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# TRANSVERSE MOMENTUM DISTRIBUTIONS OF STRANGE HADRONS PRODUCED IN $p$–$p$ COLLISIONS AT $\sqrt{s_{NN}} = 200$ GeV

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The mid-rapidity transverse momentum spectra of strange hadrons ($K^+$, $K^-$, $K^-/K^+$, $\Lambda$, $\overline{\Lambda}$, $\Xi^-$, $\Xi^+$, and $\Omega$) produced in $p$–$p$ collisions at the highest RHIC energy $\sqrt{s_{NN}} = 200$ GeV have been studied using a statistical unified thermal freeze-out model. The calculated results are found to be in good agreement with the experimental data taken from STAR and BRAHMS experiments. The fits of the transverse momentum spectra to the model calculations provide the thermal freeze-out conditions in terms of the temperature and collective flow effect parameters for different particle species. The model incorporates a longitudinal and a transverse hydrodynamic flow. The rapidity distributions of kaons and their ratios are also reproduced successfully, which reveals the presence of partial nuclear transparency effects in $p$–$p$ collisions at $\sqrt{s_{NN}} = 200$ GeV. The contributions from heavier decay resonances are also taken into account.

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## 1. INTRODUCTION

The ultra-relativistic $p$–$p$ collisions serve as an important tool to understand the high-energy collision scenarios and the particle production mechanism. These are also often used as a simple hadronic reference system to disentangle nuclear effects in $p$–A and A–A collision systems. Experiments at Relativistic Heavy Ion Collider (RHIC) continue to study the detailed properties of the strongly interacting matter formed in $p$–$p$ and Au–Au systems at colliding energies with $\sqrt{s_{NN}}$ ranging from 7.7 GeV to 200 GeV. The formation of a quark–gluon plasma (QGP) has already been pointed out by the measurements in Au–Au collisions performed at RHIC [1–3]. The high-$p_T$ hadrons are found to be important for QGP studies because they measure the jet quenching [4] effect in the QGP, while the low-$p_T$ hadrons arise from multiple scatterings and follow an exponential distribution, suggesting particle production in a thermal system [5]. In addition, the hadron spectra at intermediate $p_T$ are sensitive to effects arising from quark recombination [6] in heavy-ion collisions. It is believed that the produced hadrons carry information about the collision dynamics and the subsequent space–time evolution of the system. Hence, a precise measurement of the transverse momentum distributions of identified hadrons along with the rapidity spectra is essential for understanding the dynamics and properties of the created matter up to the final hydrodynamic freeze-out [7]. The particle momentum distributions reflect the conditions at the thermal freeze-out and the integrated effects of expansion from the beginning of the collision. Thus, transverse momentum distributions encode information about the collective transverse expansion and the thermal freeze-out temperature.

The applicability of the statistical model in small systems (e. g., $p$–$p$) has been the subject of several recent publications [8, 9]. In this paper, we use our earlier proposed unified statistical thermal freeze-out model [7], which assumes that the fireball produced in the collision reaches thermo-chemical equilibration at the final freeze-out. The model is found to be effectively suitable for the $p_T$ range that is dominated by the soft particle production. Also the particle production due to hard scatterings contributes less than a few percent at $\sqrt{s_{NN}} \leq 200$ GeV [10]. This motivates us to apply our model to $p$–$p$ collisions at $\sqrt{s_{NN}} = 200$ GeV to describe the particle production originating from the soft interactions in a thermalized system. The application of hydrodynamic models for $p$–$p$ collisions dates back to 1954, when multiple meson production was

