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PHASE DEPENDENCE OF THE UNNORMALIZED SECOND-ORDER PHOTON CORRELATION FUNCTION

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We investigate the resonant quantum dynamics of a multi-qubit ensemble in a microcavity. Both the quantum-dot subsystem and the microcavity mode are pumped coherently. We find that the microcavity photon statistics depends on the phase difference of the driving lasers, which is not the case for the photon intensity at resonant driving. This way, one can manipulate the two-photon correlations. In particular, higher degrees of photon correlations and, eventually, stronger intensities are obtained. Furthermore, the microcavity photon statistics exhibits steady-state oscillatory behaviors as well as asymmetries.

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1. INTRODUCTION

Quantum dots, or *artificial atoms*, can undergo sharp optical transitions, similar to those of real atoms [1]. These transitions can be probed by applying a coherent laser field, to a two-level system in particular. This way, a number of effects can be obtained, some of which are known from pumping of real atoms with coherent laser fields. In particular, resonance fluorescence from a coherently driven semiconductor quantum dot in a cavity was experimentally investigated in [2]. The observation of the Mollow triplet [3] from a quantum dot system was reported in [4, 5]. Dephasing of triplet-sideband optical emission of a resonantly driven InAs/GaAs quantum dot inside a microcavity was studied in Ref. [6]. Furthermore, cascaded single-photon emission from the Mollow triplet sidebands of a quantum dot as well as spectral photon correlations were obtained in [7]. A pronounced interaction between the quantum dot and the cavity has been observed even for detunings of many cavity linewidths [8]. In the small Rabi frequency regime, subnatural-linewidth single photons from a quantum dot were also obtained in [9]. Moreover, the self-homodyne measurement of a dynamic Mollow triplet in the solid state systems was performed recently [10].

When two or more quantum dots are close to each other on the emission wavelength scale, collective interactions become important [11–16]. In particular, superradiance in an ensemble of quantum dots was experimentally observed in [17] while the dynamics of quantum dot superradiance was investigated in [18]. The collective fluorescence and decoherence of a few nearly identical quantum dots and superbunched photons via a strongly pumped near-equispaced multiparticle system were respectively analyzed in [19] and [20]. Dicke states in multiple quantum dot systems were also discussed in Ref. [21]. Furthermore, sub- and superradiance phenomena in quantum dot nanolasers were investigated in [22]. The collective modes of quantum dot ensembles in microcavities were obtained in [23]. Finally, entanglement of two quantum dots was investigated in Ref. [24].

Here, we investigate the dynamics of a two-level quantum-dot ensemble inside a microcavity. However, the developed approach also applies to a real atomic sample. The microcavity mode together with the qubit subsystem are pumped with two distinct coherent electromagnetic fields. When the laser that resonantly pumps the qubit ensemble is moderately intense (the corresponding Rabi frequency is larger than the qubit–cavity coupling strength as well as the spontaneous and cavity decay rates), we find enhanced photon–photon correlations. In particular, the photon statistics displays oscillatory steady-state behaviors due to an interplay between the cavity and spontaneous emission decay rates. Furthermore, the microcavity photon statistics depends on the phase difference of the applied coherent sources, which can be a convenient mechanism

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to influence the second-order photon–photon correlations. An asymmetric steady-state behavior of the second-order photon correlation function versus the cavity-field detuning is observed due to the relative phase dependence.

We note that larger photon correlations are useful for a number of practical applications like new photon sources emitting N-photon bundles [25], in many-body phenomena with strongly interacting photons [26], and in finding the relation between photon quantum statistics and entanglement [27]. Therefore, the aim of this study is to further investigate the generation of strongly correlated photons, but with stronger photon intensities $\propto N^2$ and intense unnormalized second-order photon–photon correlations $\propto N^4$, as well as to present various tools to manipulate these correlations (here, N is the number of artificial two-level atoms involved). It is quite useful to find alternative ways to generate an intense steady-state photon flux with enhanced photon–photon correlations.

The article is organized as follows. In Sec. 2, we describe the analytic approach and the system of interest, and obtain the corresponding equations of motion. Section 3 deals with discussions of the obtained results. A summary is given in Sec. 4.

