both $\mathcal{V}_\mathbf{r}$ and $\mathcal{V}_\mathbf{R}$ to have a 1D shape of length L parallel to the z axis.

For the treatment of our problem, we use the Boltzmann equation for the one-particle electron distribution function. As is well known, due to e–e interaction, a single-channel state may be unstable, which for $\mathcal{N}=1$ gives rise to the so-called Tomonaga–Luttinger liquid [15, 16] for the drag wire. This means that the results of this paper are valid for $\mathcal{N}>1$; the case $\mathcal{N}=1$ may be not covered by the theory we work out below.

We can give the following qualitative considerations concerning the drag by an ion beam. Due to the conservation of quantities such as the energy, the transverse quantized channel number n, and the (quasi)momentum in electron–ion collisions, we have to consider in the Born approximation the transition of an electron from an $|n,p\rangle$ to an $|n,p+q_z\rangle$ state (where p is the z-component of the electron quasimomentum) and that of the ion from a $|\mathbf{P}\rangle$ to a $|\mathbf{P}-\mathbf{q}\rangle$ state according to the relation

$$\frac{p^2}{2m}+\frac{\mathbf{P}^2}{2M}=\frac{(p+q_z)^2}{2m}+\frac{(\mathbf{P}-\mathbf{q})^2}{2M}, \tag{6}$$

where m is the effective mass of the conduction electron and M is the mass of an ion. The δ-function describing the energy conservation can therefore be written as

$$\delta\left[\frac{q_z^2}{2m}\left(1+\frac{m}{M}\right)+\frac{q_z}{m}(p-mV)+\frac{q_\perp^2}{2M}\right]\approx \approx\frac{2m}{|q_z|}\delta\left[q_z-2(mV-p)\right], \tag{7}$$

where $P_z\equiv P=MV$. In what follows, we take into account that $m/M\ll 1$ and neglect m/M compared to unity and $(m/M)q_\perp^2$ compared to q_z^2. Therefore, the transferred (quasi)momentum is $q_z=2(mV-p)$ and the probability of such a transition includes the factor

$$f_n(p)[1-f_n(p+q_z)]-f_n(p+q_z)[1-f_n(p)]= =f_n(p)-f_n(2mV-p) \tag{8}$$

as well as the electron–ion Coulomb interaction matrix element squared. For the 1D situation under consideration, it has a factor proportional to

$$K_0^2(|q_z|d/\hbar)\big|_{q_z=2(mV-p)}, \tag{9}$$

where d is the distance between the ion beam and the wire and K_0 is the McDonald function (see below Eq. (19)). For it, we can use the approximate equations

$$K_0(s)\approx\ln\frac{2}{\gamma s},\quad s\ll 1, \tag{10}$$

$$K_0(s)\approx\sqrt{\frac{\pi}{2s}}e^{-s},\quad s\gg 1, \tag{11}$$

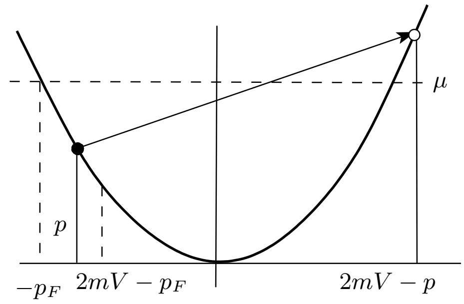

Fig. 1. Momenta from $-p_F$ to $2mV-p_F$ are involved in transitions. For $V>v_F/2$, all negative momenta p contribute to the drag current

where $\ln\gamma=0.577$. The drag current is proportional to the sum over electron quasimomenta p of the products in Eqs. (7)–(9). We consider $p<0$ and require the state p to be occupied, which leads to $-p_F<p<0$. The requirement that the final state with the momentum $2mV-p$ is empty gives $2mV-p>p_F$ if $V<v_F/2$ (Fig. 1). If $V>v_F/2$, there is no additional restriction except $-p_F<p<0$, i. e., all occupied states are involved in transitions. Therefore, if $V<v_F/2$, we obtain for the drag current

$$J_d\propto\int\limits_{-p_F}^{-p_F+2mV}dp\frac{K_0^2[2(mV-p)d/\hbar]}{mV-p}= =\int\limits_{p_F-mV}^{p_F+mV}dp\frac{K_0^2(2pd/\hbar)}{p}, \tag{12}$$

and we see that an increase in V decreases the minimal transferred momentum and increases the effective Coulomb interaction $K_0(2pd/\hbar)$. If $V>v_F/2$, we have

$$J_d\propto\int\limits_{-p_F}^{0}dp\frac{K_0^2\left[2(mV-p)d/\hbar\right]}{mV-p}= =\int\limits_{mV}^{p_F+mV}dp\frac{K_0^2(2pd/\hbar)}{p}, \tag{13}$$

and an increase in V results in a decrease in the drag current. These equations provide an adequate description of the drag current dependence on the ion beam velocity, as can be readily seen in our quantitative approach below.

