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ACCRETION OF RADIATION AND ROTATING PRIMORDIAL BLACK HOLES

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We consider rotating primordial black holes (PBHs) and study the effect of accretion of radiation in the radiation-dominated era. The central part of our analysis deals with the role of the angular momentum parameter on the evolution of PBHs. We find that both the accretion and evaporation rates decrease with an increase in the angular momentum parameter, but the rate of evaporation decreases more rapidly than the rate of accretion. This shows that the evaporation time of PBHs is prolonged with an increase in the angular momentum parameter. We also note that the lifetime of rotating PBHs increases with an increase in the accretion efficiency of radiation as in the case of nonrotating PBHs.

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1. INTRODUCTION

Primordial black holes (PBHs) are supposed to be formed during the early expansion of the Universe. These black holes may have been produced due to density fluctuations in the early universe with extremely high temperature and pressure. The mass of the PBHs can cover a wide range. There are different theories regarding the formation of PBHs: initial inhomogeneities [1,2], inflation [3,4], phase transitions [5], bubble collisions [6,7], decay of cosmic loops [8], and so on. The formation of PBHs can also play a very important role in understanding the cosmological inflation. According to Hawking, black holes emit thermal radiation due to quantum effects near the event horizon [9]. As a result of Hawking radiation, black holes can lose mass and evaporate. Smaller-mass black holes are expected to evaporate quickly. The PBHs with a longer lifetime can act as seeds for structure formation [10]. PBHs with the mass greater than 10^{15} g do not evaporate completely via Hawking radiation and the abundance of such black holes can be considered a suitable dark matter candiadate [11].

In the context of standard cosmology, early work on the study of the effect of accretion of radiation on PBHs has led to several speculations regarding the possibility of increasing the mass of a PBH [1, 12]. Cosmological consequences of the evaporation of PBHs in different eras have been studied quite well [13, 14] (see [15] for new cosmological constraints on PBHs). It has been realized during the last couple of years that the effect of accretion in the radiation-dominated era can result in longlived PBHs in the braneworld scenario [16], in the Brans–Dicke theory [17–19], and in standard cosmology [20]. The impact of accretion of phantom energy and vacuum energy on the evolution of PBH has also been discussed in [21, 22].

In this paper, we study the evolution of rotating PBHs in the context of standard cosmology by including the effect of accretion of radiation. We obtain the dependence of the evaporation time on the accretion efficiency and the angular momentum parameter. It is found that the evaporation time of the rotating PBHs is prolonged due to the increase in both the angular momentum parameter and the accretion efficiency.

2. ROTATING PBHs AND ACCRETION OF RADIATION

In the Einstein–Maxwell theory, the most general black hole solutions with nonzero charge and angular momentum are described by the Kerr–Newman space–time. Here, we consider uncharged rotating PBHs in the context of a spatially flat FRW Universe. We assume that the universe is filled with a perfect fluid described by the equation of state $p = \gamma\rho$ (where $\gamma = 1/3$

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for the radiation-dominated era and $\gamma = 0$ for the matter-dominated era). The Einstein equation is given by

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho. \quad (2.1)$$

The energy–momentum conservation equation is

$$\dot{\rho} + \left(\frac{3\dot{a}}{a}\right)(1+\gamma)\rho = 0, \quad (2.2)$$

where $a(t)$ is the scale factor. From the above equations, we find that the scale factor $a(t)$ behaves as follows: $a(t) \propto t^{1/2}$ for $t < t_1$ and $a(t) \propto t^{2/3}$ for $t > t_1$. Here, $t < t_1$ corresponds to the radiation-dominated era and $t > t_1$ corresponds to the matter-dominated era.