2. QUANTUM DYNAMICS OF A PUMPED MULTI-QUBIT SYSTEM IN A MICROCAVITY

The Hamiltonian describing a wavelength-size collection of pumped two-level artificial (or real) atomic systems having a frequency ω_0 and embedded in a microcavity of a frequency ω_c is

$$H = \hbar\Delta a^\dagger a + \hbar g(a^\dagger S^- + aS^+) + \hbar\epsilon(a^\dagger e^{i\phi_1} + ae^{-i\phi_1}) + \hbar\Omega(S^+e^{i\phi_2} + S^-e^{-i\phi_2}). \quad (1)$$

Here, both the atomic sample and the microcavity mode are interacting with coherent sources of the frequency $\omega_{L1} = \omega_{L2} \equiv \omega_L$, in a frame rotating at ω_L, and we assume that $\omega_0 = \omega_L$. In Hamiltonian (1), the first term describes the cavity free energy with $\Delta = \omega_c - \omega_L$, and the second term characterizes the interaction of the quantum dot system with the microcavity mode via the coupling g. The third term is the interaction of the microcavity mode with the first coherent light source of an amplitude ϵ and a phase ϕ_1. The last term describes the interaction of the qubit subsystem with the second laser, with Ω and ϕ_2 being the corresponding Rabi frequency and phase. The collective operators

$$S^+ = \sum_{j=1}^{N} S_j^+ = \sum_{j=1}^{N} |2\rangle_{jj}\langle 1|$$

and $S^- = [S^+]^\dagger$ obey the commutation relations of the su(2) algebra:

$$[S^+, S^-] = 2S_z, \quad [S_z, S^\pm] = \pm S^\pm.$$

Here,

$$S_z = \sum_{j=1}^{N} S_{zj} = \sum_{j=1}^{N} (|2\rangle_{jj}\langle 2| - |1\rangle_{jj}\langle 1|)/2$$

is the bare-state inversion operator and N is the number of quantum dots involved. The states $|2\rangle_j$ and $|1\rangle_j$ are the respective excited and ground state of the jth qubit. Further, $a^\dagger$ and a are the creation and the annihilation operators of the electromagnetic field (EMF), which satisfy the standard boson commutation relations $[a, a^\dagger] = 1$ and $[a, a] = [a^\dagger, a^\dagger] = 0$. We suppose here that the quantum dot system is coupled to the laser and microcavity fields with the same coupling strength, i. e., the linear extension of the quantum dot ensemble is smaller than the relevant emission wavelength.

In what follows, we are interested in the laser-dominated regime where $\Omega \gg \{g, \gamma, \kappa\}$ (here γ and κ are the respective spontaneous and cavity decay rates) and describe our system using the dressed-state formalism [11, 28]:

$$|1\rangle_j = \frac{1}{\sqrt{2}}(|\bar{1}\rangle_j + |\bar{2}\rangle_j), \quad |2\rangle_j = \frac{1}{\sqrt{2}}(|\bar{2}\rangle_j - |\bar{1}\rangle_j). \quad (2)$$

Before applying transformation (2), we perform the substitution $a^\dagger e^{i\phi_1} = \tilde{a}^\dagger$ and $S^+e^{i\phi_2} = \tilde{S}^+$ and drop the tilde afterwards. Restricting ourselves to values $\Delta \ll \Omega$ and the secular approximation, we arrive at the master equation describing our system:

$$\frac{d}{dt}\rho(t) + i[H_0, \rho] = -\Gamma_0[R_z, R_z\rho] - \Gamma\{[R^+, R^-\rho] + [R^-, R^+\rho]\} - \kappa[a^\dagger, a\rho] + \text{H.c.} \quad (3)$$

Here,

$$H_0 = \Delta a^\dagger a + R_z(g_0^* a^\dagger + g_0 a) + \epsilon(a^\dagger + a),$$

where $g_0 = ge^{i\phi}/2$ and $g_0^* = ge^{-i\phi}/2$ with $\phi = \phi_1 - \phi_2$. Next, $\Gamma_0 = \gamma/4$ and $\Gamma = (\gamma + \gamma_d)/4$, with 2γ being the single-qubit spontaneous decay rate and γ_d the quantum-dot dephasing rate. The new quasispin operators

$$R^+ = \sum_{j=1}^{N} |\bar{2}\rangle_{jj}\langle\bar{1}|, \quad R^- = [R^+]^\dagger,$$

$$R_z = \sum_{j=1}^{N} (|\bar{2}\rangle_{jj}\langle\bar{2}| - |\bar{1}\rangle_{jj}\langle\bar{1}|)$$

operate in the dressed-state picture. They obey the commutation relations

$$[R^+, R^-] = R_z, \quad [R_z, R^\pm] = \pm 2R^\pm.$$

We note the dependence of the coupling strength g_0 on the phase difference of the applied coherent sources. Additional and different phase dependent effects can be found in [13, 14].