One more comment concerning conservation law (6) is called for. The point is that p is the z-component of the electron quasimomentum rather than the true momentum. This means that it is conserved within the accuracy of $\hbar$ times the additional vector of the reciprocal lattice. It can be verified, however, that in the case of a simple electron spectrum (one minimum in the center of the Brillouin zone), the conservation law is given by Eq. (6).

2. INTERACTION OF AN ION BEAM WITH ELECTRONS OF A NANOSTRUCTURE

For simplicity, we assume the width of the beam to be constant (actually, it may slightly vary in the course of beam propagation). Then we can write the distribution of the ions within the beam as

$$F_{\mathbf{P}} = N(2\pi\hbar)^3\delta(P_x)\delta(P_y)\delta(P_z - P), \qquad (14)$$

where N is the ion concentration.

The collision term of the Boltzmann equation for 1D electrons and 3D ions in the Born approximation is given by

$$\left(\frac{\partial f_{np}}{\partial t}\right)_{coll} \equiv I\{f,F\} = \int \frac{\mathcal{V}_{\mathbf{R}} d^3P}{(2\pi\hbar)^3} \int \frac{\mathcal{V}_{\mathbf{R}} d^3q}{(2\pi\hbar)^3} \times \\ \times \frac{2\pi}{\hbar} \left|\langle p, n, \mathbf{P}|U|p+q_z, n, \mathbf{P}-\mathbf{q}\rangle\right|^2 \times \\ \times \delta(\epsilon_{np} + E_{\mathbf{P}} - \epsilon_{n,p+q_z} - E_{\mathbf{P}-\mathbf{q}}) \times \\ \times \left[f_{np}(1-f_{n,p+q_z})F_{\mathbf{P}} - f_{n,p+q_z}(1-f_{np})F_{\mathbf{P}-\mathbf{q}}\right], \qquad (15)$$

where

$$\epsilon_n(p) = \epsilon_n(0) + p^2/2m. \qquad (16)$$

Here, n is the number of the channel, i. e., of the miniband of 1D transverse quantization (according to the assumption made above, this number does not change in the course of electron transitions), $\mathbf{q}$ is the transferred (quasi)momentum, and

$$U = \frac{2}{1+\kappa}\frac{ee_I}{|\mathbf{R}-\mathbf{r}|} \qquad (17)$$

describes the Coulomb interaction of an ion with a charge e_I and an electron in the wire, with κ being the dielectric constant of the wire. For the matrix element in Eq. (15), we have

$$\langle p, n, \mathbf{P}|U|p+q_z, n, \mathbf{P}-\mathbf{q}\rangle = \int\limits_{\mathcal{V}_{\mathbf{r}}} d^3r \int\limits_{\mathcal{V}_{\mathbf{R}}} d^3R\, \psi_n^*(\mathbf{r}_\perp) \times \\ \times \Psi_{\mathbf{P}}^* \frac{2ee_I}{L(1+\kappa)|\mathbf{r}-\mathbf{R}|} \psi_n(\mathbf{r}_\perp)\Psi_{\mathbf{P}-\mathbf{q}} \exp\left(\frac{iq_z z}{\hbar}\right). \qquad (18)$$

Because

$$\int \frac{dZ dz}{L|\mathbf{r}-\mathbf{R}|} \exp\left[\frac{iq_z(z-Z)}{\hbar}\right] = \\ = 2K_0\left(\frac{|q_z||\Delta\mathbf{r}_\perp|}{\hbar}\right), \qquad (19)$$

where

$$|\Delta\mathbf{r}_\perp| \equiv \sqrt{(x-X)^2 + (y-Y)^2},$$

we can write

$$\langle p, n, \mathbf{P}|U|p+q_z, n, \mathbf{P}-\mathbf{q}\rangle = \\ = \frac{4ee_I}{(1+\kappa)\mathcal{V}_{\mathbf{R}}} \int d\mathbf{R}_\perp \int d\mathbf{r}_\perp |\psi_n(\mathbf{r}_\perp)|^2 \times \\ \times \exp\left(-\frac{i\mathbf{q}_\perp \mathbf{R}_\perp}{\hbar}\right) K_0(|q_z||\Delta\mathbf{r}_\perp|/\hbar). \qquad (20)$$

The Boltzmann equation for electrons is

$$v\frac{\partial f_{np}}{\partial z} = -\left(\frac{\partial f_{np}}{\partial t}\right)_{coll}, \qquad (21)$$

where

$$v = \frac{d\epsilon_{np}}{dp} = \frac{p}{m} \qquad (22)$$

is the electron velocity.