Here, we consider the effect of accretion on the lifetime of the PBHs. Due to accretion in the radiation-dominated era, the mass of a PBH increases and the accretion rate (which is taken to be proportional to the product of the surface area of the PBH and the energy density of the radiation [23]) is given by

$$\dot{M}_{acc} = 4\pi f R_{BH}^2 \rho_R, \quad (2.3)$$

where ρ_R is the radiation energy density of the surrounding of the black hole, f is the accretion efficiency, and R_{BH} is the radius of the outer horizon of the rotating black hole with the mass M and is given by $R_{BH} = r_+ = M + \sqrt{M^2 - a^2}$, with the rotation parameter $a\,(= J/M)$ and the angular momentum J. The rotating black hole solution satisfies the inequality $M^2 \geq a^2$ in order to avoid a naked singularity. The precise value of f is not known. The accretion efficiency could in principle depend on complex physical processes such as the mean free paths of the particles comprising the radiation surrounding the PBHs.

We can calculate ρ_R from the Einstein equation as $\rho_R = 3/32\pi t^2$ (we take $G = 1$). Using these values, we obtain

$$\dot{M}_{acc} = \frac{3f}{8t^2}\left(M + \sqrt{M^2 - a^2}\right)^2. \quad (2.4)$$

This expression can be integrated to obtain the value of M_{acc}, and it reduces to the nonrotating case in the limit $a = 0$. We note that when a^2 is comparable to M^2, the rate of change of mass reduces to one fourth of the corresponding value obtained in the nonrotating case. In order to understand the exact effect of the angular momentum parameter on the rate of accretion in the radiation-dominated era, in Fig. 1 we numerically plot the variation of mass with a change in the angular momentum parameter a for a particular PBH formed at $t = 10^{-22}$ s and having accretion efficiency 0.5. In

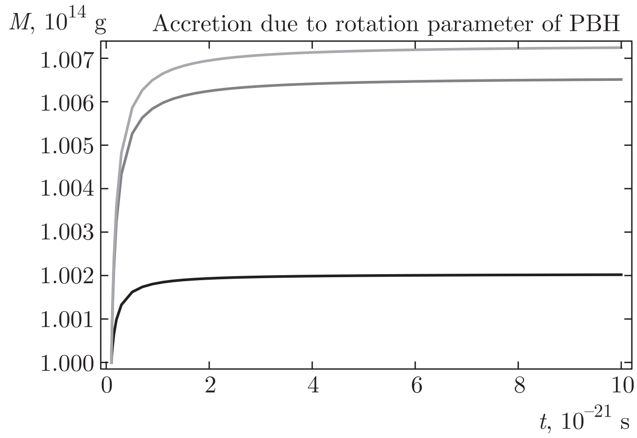

Fig. 1. Variation of the PBH mass for $a = M_i$ (lower), $M_i/2$ (middle), $M_i/4$ (upper); $f = 0.5$

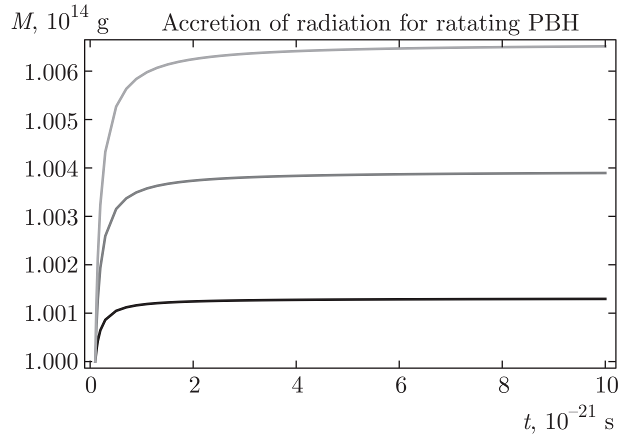

Fig. 2. Variation of the PBH mass for $f = 0.1$ (lower), 0.3 (middle), 0.5 (upper), $a = M_i/2$

our analysis, we assume the initial mass M_i of PBHs to be of the same order as the horizon mass [24, 25]. We can see from Fig. 1 that the rate of accretion decreases as the angular momentum parameter increases.

In Fig. 2, we plot the variation of mass of a particular rotating PBH formed at $t = 10^{-22}$ s with the angular momentum parameter $a = M_i/2$ for different accretion efficiencies f. We again find that the PBH mass increases with an increase in f, as in the nonrotating case.