In the next subsection, we obtain the equations of motion of the variables of interest in order to calculate the second-order microcavity photon correlation function $g^{(2)}(0) = \langle a^\dagger a^\dagger a a\rangle/(\langle a^\dagger a\rangle)^2$. Values of $g^{(2)}(0)$ smaller than unity describe sub-Poissonian photon statistics and manifest a quantum effect. In the Poissonian photon statistics, $g^{(2)}(0) = 1$, $g^{(2)}(0) > 1$ characterizes super-Poissonian photon statistics. In particular, for thermal light we have $g^{(2)}(0) = 2$ and we are therefore interested in correlations larger than two, $g^{(2)}(0) > 2$.

2.1. Equations of motion

Using Eq. (3), we can obtain the following equations of motion in order to calculate the microcavity photon intensity and their second-order photon–photon correlations:

$$\begin{aligned}
\frac{d}{dt}\langle a^\dagger a\rangle &= i\epsilon(\langle a\rangle - \langle a^\dagger\rangle) + ig_0\langle R_z a\rangle - \\
&\quad - ig_0^*\langle R_z a^\dagger\rangle - 2\kappa\langle a^\dagger a\rangle, \\
\frac{d}{dt}\langle a^\dagger\rangle &= i\epsilon + ig_0\langle R_z\rangle - (\kappa - i\Delta)\langle a^\dagger\rangle, \\
\frac{d}{dt}\langle R_z a\rangle &= -i\epsilon\langle R_z\rangle - ig_0^*\langle R_z^2\rangle - \\
&\quad - (4\Gamma + \kappa + i\Delta)\langle R_z a\rangle, \\
\frac{d}{dt}\langle a^{\dagger^2} a^2\rangle &= 2i\epsilon(\langle a^\dagger a^2\rangle - \langle a^{\dagger^2} a\rangle) + \\
&\quad + 2ig_0\langle R_z a^\dagger a^2\rangle - 2ig_0^*\langle R_z a^{\dagger^2} a\rangle - 4\kappa\langle a^{\dagger^2} a^2\rangle, \\
\frac{d}{dt}\langle R_z a^\dagger a^2\rangle &= i\epsilon(\langle R_z a^2\rangle - 2\langle R_z a^\dagger a\rangle) + \\
&\quad + ig_0\langle R_z^2 a^2\rangle - 2ig_0^*\langle R_z^2 a^\dagger a\rangle - \\
&\quad - (3\kappa + 4\Gamma + i\Delta)\langle R_z a^\dagger a^2\rangle, \\
\frac{d}{dt}\langle a^\dagger a^2\rangle &= i\epsilon(\langle a^2\rangle - 2\langle a^\dagger a\rangle) + ig_0\langle R_z a^2\rangle - \\
&\quad - 2ig_0^*\langle R_z a^\dagger a\rangle - (3\kappa + i\Delta)\langle a^\dagger a^2\rangle, \\
\frac{d}{dt}\langle R_z^2 a^2\rangle &= -2i\epsilon\langle a R_z^2\rangle - 2ig_0^*\langle R_z^3 a\rangle + \\
&\quad + 16\Gamma j(j+1)\langle a^2\rangle - (2\kappa + 12\Gamma + 2i\Delta)\langle a^2 R_z^2\rangle, \\
\frac{d}{dt}\langle R_z^2 a^\dagger a\rangle &= i\epsilon(\langle a R_z^2\rangle - \langle a^\dagger R_z^2\rangle) + ig_0\langle R_z^3 a\rangle - \\
&\quad - ig_0^*\langle R_z^3 a^\dagger\rangle + 16\Gamma j(j+1)\langle a^\dagger a\rangle - \\
&\quad - (2\kappa + 12\Gamma)\langle a^\dagger a R_z^2\rangle, \\
\frac{d}{dt}\langle R_z^3 a\rangle &= -i\epsilon\langle R_z^3\rangle - ig_0^*\langle R_z^4\rangle - (24\Gamma + \kappa + i\Delta) \times \\
&\quad \times \langle R_z^3 a\rangle + 16\Gamma(3j(j+1) - 1)\langle a R_z\rangle, \\
\frac{d}{dt}\langle a^2\rangle &= -2i\epsilon\langle a\rangle - 2ig_0^*\langle R_z a\rangle - (2\kappa + 2i\Delta)\langle a^2\rangle, \\
\frac{d}{dt}\langle R_z a^2\rangle &= -2i\epsilon\langle R_z a\rangle - 2ig_0^*\langle R_z^2 a\rangle - \\
&\quad - (2\kappa + 4\Gamma + 2i\Delta)\langle R_z a^2\rangle, \\
\frac{d}{dt}\langle R_z a^\dagger a\rangle &= i\epsilon(\langle a R_z\rangle - \langle a^\dagger R_z\rangle) + ig_0\langle R_z^2 a\rangle - \\
&\quad - ig_0^*\langle R_z^2 a^\dagger\rangle - (2\kappa + 4\Gamma)\langle a^\dagger a R_z\rangle, \\
\frac{d}{dt}\langle R_z^2 a\rangle &= -i\epsilon\langle R_z^2\rangle - ig_0^*\langle R_z^3\rangle - \\
&\quad - (12\Gamma + \kappa + i\Delta)\langle R_z^2 a\rangle + 16\Gamma j(j+1)\langle a\rangle.
\end{aligned} \tag{4}$$