To calculate the current in the wire, we iterate the Boltzmann equation for the electrons of the wire in the term describing collisions between electrons of the wire and ions. In the zeroth approximation, we can choose the electron distribution function in the collision term to be the equilibrium one. In what follows, $f_{np} \equiv f_F(\epsilon_{np} - \mu)$ is assumed, where f_F is the Fermi distribution function and μ is the Fermi level. The first iteration of Eq. (21) gives the nonequilibrium part of the distribution function in the form

$$\Delta f_{np} = -\left(z \pm \frac{L}{2}\right)\frac{1}{v_{np}} I\{f,F\} \qquad (23)$$

with the two signs corresponding to $p > 0$ and $p < 0$. Here, $I\{f,F\}$ is a shorthand notation for the collision term. Using the particle conservation property of the scattering integral

$$\sum_n \int dp\, I\{f,F\} = 0, \qquad (24)$$

we obtain the drag current J_d in the form (cf. Ref. [17])

$$J_d = -2eL\sum_n \int\limits_0^\infty \frac{dp}{2\pi\hbar} I\{f,F\}. \qquad (25)$$

With the distribution function given by Eq. (14), we have

$$J_d = -2eN\mathcal{V}_{\mathbf{R}}^2 L \sum_n \int_0^\infty \frac{dp}{2\pi\hbar} \int \frac{d^3q}{(2\pi\hbar)^3} \times \\ \times \frac{2\pi}{\hbar} \Big\{ |\langle p, n, P\mathbf{e}_z | U | p+q_z, n, P\mathbf{e}_z - \mathbf{q}\rangle|^2 \times \\ \times \delta(\epsilon_{np} + E_{P\mathbf{e}_z} - \epsilon_{n,p+q_z} - E_{P\mathbf{e}_z - \mathbf{q}}) f_{np}(1 - f_{n,p+q_z}) - \\ - |\langle p, n, P\mathbf{e}_z + \mathbf{q} | U | p+q_z, n, P\mathbf{e}_z\rangle|^2 \times \\ \times \delta(\epsilon_{np} + E_{P\mathbf{e}_z + \mathbf{q}} - \epsilon_{n,p+q_z} - E_{P\mathbf{e}_z}) \times \\ \times f_{n,p+q_z}(1 - f_{np}) \Big\}, \quad (26)$$

where $\mathbf{e}_z$ is the unit vector along the z axis. In the first term in the integrand, we change $\mathbf{q} \to -\mathbf{q}$ and shift the integration variable p by q_z. Then

$$J_d = -2eN\mathcal{V}_{\mathbf{R}}^2 L \sum_n \int \frac{d^3q}{(2\pi\hbar)^3} \int_0^{q_z} \frac{dp}{2\pi\hbar} \times \\ \times \frac{2\pi}{\hbar} |\langle p, n, P\mathbf{e}_z + \mathbf{q} | U | p+q_z, n, P\mathbf{e}_z\rangle|^2 \times \\ \times \delta(\epsilon_{np} + E_{P\mathbf{e}_z + \mathbf{q}} - \epsilon_{n,p-q_z} - E_{P\mathbf{e}_z}) \times \\ \times f_{n,p-q_z}(1 - f_{np}), \quad (27)$$

and hence the drag current is

$$J_d = -J_0 \frac{2MS_{\mathbf{R}}}{m\pi^2\hbar^2} \sum_n \int dq_z \int_0^{q_z} dp\, f_{n,p-q_z}(1 - f_{np}) \times \\ \times \int d\mathbf{q}_\perp g(\mathbf{q}_\perp, |q_z|) \times \\ \times \delta\left[q_\perp^2 - q_z^2\left(\frac{M}{m} - 1\right) + 2q_z\left(\frac{Mp}{m} + P\right)\right].$$

Here, we introduce

$$J_0 = \frac{e(2ee_I)^2 L N m S_{\mathbf{R}}}{(1+\kappa)^2 \pi\hbar^3}$$

and a dimensionless quantity $g(\mathbf{q}_\perp, |q_z|)$ according to

$$S_{\mathbf{R}}^2 g(\mathbf{q}_\perp, |q_z|) = \left| \int d\mathbf{R}_\perp d\mathbf{r}_\perp \exp\left(-\frac{i\mathbf{q}_\perp \mathbf{R}_\perp}{\hbar}\right) \times \right. \\ \left. \times |\psi_n(\mathbf{r}_\perp)|^2 K_0\left(\frac{|q_z||\Delta\mathbf{r}_\perp|}{\hbar}\right) \right|^2, \quad (28)$$

where $S_{\mathbf{R}}$ is the cross-sectional area of the ion beam.