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first observed at the Brookhaven Cosmotron in $n$–$p$ collisions [11]. This observation of multi-particle production occurring in $p$–$p$ collisions led Fermi and Landau to develop the statistical [12] and hydrodynamical [13] approaches to multi-particle production. Belenkij and Landau observed that although the statistical model of Fermi is sufficient to describe the particle numbers in terms of only the temperature and the chemical potential, this model has to be extended to hydrodynamics when particle spectra are considered. They also noted that the domain of the applicability of ideal relativistic hydrodynamics coincides with the domain of the applicability of thermodynamical models in high-energy $p$–$p$ collisions [13]. Hydrodynamic models [14, 15] that include radial flow successfully describe the measured $p_T$ distributions in Au–Au collisions at $\sqrt{s_{NN}} = 130$ GeV [16]. The $p_T$ spectra of identified charged hadrons below 2 GeV in central collisions have been well reproduced in some models by two simple parameters: the transverse flow velocity $\beta_T$ and the thermal freeze-out temperature $T$ under the assumption of thermalization. Some statistical thermal models have successfully described the particle abundances at low $p_T$ [17]. It has been shown earlier [18] that our model can simultaneously explain the rapidity and transverse momentum distributions of hadrons in Au–Au collisions at the highest RHIC energy $\sqrt{s_{NN}} = 200$ GeV. We have also employed this model to successfully reproduce the transverse momentum distributions of hadrons produced in the central Pb–Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV [19]. Here, we use the same model to reproduce the transverse momentum spectra of strange hadrons produced in $p$–$p$ collisions at 200 GeV. This is done in order to see whether such a system has reached its final thermo-chemically equilibrated stage as assumed in our model. Also, we want to address the collectivity, if any, in the produced system and the behavior of the thermal freeze-out temperature with the hadron mass.

## 2. BRIEF DESCRIPTION OF THE MODEL

In a hydrodynamic description taking the flow in the transverse and longitudinal directions into account, the final-state particles leave the hadronic medium at the time of freeze-out. The momentum distributions of hadrons, emitted from within an expanding fireball in the state of local thermal equilibrium, are characterized by the Lorentz-invariant Cooper–Frye formula [20]

$$E\frac{d^3n}{d^3P} = \frac{g}{(2\pi)^3}\int f\left(\frac{p^\mu u^\mu}{T}, \lambda\right) p^\mu d\Sigma_\mu, \qquad (1)$$

where $\Sigma_f$ represents a 3-dimensional freeze-out hypersurface and $g = 2J + 1$ is the degree of degeneracy of the expanding relativistic hadronic gas. Also, $\mu$ is the chemical potential of the given hadronic species. In recent works [21–23], it has been clearly shown that there is strong evidence of the increasing baryon chemical potential $\mu_B$ along the rapidity axis at RHIC, which can be written as [22, 23]

$$\mu_B = a + by_0^2, \qquad (2)$$

where $y_0 = cz$ is the rapidity of the expanding hadronic fluid element along the beam axis ($z$ axis) and $c$ is the constant of proportionality. This simple linear dependence of $y_0$ on $z$ ensures that under the transformation $z \to -z$, we have $y_0 \to -y_0$, thereby preserving the symmetry of the hadronic fluid flow about $z = 0$ along the rapidity axis in the center-of-mass frame of the colliding nuclei. The scaling of the chemical potential described by Eq. (2) is inspired by the experimental data at the highest RHIC energy. The BRAHMS collaboration data at $\sqrt{s_{NN}} = 200$ GeV has a clear dependence of the baryon chemical potential on rapidity, which is revealed through the $\overline{p}/p$ ratio changing with rapidity. Hence, this effect has been incorporated by considering a thermal model that assumes that the rapidity axis is populated with hot regions (fireballs) moving along the beam axis with a monotonically increasing rapidity $y_0$. This essentially emerges from the situation where the colliding nuclei exhibit transparency (at least partial). Hence the regions away from the mid-region ($z \approx 0$) also consist of the constituent partons of the colliding nucleons, which suffer less rapidity loss due to the partial nuclear transparency. Due to this, these regions have an excess of quarks over antiquarks and hence maintain larger baryon chemical potentials on either side of the mid-region in a symmetric manner. For this reason, a quadratic-type dependence of the baryon chemical potential $\mu_B$ on $y_0$ has been considered in Eq. (2) so as to make it invariant under the transformation $y_0 \to -y_0$, as the system properties are to remain invariant under the above transformation.