System of equations (4) is not complete. Additional equations are necessary for the qubit subsystem operators $\langle R_z\rangle$, $\langle R_z^2\rangle$, and so on. However, we represent the steady-state expectation values of the field correlators $\langle a^\dagger a\rangle$ and $\langle a^{\dagger^2} a^2\rangle$ via the quantum-dot operators alone. The expectation values of the quantum dot operators are evaluated in a different way, as is described in the next subsection. We note that in deriving the above system of equations, we used the relation $R_z^2/4 + (R^+R^- + R^-R^+)/2 = j(j+1)$, where $j = N/2$.

2.2. Qubit subsystem correlations

As mentioned in the preceding subsection, the steady-state values of field correlators and the qubit-field correlators can be expressed via the expectation values of the dressed-state inversion $\langle R_z^n\rangle$, $\{n \in 1, 2, 3, 4\}$. These qubit-subsystem operators can be obtained from master equation (3) by observing that any diagonal form of operators $R^{+m} R_z^n R^{-m}$, $\{m, n \in 0, 1, \ldots\}$, commutes with H_0. Therefore, the steady-state values of these operators are determined only by the dissipation part of the master equation. It is not difficult to show that the steady-state solution of the qubit-subsystem master equation is [11]

$$\rho_q = \frac{\hat{I}}{N+1}, \tag{5}$$

where $\hat{I}$ is the unity operator. We consider an atomic coherent state $|n\rangle$, which is a symmetrized

N-atom state with $N-n$ particles in the lower dressed state $|\tilde{1}\rangle$ and n atoms excited to the upper dressed state $|\tilde{2}\rangle$. We can calculate the expectation values of any atomic correlators of interest using the relations $R^+|n\rangle = \sqrt{(N-n)(n+1)}|n+1\rangle$, $R^-|n\rangle = \sqrt{n(N-n+1)}|n-1\rangle$, and $R_z|n\rangle = (2n-N)|n\rangle$. In particular, the steady-state expectation values of collective dressed-state inversion operator can be easily evaluated as

$$\langle R_z^2\rangle = \frac{N}{3}(N+2), \qquad \langle R_z^4\rangle = \frac{N}{15}(N+2)(3N^2+6N-4), \tag{6}$$

while $\langle R_z\rangle = \langle R_z^3\rangle = 0$.

In the next section, we discuss the microcavity photon statistics.