We obtain

$$J_d = J_0 \frac{2MS_{\mathbf{R}}}{m\pi^2\hbar^2} \sum_n \int_0^\infty dq_z \int_0^{q_z} dp\, f_{n,p-q_z}(1 - f_{np}) \times \\ \times \int d\mathbf{q}_\perp g(\mathbf{q}_\perp, q_z) \times \\ \times \left\{ \delta\left[q_\perp^2 - q_z^2\left(\frac{M}{m} - 1\right) + 2q_z\left(\frac{Mp}{m} - P\right)\right] - \right. \\ \left. - \delta\left[q_\perp^2 - q_z^2\left(\frac{M}{m} - 1\right) + 2q_z\left(\frac{Mp}{m} + P\right)\right] \right\}. \quad (29)$$

2.1. Linear response

In the linear response regime

$$V \ll T/p_{nF}, \quad \text{where} \quad p_{nF} = \sqrt{2m[\mu - \epsilon_n(0)]}, \quad (30)$$

the difference of δ-functions in Eq. (29) can be expanded as (we again take into account that $M/m \gg 1$)

$$\delta\left[q_\perp^2 - q_z^2 \frac{M}{m} + 2q_z\left(\frac{Mp}{m} + P\right)\right] - \\ - \delta\left[q_\perp^2 - q_z^2 \frac{M}{m} + 2q_z\left(\frac{Mp}{m} - P\right)\right] = \\ = \frac{m}{2M|q_z|}\left[\delta\left(\frac{mq_\perp^2}{2Mq_z} - \frac{q_z}{2} + p + mV\right) - \right. \\ \left. - \delta\left(\frac{mq_\perp^2}{2Mq_z} - \frac{q_z}{2} + p - mV\right)\right] = \\ = mV \frac{m}{M|q_z|} \frac{\partial}{\partial p} \delta\left(\frac{mq_\perp^2}{2Mq_z} - \frac{q_z}{2} + p\right).$$

Then the integration by parts gives

$$J_d = J_0 \frac{2S_{\mathbf{R}}}{\pi^2\hbar^2} mV \sum_n \int_0^\infty \frac{dq_z}{q_z} \int_0^{q_z} dp \int d\mathbf{q}_\perp g(\mathbf{q}_\perp, q_z) \times \\ \times \delta\left(\frac{mq_\perp^2}{2Mq_z} - \frac{q_z}{2} + p\right) \frac{\partial}{\partial p} f_{n,p-q_z}(1 - f_{np}). \quad (31)$$

Using

$$\frac{\partial}{\partial p} f_{n,p-q_z}(1 - f_{np}) = (1 - f_{np}) \times \\ \times \delta(q_z - p - p_{nF}) + f_{n,p-q_z} \delta(p - p_{nF}), \quad (32)$$

we have

$$J_d = J_0 \frac{4S_{\mathbf{R}}}{\pi^2\hbar^2} mV \sum_n \int_{p_{nF}}^\infty dq_z \int d\mathbf{q}_\perp g(\mathbf{q}_\perp, q_z) \times \\ \times \left[\delta\left(q_z^2 - 2p_{nF}q_z + \frac{mq_\perp^2}{M}\right)(1 - f_{n,q_z - p_{nF}}) + \right. \\ \left. + \delta\left(q_z^2 - 2p_{nF}q_z - \frac{mq_\perp^2}{M}\right) f_{n,p_{nF} - q_z}\right]. \quad (33)$$

Eliminating the δ-functions yields

$$J_d = J_0 \frac{2S_{\mathbf{R}}}{\pi^2\hbar^2} mV \sum_n \int d\mathbf{q}_\perp \times \\ \times \left\{ g(\mathbf{q}_\perp, p_{nF}+p_1) \frac{1-f_{n,p_1}}{p_1} + g(\mathbf{q}_\perp, p_{nF}+p_2)\frac{f_{np_2}}{p_2} \right\},$$

where $p_1 = \sqrt{p_{nF}^2 - mq_\perp^2/M}$ and $p_2 = \sqrt{p_{nF}^2 + mq_\perp^2/M}$. The expression for J_d can be simplified as

$$J_d = J_0 \frac{4S_{\mathbf{R}}}{\pi^2\hbar^2} mV \times \\ \times \sum_n \frac{1}{p_{nF}} \int d\mathbf{q}_\perp \frac{g(\mathbf{q}_\perp, 2p_{nF})}{\exp\left(q_\perp^2/2MT\right)+1}. \quad (34)$$

If the ion beam cross section is of a circular form with radius a, we have

$$g(\mathbf{q}_\perp, 2p_{nF}) = \left(\frac{2\hbar J_1(aq_\perp/\hbar)}{aq_\perp}\right)^2 K_0^2\left(\frac{2p_{nF}d}{\hbar}\right), \quad (35)$$

where $J_1(x)$ is the Bessel function of the first order and d is the distance between the central lines of the ion flux and the wire.