The transverse velocity component of the hadronic fireball $\beta_T$ is assumed to vary with the transverse coordinate $r$ as [24]

$$\beta_T(r) = \beta_T^s \left(\frac{r}{R}\right)^n, \qquad (3)$$

where $r \le R$ (the term $r/R$ accounts for the change in the velocity as a function of the radial distance). Also, $n$ is a velocity profile index and $\beta_T^s$ is the hadronic fluid surface transverse expansion velocity and is fixed in the model by using the parameterization [7]

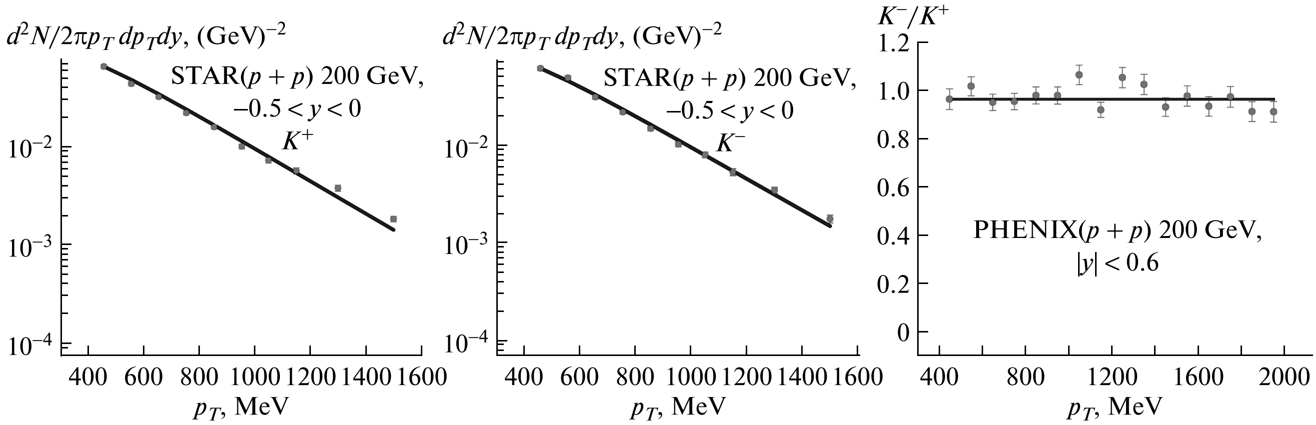


**Fig. 1.** Transverse momentum spectra of $K^+$, $K^-$, and $K^-/K^+$. Errors are statistical and systematic combined (in $K^-$ and $K^+$) and statistical only in $K^-/K^+$

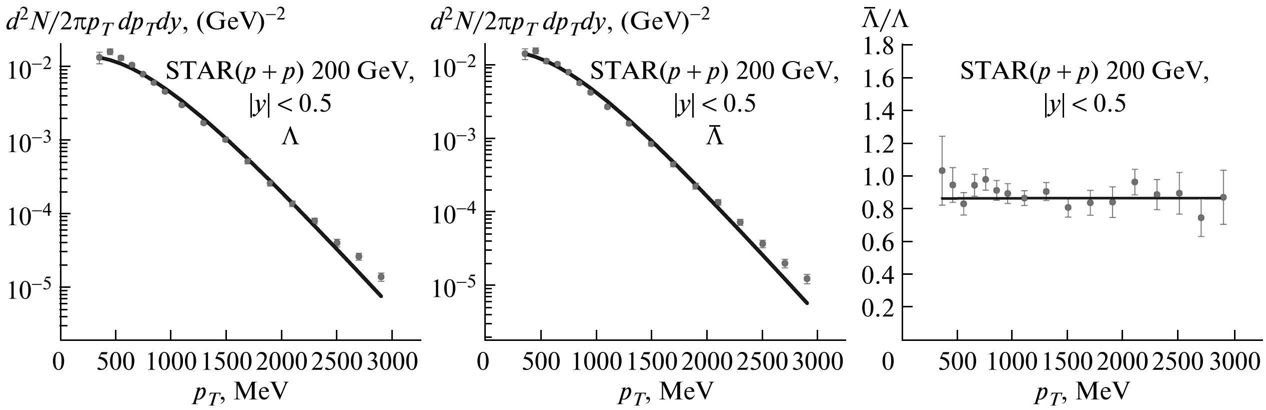


**Fig. 2.** Transverse momentum spectra of $\Lambda$, $\overline{\Lambda}$, and $\overline{\Lambda}/\Lambda$. Errors are statistical and systematic combined