3. RESULTS AND DISCUSSION

The general expression for the second-order photon correlation function is too cumbersome and we present it analytically for a few particular cases. If $g_0 = 0$, we have $g^{(2)}(0) = 1$; if $\{\epsilon, \Delta\} = 0$, then

$$g^{(2)}(0) = \frac{3(4+\kappa/\Gamma)^2}{(4+3\kappa/\Gamma)(6+\kappa/\Gamma)\langle R_z^2\rangle^2}\left\{\frac{\langle R_z^4\rangle}{1+24\Gamma/\kappa} + \frac{8j(j+1)\langle R_z^2\rangle}{4+\kappa/\Gamma} + \frac{16(3j(j+1)-1)\langle R_z^2\rangle}{(1+4\Gamma/\kappa)(24+\kappa/\Gamma)}\right\}. \tag{7}$$

We can see here that the second-order correlation function does not depend on the microcavity–dot coupling strength g_0. This is the case also for $\epsilon = 0$ and $\Delta \neq 0$. However, in general, when $\epsilon \neq 0$, the microcavity photon correlation function depends on g_0. We note the two-photon correlator $\langle a^\dagger a^\dagger a a\rangle \propto N^4$ and the photon intensity $\langle a^\dagger a\rangle \propto N^2$, which means that we have an enhancement of these correlations due to collectivity.

In what follows, using system of equations (4) and for some particular cases of expression (7), we describe the microcavity second-order photon correlation function in detail for various parameters of interest. We proceed by considering that the microcavity mode is not additionally pumped, i. e., $\epsilon = 0$. Figure 1 shows the second-order correlation function as a function of κ/Γ for various cavity detunings and numbers of quantum dots involved. At the exact resonance $\Delta/\Gamma = 0$, we see larger photon correlations, $g^{(2)}(0) = 3$, while their intensity is also enhanced due to collectivity. The picture is different in the off-resonance case. For $\kappa \ll \Delta \neq 0$, the photon statistics is similar to that of a thermal light, with $g^{(2)}(0) = 2$. But for intermediate

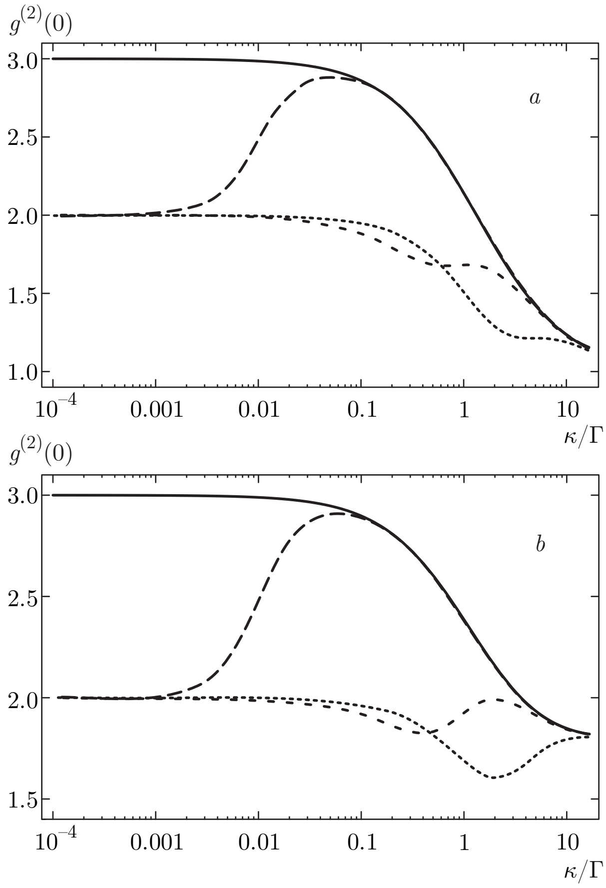

Fig. 1. Steady-state dependence of the microcavity second-order photon correlation function $g^{(2)}(0)$ on κ/Γ for $\epsilon = 0$. The solid line is for $\Delta/\Gamma = 0$, the long-dashed line is for $\Delta/\Gamma = 0.01$, the short-dashed curve corresponds to $\Delta/\Gamma = 1$, and the dotted line to $\Delta/\Gamma = 4$. (a) $N = 1$ and (b) $N = 20$

detunings, we see an oscillatory behavior of the second-order correlation function due to the interplay of κ and Γ. As the detuning is increased further, the two-photon correlation shows a dip because of the off-resonant driving (see Fig. 1). Figure 1b does not change if the number of quantum dots is increased further.