Below, we discuss the special case where $g(\mathbf{q}_\perp, 2p_{nF})$ is independent of $\mathbf{q}_\perp$ in more detail. For instance, this is the case if

$$\sqrt{MT} \ll \hbar/a. \quad (36)$$

Then we obtain

$$J_d = J_0 \frac{4a^2 \ln 4}{\hbar^2} MT \frac{V}{v_F} K_0^2\left(\frac{2p_F d}{\hbar}\right), \quad (37)$$

where $v_F = p_F/m$ is the Fermi velocity and

$$J_0 = \frac{e(2ee_I)^2 LNma^2}{(1+\kappa)^2\hbar^3}. \quad (38)$$

In the opposite case where

$$\sqrt{MT} \gg \hbar/a, \quad (39)$$

the drag is independent of temperature. For the values $M = 10^{-22}$ g (Ga), $T = 4$ K, and $a = 10^{-5}$ cm, this inequality can easily be satisfied. Then, we have

$$J_d = J_0 \frac{8V}{v_F} K_0^2\left(\frac{2p_F d}{\hbar}\right). \quad (40)$$

It is interesting to calculate the ratio J_d/J_I in this case:

$$\frac{J_d}{J_I} = \frac{e}{e_I}\frac{32(ee_I)^2 Lm}{(1+\kappa)^2\pi\hbar^3 v_F} K_0^2\left(\frac{2p_F d}{\hbar}\right). \quad (41)$$

Here, we can use Eqs. (10) and (11) for $K_0(s)$.

For an estimate, we assume the values $L = 10^{-4}$ cm, $m = 7\cdot 10^{-29}$ g, $v_F = 2\cdot 10^7$ cm/s, $\kappa = 10$, and $p_F d/\hbar = 2$, whence $K_0^2(2p_F d/\hbar) = 1.3\cdot 10^{-4}$. Then, for $J_I = 10^{-8}$ A, we have $J_d = 2\cdot 10^{-9}$ A and the corresponding drag voltage

$$\mathcal{V}_d \approx 20\,\mu\text{V}. \quad (42)$$

Naturally, if J_I increases, $\mathcal{V}_d$ also increases in proportion to J_I.

2.2. Nonlinear case

We consider the simplest case of low temperatures assuming that

$$V \gg T/p_F. \quad (43)$$

In our further calculation, we assume that $T = 0$. Then, the integration due to the Fermi functions in Eq. (29) is restricted and we obtain (the first or the second δ-function contributes for $V > 0$ and $V < 0$, respectively, and therefore the drag current changes its sign with V, as it should)

$$J_d = J_0 \frac{a^2}{\pi\hbar^2} \sum_n \left(\int\limits_{p_{nF}}^{2p_{nF}} \frac{dq_z}{q_z} \int\limits_{p_{nF}}^{q_z} dp + \right. \\ \left. + \int\limits_{2p_{nF}}^{\infty} \frac{dq_z}{q_z} \int\limits_{q_z - p_{nF}}^{q_z} dp \right) \int d\mathbf{q}_\perp g(\mathbf{q}_\perp, q_z) \times \\ \times \delta\left[p - mV - \frac{q_z}{2}\left(1 - \frac{m}{M}\right) + \frac{mq_\perp^2}{2q_z M}\right].$$

The result valid for

$$V < v_F/2 \quad (44)$$

is

$$J_d = J_0 \frac{a^2}{\pi\hbar^2} \sum_n \int d\mathbf{q}_\perp \theta\left(4Pp_{nF} - q_\perp^2\right) \times \\ \times \int\limits_{p_-}^{p_+} \frac{dq_z}{q_z} g(\mathbf{q}_\perp, q_z), \quad (45)$$

where θ is the step function and

$$p_\pm = p_{nF} \pm mV + \sqrt{(p_{nF} \pm mV)^2 \mp mq_\perp^2/M}$$

(other cases are considered in Appendix A).