It is also worth noting that the upper bound of the accretion efficiency f is not fixed, but varies with the angular momentum parameter a. For a^2 approaching M^2, the upper bound for f is 8/3 and for $a = 0$, it reduces to the standard limit $f < 2/3$. In the case of a nonrotating black hole in standard cosmology, it has been shown that the accretion can be effective in increasing the mass of the black hole and thereby increasing the lifetime of PBHs.

It is worth mentioning here that in the hydrodynamic picture of the formation of PBHs during expansion of the early Universe [1, 26, 27], it has been shown by numerical calculations that the pressure gradient plays an important role in impeding the formation of PBHs. The rate of accretion of PBHs can be reduced drastically by the pressure gradient. In case of the relativistic equation of state, initial perturbations have to be large enough in order to allow the formation of PBHs. In the present context, we have not considered the effect of pressure gradient on the accretion efficiency. Such a consideration will need a full numerical computation, which is beyond the scope of our paper.

3. EVAPORATION OF ROTATING PBHs

We now consider the evaporation of rotating PBHs due to Hawking radiation. The rate of change of mass due to evaporation is given by

$$\dot{M}_{evap} = -4\pi R_{BH}^2 \sigma_H T_{BH}^4, \tag{3.1}$$

where σ_H is Stefan's constant multiplied by the number of the degrees of freedom of radiation and T_{BH} is the Hawking temperature for the rotating uncharged black hole, given by

$$T_{BH} = \frac{\sqrt{M^2 - a^2}}{4\pi M\left(M + \sqrt{M^2 - a^2}\right)}. \tag{3.2}$$

Hence, one gets

$$\dot{M}_{evap} = -\frac{\sigma_H}{64\,\pi^3}\,\frac{\left(M^2 - a^2\right)^2}{M^4\left(M + \sqrt{M^2 - a^2}\right)^2}. \tag{3.3}$$

We can see that when a^2 becomes comparable to M^2, the rate of change of mass during evaporation becomes negligibly small. Therefore, the rate of evaporation decreases with an increase in the angular momentum parameter. In principle, we should also consider the rate of change of the black hole angular momentum due to the emission of particles together with the rate of change of mass. However, we only discuss the rate of change of mass due to evaporation for simplicity.

The total rate of change of mass including both accretion and evaporation for the rotating PBH is given by

Table 1. An estimate of the evaporation time with a change in the accretion efficiency f at a fixed angular momentum parameter

$t_i = 10^{-23}$ s; $M_i = 10^{15}$ g; $a^2 = 10^{19}$	
f	t_{evap}, 10^{13} s
0	3.333
0.2	3.363
0.4	3.394
0.5	3.409
0.6	3.425

$$\dot{M} = \frac{3f}{8t^2}\left(M + \sqrt{M^2 - a^2}\right)^2 - \\ - \frac{\sigma_H}{64\pi^3}\,\frac{\left(M^2 - a^2\right)^2}{M^4\left(M + \sqrt{M^2 - a^2}\right)^2}. \tag{3.4}$$

It follows from the above equation that during the early period of evolution, the accretion term becomes dominant and evaporation dominates at a later time. We can assume that accretion occurs until a time $t = t_c$ when the accretion and evaporation rates become equal, and then evaporation plays its role beyond t_c.

From Eq. (3.4), we obtain the expression for the time $t = t_c$ in terms of the maximum mass M_c and the accretion efficiency,

$$t_c = \left(\frac{3f}{32}\right)^{1/2} \times \\ \times \alpha^{-1/2}\left(\frac{M_c^2\left(M_c + \sqrt{M_c^2 - a^2}\right)^2}{M_c^2 - a^2}\right), \tag{3.5}$$

where M_c is the mass obtained from the accretion equation $M_c = M_{max}$.

Generally, PBHs are formed in the radiation-dominated era and during their evolution in that era, they obey the evolution equation given in (3.4). But in the matter-dominated era, due to less dense surroundings, there is no appreciable absorption of matter–energy by the PBHs. Hence, in the matter-dominated era, only the second term in the right-hand side of (3.4) contributes.