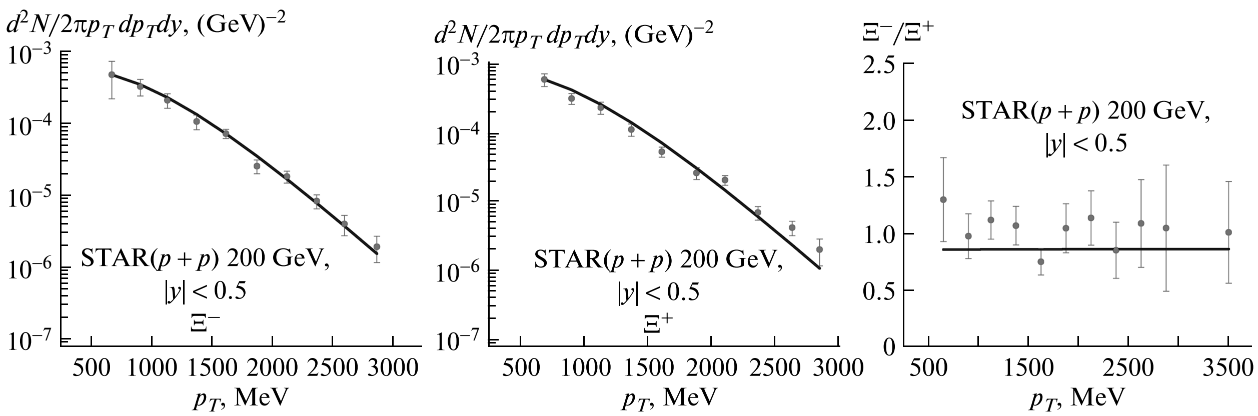


**Fig. 3.** Transverse momentum spectra of $\Xi^-$, $\Xi^+$, and $\Xi^-/\Xi^+$. Errors are statistical and systematic combined

**Table 1.** Freeze-out conditions of strange hadrons obtained from their $p_T$ spectra

| Particle | $\beta_T^0$ | $T$, MeV | $n$ | $\chi^2$/DoF |
|---|---|---|---|---|
| $K^+$ | 0.41 | 175 | 1.0 | 3.50 |
| $K^-$ | 0.40 | 176 | 1.0 | 1.30 |
| $K^-/K^+$ | 0.41 | 175 | 1.0 | 1.34 |
| $\Lambda$ | 0.34 | 188 | 1.36 | 3.83 |
| $\overline{\Lambda}$ | 0.33 | 189 | 2.20 | 4.30 |
| $\overline{\Lambda}/\Lambda$ | 0.35 | 189 | 1.0 | 0.90 |
| $\Xi^-$ | 0.35 | 189 | 1.0 | 2.0 |
| $\Xi^+$ | 0.33 | 189 | 1.0 | 1.66 |
| $\Xi^+/\Xi^-$ | 0.34 | 188 | 1.0 | 0.85 |
| $\Omega$ | 0.34 | 190 | 1.0 | 0.16 |

**Table 2.** Fit parameters of $K^+$, $K^-$, and $K^-/K^+$ obtained from their rapidity spectra

| Particle | $a$, MeV | $b$, MeV | $\sigma$, fm | $\chi^2$/DoF |
|---|---|---|---|---|
| $K^+$ | 12.2 | 18.0 | 4.25 | 1.0 |
| $K^-$ | 12.0 | 18.0 | 4.25 | 1.0 |
| $K^-/K^+$ | 12.0 | 18.2 | 4.45 | 1.18 |

$$\beta_T^s = \beta_T^0 \sqrt{1-\beta_z^2}\,. \tag{4}$$

The transverse fireball radius $R$ is parameterized as [7, 25]

$$R = r_0 \exp\left(-\frac{z^2}{\sigma^2}\right), \tag{5}$$

where the parameter $r_0$ fixes the transverse size of the hadronic matter and $\sigma$ fixes the width of the matter distribution in the transverse direction [7]. The contributions of various heavier hadronic resonances [22, 26], which decay after the freeze-out has occurred, are also taken into account in our analysis. We also impose the criterion of exact strangeness conservation such that the net strangeness in each fireball is separately equal to zero.