To better understand the steady-state behaviors of the photon–photon correlation function for $\epsilon \neq 0$, we plot $g^{(2)}(0)$ again in Fig. 2. In this case, we observe a dependence of the normalized second-order correlation function $g^{(2)}(0)$ on the phase difference ϕ of the applied coherent sources. It is easy to show that the microcavity photon intensity (see Eqs. 4)

$$\langle a^\dagger a\rangle = \frac{\epsilon^2}{\kappa^2+\Delta^2} + \frac{(\kappa+4\Gamma)|g_0|^2\langle R_z^2\rangle}{[(\kappa+4\Gamma)^2+\Delta^2]\kappa} \tag{8}$$

does not depend on ϕ in this particular case. Therefore, the phase difference appears in the unnormalized

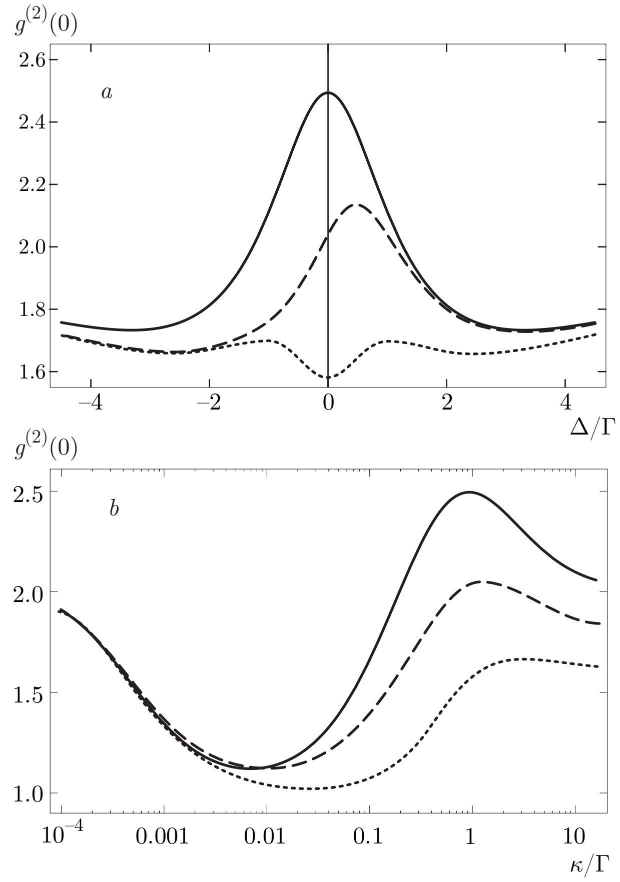

Fig. 2. Steady-state dependence of the microcavity second-order photon correlation function $g^{(2)}(0)$ on (a) Δ/Γ and (b) κ/Γ. The solid line is for $\phi = 0$, the long-dashed line corresponds to $\phi = \pi/4$, and the short-dashed curve to $\phi = \pi/2$. Here, $\epsilon/\Gamma = 20$, $g/\Gamma = 10$, and $N = 20$. Other parameters are: (a) $\kappa/\Gamma = 1$ and (b) $\Delta/\Gamma = 0.01$

second-order correlator $\langle a^\dagger a^\dagger aa \rangle$. This happens due to the possibility of the scattering of two photons from different applied coherent sources giving rise to interference, i. e., phase-dependent effects. In Fig. 2a, we can see an asymmetric steady-state behavior of the second-order correlation function for $\phi = \pi/4$. Furthermore, the maximum at $\Delta/\Gamma = 0$ for $\phi = 0$ turns into a minimum for $\phi = \pi/2$ (see the solid and short-dashed curves in Fig. 2a). Thus, the relative phase between the applied coherent sources can be a convenient tool to manipulate the photon statistics. In particular, we can generate coherent light, $g^{(2)}(0) \approx 1$, despite spontaneous incoherent photon scattering into the cavity mode (see Fig. 2b). Again, the photon intensity and second-order correlations are enhanced due to collectivity.

4. SUMMARY

In summary, we have investigated the interaction of a collection of laser-pumped artificial atoms embedded in a leaking optical microcavity. In particular, we were interested in photon statistics of photons scattered into the cavity mode. We have found that the photon statistics depends on the phase difference between the coherent sources pumping the quantum dot system and the cavity mode. Various steady-state behaviors of photon correlations were shown to occur.

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