For $V \ll v_{nF}$, the integration variable q_z is in the vicinity of $2p_{nF}$ and we have

$$J_d = J_0 \frac{a^2}{2\pi\hbar^2} \sum_n \int d\mathbf{q}_\perp \theta\left(4Pp_{nF} - q_\perp^2\right) \times \times \left(4\frac{mV}{p_{nF}} - \frac{m}{M}\frac{q_\perp^2}{p_{nF}^2}\right) g(\mathbf{q}_\perp, 2p_{nF}). \quad (46)$$

Equation (46) can be substantially simplified if $g(\mathbf{q}_\perp, 2p_{nF})$ is independent of $\mathbf{q}_\perp$; this is the case if the ion flux cross section characteristic width a obeys the inequality

$$\sqrt{Pp_{nF}}\, a/\hbar \ll 1. \quad (47)$$

Then,

$$J_d = J_1 \sum_n g(2p_{nF}), \quad (48)$$

where

$$J_1 = J_0 \frac{(2mV)^2 a^2}{2\hbar^2} \frac{M}{m}. \quad (49)$$

This expression is valid for $V > 0$, i. e., when the ion flux is directed "to the right". Then, the momentum transferred to the electron system in the wire is also directed to the right and the current (since $e < 0$) flows in the opposite direction regardless of the sign of the dragging ion charge.

Assuming that the distance d between the ion flux and the wire is much larger than the characteristic cross-sectional length of the wire and the flux, we can write

$$g(\mathbf{q}_\perp, 2p_{nF}) \approx K_0^2\left(\frac{2p_{nF}d}{\hbar}\right) \quad (50)$$

and

$$J_d = J_1 \sum_n K_0^2\left(\frac{2p_{nF}d}{\hbar}\right). \quad (51)$$

Using approximation (11) for the function K_0, we obtain

$$J_d = J_1 \sum_n \frac{1}{k_{nF}d}\exp(-4k_{nF}d), \quad (52)$$

where $k_{nF} = p_{nF}/\hbar$.

In the case $a\sqrt{Pp_{nF}}/\hbar \gg 1$, we have

$$J_d = 4J_0MV \sum_n \frac{1}{p_{nF}} K_0^2\left(\frac{2p_{nF}d}{\hbar}\right) \quad (53)$$

and therefore the drag current is a linear function of V.

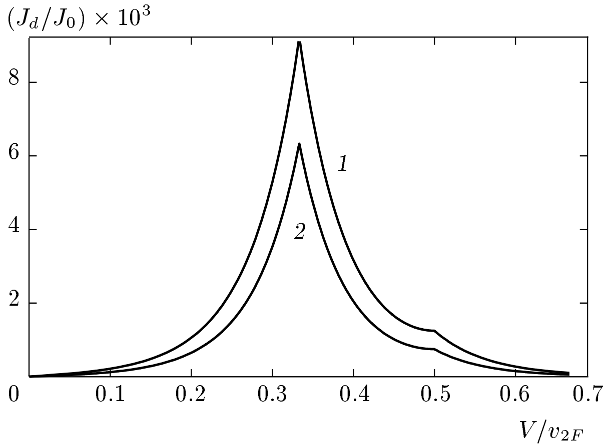

Fig. 2. Drag current dependence on the ion beam velocity for two parameter values $k_{2F}d = 3$ (curve *1*) and $k_{2F}d = 3.2$ (curve *2*). We take $v_{2F} = kv_{1F}$ and $k = 3/2$. The first peak corresponds to $V/v_{2F} = 1/2k$ (i. e., $V/v_{1F} = 1/2$) and the second peak (to the right) corresponds to $V/v_{2F} = 1/2$. Here, $J_0 = e(2ee_I)^2LNma^2/(1+\kappa)^2\hbar^3$

3. CONCLUDING REMARKS

We have developed a theory of Coulomb drag of electrons in a 1D ballistic nanostructure by an ion beam. This provides an example of drag of quasiparticles of a nanostructure by particles of the beam. It is worth mentioning that such a beam may consist not only of heavy ions but also of free electrons. The free electron mass is usually bigger than the effective mass of conduction electrons, and hence the approximations adopted in our calculation, $M \gg m$, may remain valid in this case.

The experimental setup should permit varying the velocity V within rather wide limits. We see, however, that to achieve a large drag effect, we should choose the value of V near $v_{nF}/2$ (Fig. 2). Here, we wish to note that this velocity is preferred regardless of the ion beam shape or the distance from the nanostructure (see Appendix B). This means in particular that the ion beam drag may be a useful tool in nanostructure spectroscopy.