## 3. RESULTS AND DISCUSSION

In our analysis of the transverse momentum spectra of hadrons shown in Figs. 1–4, the best fit is obtained

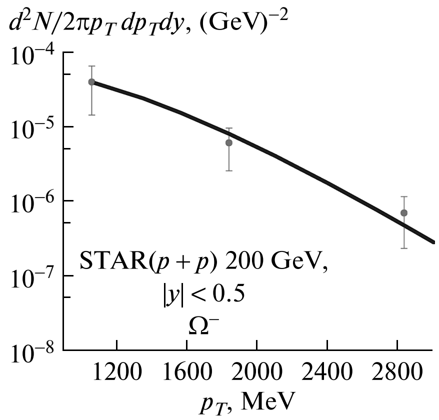


**Fig. 4.** Transverse momentum spectrum of $\Omega$. Errors are statistical and systematic combined

by minimizing the distribution of $\chi^2$ given by [27]

$$\chi^2 = \sum_i \frac{(R_i^{exp} - R_i^{theor})^2}{\epsilon_i^2}, \tag{6}$$

where $R_i^{exp}$ is the measured value of the yield with its uncertainty $\epsilon_i$ and $R_i^{theor}$ is the value from the model calculations. The $\chi^2$/DoF is minimized with respect to the variables $T$ and $\beta_T^0$ whereas the values of $a$, $b$ and $\sigma$ are obtained by fitting the available rapidity distributions of kaons (Fig. 5) by using the standard relation as

$$\frac{dn}{dy} = \int \left(E\frac{d^3n}{d^3p}\right) dp_T. \tag{7}$$

The various fit parameters obtained from these spectra are given in Tables 1 and 2. The experimental data taken from the STAR and BRAHMS experiments [28, 29] is shown by filled circles while our model calculations are shown by solid black curves in all cases.

In the present analysis, we consider the (maximum) $p_T$ range up to 3.0 GeV only (in the case of $\Lambda$, $\overline{\Lambda}$, $\Xi^-$, and $\Xi^+$) because the statistical hydrodynamic calculations cannot describe the hadron spectra at such large transverse momenta. The hadrons detected in this region are a result of the hard processes originating from the direct fragmentation of high-energy partons of the colliding beams and therefore are not able to thermalize through the process of multiple collisions [19]. We therefore turn to the low-$p_T$ softer hadrons, which are assumed to be reasonably thermalized and form the bulk of the secondary matter produced. It is seen that the model curves virtually cross all data points within the error bars.

It is seen from Fig. 5 that the kaon ratios fall below unity at mid-rapidity as well as at higher rapidi-