To improve the analysis, we construct Tables 1 and 2 showing the variation of the evaporation time with respect to the accretion efficiency and angular momentum parameter.

Table 2. A rough estimate of the evaporation time with a change in the angular momentum parameter

$t_i = 10^{-22}$ s; $M_i = 10^{14}$ g; $f = 0.5$	
a^2	t_{evap}
0	$\sim 10^{13}$ s
$10^{-9}\,M_i^2$	$\sim 10^{13}$ s
$10^{-7}\,M_i^2$	$\sim 10^{18}$ s
$10^{-5}\,M_i^2$	$\sim 10^{22}$ s
$10^{-3}\,M_i^2$	$\sim 10^{24}$ s
$10^{-1}\,M_i^2$	$\sim 10^{28}$ s

We can see from Table 1 that the lifetime for a rotating PBH becomes longer when the effect of accretion of radiation is included. It can be verified that in the limit $a = 0$, the above expression for t_c reduces to that for a nonrotating PBH, $t_c = (3f/2)^{1/2}\alpha^{-1/2}M_c^2$.

We note from Table 2 that the lifetime of a PBH increases with an increase in the angular momentum parameter, which happens due to a more rapid decrease in the evaporation rate than the accretion rate with the increase in the angular momentum parameter. However, for small values of a^2, the evaporation time does not change significantly.

As shown in Table 2, the PBHs formed at a particular time (10^{-22} s) with the initial mass of 10^{14} g, which are supposed to be evaporated by now, could have a lifetime greater than the present age $t_0 = 1.4\cdot 10^{10}$ years $= 4.42\cdot 10^{17}$ s depending on the value of the angular momentum parametrer. In other words, rotation makes it possible for the PBHs evaporating now to be formed at earlier times with smaller initial masses.

A similar kind of analysis for rotating PBHs has been discussed in the nice work of Page [28, 29]. In these papers, he has shown that the angular momentum itself decreases with time nearly in the same order as the mass of the PBH (see also [30]). Although this consideration changes our numerical results, still large values of a^2 give a significantly longer lifetime to PBHs. This is because a large value of a^2 does not suddenly dilute as a small value. This fact gives rise to a difference in the lifetime of rotating PBHs with small and large values of a^2. In Page's work, it has been found that rotation does not significantly affect the lifetime of PBHs and after a short period, every rotating PBH becomes a Schwarzschild type black hole.

From the comparison of both works, we can see that there is no conflict between the present work and that of Page, but there is a slight difference in the analysis. This difference arises due to two reasons. First, Page used the controlling parameter for the angular momentum as $a_\star = J/M^2$, whereas we use $a = J/M = a_\star M$ as the controlling parameter for angular momentum. Because M is changing with time, varying a and $a_\star$ does not give the same numerical result. Second, Page's calculation was basically for rotating PBHs emitting massless particles and he also found that if rotating PBHs emit more and more massive particles, then they would live longer. Because we are considering complete evaporation of PBHs (by using the Hawking evaporation equation along with the accretion equation) irrespective of the particle type they emitted, the lifetime of PBHs becomes longer than Page's prediction.

4. SUMMARY AND DISCUSSION

We have considered the effect of accretion of radiation on rotating PBHs in a homogeneous and isotropic FRW Universe. We find that the increase in the angular momentum parameter decreases both the accretion and evaporation rate, but the rate of evaporation decreases more rapidly than the rate of accretion. This shows that rotation increases the lifetime of PBHs. It is also noted that the mass of the PBH increases with the accretion efficiency as in the nonrotating case. Because Hawking radiation is supposed to carry away angular momentum, it is worthwhile to have a detailed analysis of the evolution of the rotating PBHs taking the rate of change of angular momentum into account in the context of emission of massless or nearly massless particles with different spins. Here, we have not considered the effect of back reaction of the PBH evaporation [31], which is supposed to modify the radius of the horizon and the Hawking temperature of the black hole [32]. It is expected that such effects might affect the evolution of PBHs. It is worth investigating these issues further in the context of PBHs with and without rotation.

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