APPENDIX A

Evaluation of the drag current for various ratios $\alpha = V/v_F$

We introduce the dimensionless parameters

$$\alpha = mV/p_{nF} = V/v_{nF}$$

and

$$b = mq_\perp^2/p_{nF}^2 M$$

and write q instead of $q(1-m/M)$ in the argument of the δ-function,

$$J_d = J_0 \sum_n \left(\frac{p_{nF}a}{\hbar}\right)^2 \times \\ \times \frac{1}{\pi}\left(\int_1^2 dq \int_1^q dp + \int_2^\infty dq \int_{q-1}^q dp\right) \times \\ \times \frac{1}{q}\int d\mathbf{q}_\perp g(p_{nF}\mathbf{q}_\perp, p_{nF}q)\delta\left(p-\alpha-\frac{q}{2}+\frac{b}{2q}\right), \quad (A.1)$$

where

$$J_0 = e(2ee_I)^2 LNma^2/(1+\kappa)^2\hbar^3.$$

For $\alpha < 1/2$, we obtain

$$J_d = J_0 \sum_n \left(\frac{p_{nF}a}{\hbar}\right)^2 \frac{1}{\pi}\int d\mathbf{q}_\perp \theta(4\alpha - b) \times \\ \times \int_{A_+(-\alpha,-b)}^{A_+(\alpha,b)} dq \frac{g(p_{nF}\mathbf{q}_\perp, p_{nF}q)}{q}, \quad (A.2)$$

where $A_\pm(\alpha,b) = 1+\alpha \pm \sqrt{(1+\alpha)^2 - b}$.

If $1/2 < \alpha < 1$, we obtain

$$J_d = J_0 \sum_n \left(\frac{p_{nF}a}{\hbar}\right)^2 \frac{1}{\pi}\int d\mathbf{q}_\perp \times \\ \times \left\{\theta(2\alpha-1-b)\int_{A_+(\alpha-1,b)}^{A_+(\alpha,b)} dq + \theta(b-2\alpha+1)\theta(4\alpha-b) \times \right. \\ \left. \times \int_{A_+(-\alpha,-b)}^{A_+(\alpha,b)} dq\right\} \frac{g(p_{nF}\mathbf{q}_\perp, p_{nF}q)}{q}. \quad (A.3)$$

We do not give the explicit expressions for larger values of V/v_F here, but present a simple expression for the drag current valid for $\hbar/a \ll mV\sqrt{M/m}$:

$$J_d = 4J_0 \int_\alpha^\infty \frac{dz}{z} K_0^2\left(\frac{2p_F z d}{\hbar}\right) \times \\ \times \left\{\left[\exp\left(\frac{([z-\alpha]^2-1)p_F^2}{2mT}\right)+1\right]^{-1} - \right. \\ \left. - \left[\exp\left(\frac{([z+\alpha]^2-1)p_F^2}{2mT}\right)+1\right]^{-1}\right\}. \quad (A.4)$$

This expression reduces to Eq. (40) and Eq. (53) in the corresponding limit cases. For $mV \gg T/v_F$, the difference of the Fermi functions restricts the integration region such that we have

$$J_d = 4J_0 \int_{1-\alpha}^{1+\alpha} \frac{dz}{z} K_0^2\left(\frac{2p_F z d}{\hbar}\right) \quad (A.5)$$

for $\alpha < 1/2$ and

$$J_d = 4J_0 \int_{\alpha}^{1+\alpha} \frac{dz}{z} K_0^2\left(\frac{2p_F z d}{\hbar}\right) \quad (A.6)$$

for $\alpha > 1/2$. The drag current calculated according to these simple formulas practically coincides with that calculated from the exact expressions, presented in Fig. 2 for $\mathcal{N} = 2$.

APPENDIX B

Preferred velocity of the beam

We introduce the notation

$$j = \frac{J_d \pi^2\hbar^2}{2J_0 S_{\mathbf{R}} m}. \quad (B.1)$$

We differentiate Eq. (29) with respect to mV and determine the sign of the derivative. In the integrand, we then obtain the sum of δ-functions,

$$-\frac{d}{dp}\left[\delta\left(\frac{q_\perp^2 m}{M} - q_z^2 + 2q_z(p-mV)\right) + \right. \\ \left. + \delta\left(\frac{q_\perp^2 m}{M} - q_z^2 + 2q_z(p+mV)\right)\right]. \quad (B.2)$$