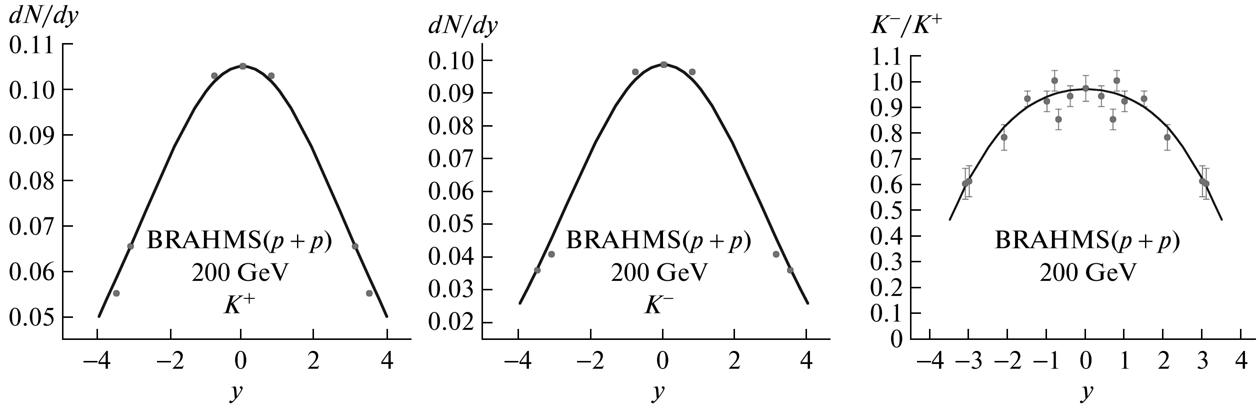


**Fig. 5.** Rapidity spectra of $K^+$, $K^-$ and $K^-/K^+$. Errors are statistical only

ties. This indicates that the chemical potential at mid-rapidity is not vanishing but attains a small but significant value, which is also evident from the values of $a$ shown in Table 2. This small but significant value of the chemical potential at mid-rapidity can be attributed to the partial transparency effects present in $p$–$p$ collisions at $\sqrt{s_{NN}} = 200$ GeV.

Also the transverse flow parameter $\beta_T^0$ obtained by fitting the $p_T$ distributions of heavier strange baryons attains almost a constant value $0.33 \le \beta_T^0 \le 0.35$, whereas the lighter kaons show a comparatively larger flow effect, as shown in Table 1. The comparatively larger flow and lower freeze-out temperature for kaons is indicative of the delayed freeze-out for these lighter particles. These smaller values of flow parameters present in the $p$–$p$ system, compared to those obtained in Au–Au collisions [18], are a result of the lower particle production in these collisions. These smaller values of collective flow and the lower particle production led us to conclude that the hadronic system formed in the $p$–$p$ collisions at 200 GeV may not have reached a complete thermo-chemical equilibration as assumed in our model. This is also evident from the relatively larger values of $\chi^2$/DoF for the above-studied hadrons. The relatively smaller values of $\chi^2$/DoF for $\Omega$, $\Xi^+/\Xi^-$, and $\overline{\Lambda}/\Lambda$ are due to their relatively larger error bars. The thermal/kinetic freeze-out temperatures of the strange baryons are also found to attain almost a constant value $188 \le T \le 190$, which is contrary to systematic freeze-out of these particles as observed in Au–Au collisions at RHIC [18]. The almost constant values of these freeze-out parameters indicate that the fireball produced in the system does not have enough time to expand before the freeze-out because of the smaller reaction volume formed in these collisions. Consequently, these strange baryons prefer to freeze out at higher constant temperatures and thus do not have enough time to develop stronger collective effects. Also the late freeze-out of kaons is understood to be due to their relatively larger cross-section with the hadronic matter.

## 4. CONCLUSION

The transverse momentum spectra of the strange hadrons ($K^+$, $K^-$, $\Lambda$, $\overline{\Lambda}$, $\Xi^-$, $\Xi^+$, and $\Omega$) along with their ratios and the available rapidity distribution of $K^+$, $K^-$, and $K^-/K^+$ at $\sqrt{s_{NN}} = 200$ GeV are fitted by using our statistical unified thermal freeze-out model. The results extracted from the rapidity distributions of kaons show the presence of a small but significant amount of chemical potential at mid-rapidity, which indicates the effects of only partial transparency in $p$–$p$ collisions at $\sqrt{s_{NN}} = 200$ GeV. Also the smaller values of collective flow, compared to those in Au–Au collisions, indicate that the hydrodynamic system formed in $p$–$p$ collisions may not be fully equilibrated because of the rapid expansion of the smaller reaction volume formed in these collisions. No systematic freeze-out of the strange baryons is seen as was observed in Au–Au collisions at RHIC. Instead, the strange baryons are found to maintain a constant value of the thermal freeze-out temperature and collective flow velocity. However, kaons are seen to freeze out later than the strange baryons.

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