We integrate over p by parts to obtain

$$\frac{dj}{dV} = \int_0^\infty dq_z \int d\mathbf{q}_\perp g(\mathbf{q}_\perp, q_z) \times \\ \times \int_0^{q_z} dp \left[\delta\left(\frac{q_\perp^2 m}{M} - q_z^2 + 2q_z(p-mV)\right) + \right. \\ \left. + \delta\left(q_\perp^2 m/M - q_z^2 + 2q_z(p+mV)\right)\right] \times \\ \times \frac{d}{dp}\left[f_{n,p-q_z}(1-f_{n,p})\right] - \\ - \int_0^\infty dq_z \int d\mathbf{q}_\perp g(\mathbf{q}_\perp, q_z)(1-f_{q_z}) \times \\ \times \left[\delta\left(\frac{q_\perp^2 m}{M} + q_z^2 - 2q_z mV\right) + \right. \\ \left. + \delta\left(\frac{q_\perp^2 m}{M} + q_z^2 + 2q_z mV\right)\right]. \quad (B.3)$$

We recall the strong Fermi degeneracy of the electron system, such that $1 - f_0 = 0$, $f_0 = 1$. Using Eq. (32), we then have

$$\frac{dj}{dV} = \int_{2p_{nF}}^{\infty} dq_z \int d\mathbf{q}_\perp g(\mathbf{q}_\perp, q_z) \times \\ \times \left[\delta\left(\frac{q_\perp^2 m}{M} + q_z^2 - 2q_z(p_{nF} + mV) \right) + \right. \\ \left. + \delta\left(\frac{q_\perp^2 m}{M} + q_z^2 - 2q_z(p_{nF} - mV) \right) \right] - \int_{p_{nF}}^{\infty} dq_z \times \\ \times \int d\mathbf{q}_\perp g(\mathbf{q}_\perp, q_z) \delta\left(\frac{q_\perp^2 m}{M} + q_z^2 - 2q_z mV \right). \quad \text{(B.4)}$$

We again use the relation $f_{q_z - p_{nF}} = 1$ for $q_z > p_{nF}$ and take into account that $V > 0$, which implies that the last δ-function in Eq. (B.3) does not contribute. The second δ-function in the first integral in the previous expression does not contribute as well, and we arrive at

$$\frac{dj}{dV} = I_+ - I_-,$$

where we introduce the notation

$$I_+ = \int d\mathbf{q}_\perp \int_{2p_{nF}}^{\infty} dq_z g(\mathbf{q}_\perp, q_z) \times \\ \times \delta\left(\frac{q_\perp^2 m}{M} + q_z^2 - 2q_z(p_{nF} + mV) \right), \quad \text{(B.5)}$$

$$I_- = \int d\mathbf{q}_\perp \int_{p_{nF}}^{\infty} dq_z g(\mathbf{q}_\perp, q_z) \times \\ \times \delta\left(\frac{q_\perp^2 m}{M} + q_z^2 - 2q_z mV \right). \quad \text{(B.6)}$$

If $V/v_{nF} < 1/2$, we obtain

$$I_+ = \int_{q_\perp^2 < 4Mp_{nF}V} \frac{d\mathbf{q}_\perp g(\mathbf{q}_\perp, q_1)}{2\sqrt{(p_{nF} + mV)^2 - mq_\perp^2/M}}, \quad \text{(B.7)}$$

where

$$q_1 = p_{nF} + mV + \sqrt{(p_{nF} + mV)^2 - mq_\perp^2/M},$$

and $I_- = 0$, and the drag current is an increasing function of the beam velocity V.

The integral I_- takes nonzero values only if $V/v_{nF} > 1/2$. If $V/v_{nF} < 1$, we have

$$I_- = \\ = \int_{q_\perp^2 < Mp_{nF}(2mV - p_{nF})/m} \frac{d\mathbf{q}_\perp g(\mathbf{q}_\perp, q_2)}{2\sqrt{(mV)^2 - mq_\perp^2/M}}, \quad \text{(B.8)}$$

where

$$q_2 = mV + \sqrt{(mV)^2 - mq_\perp^2/M}.$$

In this region, I_- becomes larger than I_+ (the latter being practically zero due to the exponential dependence on q_1) and the drag current turns into a decreasing function of the velocity V.

If $V/v_{nF} > 1$,

$$I_- = \int_{q_\perp^2 < Mp_{nF}V} \frac{d\mathbf{q}_\perp g(\mathbf{q}_\perp, q_2)}{2\sqrt{(mV)^2 - mq_\perp^2/M}} - \quad \text{(B.9)}$$

$$- \int_{q_\perp^2 > Mp_{nF}(mV - p_{nF})/m} \frac{d\mathbf{q}_\perp g(\mathbf{q}_\perp, q_3)}{2\sqrt{(mV)^2 - mq_\perp^2/M}}, \quad \text{(B.10)}$$

where

$$q_3 = mV - \sqrt{(mV)^2 - mq_\perp^2/M}.$$

We thus see that the drag current has a maximum as a function of the beam velocity in the vicinity of

$$V = v_{nF}/2.$$

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