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PHASE SLIPS, DISLOCATIONS, HALF-INTEGER VORTICES, TWO-FLUID HYDRODYNAMICS, AND THE CHIRAL ANOMALY IN CHARGE AND SPIN DENSITY WAVES

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Abstract. This brief review recalls some chapters in theory of density waves in quasi-1D electronic systems which may have appeared after inspirations from studies of I. E. Dzyaloshinskii and collaborations with him. First we address the spin density waves which rich order parameter allows for an unusual object of a complex topological nature: a half-integer dislocation combined with a semi-vortex of the staggered magnetization. It becomes energetically preferable with respect to an ordinary dislocation due to the high Coulomb energy at low concentration of carriers. Generation of these objects should form a sequence of π-phase slips in accordance with experimental doubling of phase-slips rate. Next, we revise the commonly employed time-dependent Ginzburg–Landau (TDGL) approach which is shown to suffer from a violation of the charge conservation law resulting in nonphysical generation of particles which is particularly pronounced for electronic vortices in the course of their nucleation or motion. The suggested consistent theory exploits the chiral transformations taking into account the principle contribution of the fermionic chiral anomaly to the effective action. The derived equations clarify partitions of charges, currents and rigidity among subsystems of the condensate and normal carriers and the gluing electric field. Being non-analitical with respect to the order parameter, contrarily the conventional TDGL type, the resulting equations still allow for a numerical modeling of transient processes related to space- and spatiotemporal vorticity in density waves (DWs).

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1. Introduction.

1.1. Inspirations from I. E. Dzyaloshinskii.

The authors had a chance to publish together with I. E. Dzyaloshinskii (I. E. D. in the following) the article [1] on doubly-quasi-periodic solitonic lattices emerging in a 1D electronic system under simultaneous effects of the charge doping away from the half band filling and of the spin polarization. This publication was in the course of our studies in theory of charge DWs (CDWs) which had been started in 1976 by one of us (S. B., a thankful disciple of I. E. D.) under the inspiration and initially with participation [2,3] of I. E. D. The trick of the chiral transformations having been emploied in [2] (see below), actually suggested by I. E. D., provided a handy frame to study adiabatic models of CDWs which later would lead S. B. to notice an instability of normal electrons or holes, excited or injected to a CDW, towards formation of solitons with the electron buried at the solitons' midgap state, see [4] for a review. The resulting physics of microscopic solitons and their arrays [5] can be traced back to I. E. D. invention of the commensurability locking (published neglectfully only in conference proceedings [6]) and forth to his work on exact solutions of many-body lattice models [7–10]. Much later, an indirect inspiration came to us [11, 12] from the article [13], where I. E. D. had noticed that

a presence of a dislocation in a crystal possessing the antiferromagnetic spin order enforces a curious half-integer vortex of spins' rotations.

In this short review, we shall firstly describe combined topological defects — half-integer vortices of displacements and spin rotations in incommensurate spin DWs (SDWs) which might necessarily appear under applications of the driving electric field. These considerations are related to I. E. D. study [13] and are coherent with a persistent interest in fractional vortices: from helium A ([14, 15] about the same time as [13]) and triplet superconductors (reviews [16, 17]) to FFLO phase [18, 19] and Bose condensate of polaritons [20] (see also Ref. [21] for a more recent review). This direction in general is in line with long standing interests of I. E. D. in topological defects, recall [22] and particularly [23].

Next, we shall quote and augment for modelling quite a recent development [24] on construction of the chirally invariant (recall [2]) description of transient processes in a CDW or SDW in presence of normal carriers. This formulation is free from a drawback of the non-conservation of condensed particles which we show to be inherent to the traditional TDGL approach.

1.2. Significance of static and transient topological configurations in incommensurate sliding density waves. The DWs, see reviews [25–28], are seen as superstructures, usually weak (with an amplitude $A \ll 1$) and hence harmonic ($A\cos[\mathbf{q}_0\cdot\mathbf{r}+\varphi(\mathbf{r})]$, q_0 and φ are the superstructure wave vector and phase). The DWs are produced by modulations of electronic charges and atomic displacements or of electronic spins for CDWs or SDWs, correspondingly. The spectacular phenomena is the collective Fröhlich conductivity due to the overall sliding with the collective current proportional to the phase velocity, $j_c \propto \partial_t\varphi$. The DW sliding is ultimately related to the current conversion process which necessarily involves topological defects [29] like solitons, dislocation lines/loops, and the transient processes of phase slips which develop as the edge dislocation line (D-line) proliferating/expanding across the sample [30,31] or as the plane of the DW amplitude vanishing across a narrow sample [32–35].

The DW deformations generate a local charge density $n_c \propto \partial_x\varphi$ which brings about a high cost of the Coulomb energy. The Coulomb enhancement of the dislocation energy plays an intriguing role in SDWs bringing to life a kind of a mixed topological object: a half-integer dislocation combined with a semi-vortex of a staggered magnetization vector. The phase slips, necessary for the current conversion or the depinning, should proceed via propulsion of these combined objects which provides a natural interpretation for a confusing 2-fold enhancement of a frequency generated by moving SDWs in comparison with CDWs.

Beyond transient dislocations contributing to phase slips, static arrays of dislocations can appear in lateral geometries when the electric field is applied transversely to the direction x of the CDW sliding. Indeed, the interaction energy $e\Phi\partial_x\varphi/\pi$ of the deformed CDW with the electric potential Φ resembles the one for a superconductor under a magnetic field described by the vector potential with the component $A_x \equiv \Phi$. Then the transverse to the chains electric field $E_y = -\partial_y\Phi$ acts upon the CDW phase analogously to the action of the transverse magnetic field in a superconducting film. Hence, an array of static dislocations should appear above the electric field threshold as the vortex lattice above the critical field H_{c1} in a superconductor.

Figures 1 and 2 illustrate the sequences of phase slips and the static dislocations. The plots were obtained from numerical modelling of a TDGL type equations as it has been described elsewere [24, 36, 37].

2. Combined half-integer dislocation and the magnetization vortex in a spin density wave.

CDW and SDW are characterized by scalar and vector order parameters: $\eta_{cdw}(\mathbf{R}) = A\exp(i\varphi)$ and $\boldsymbol{\eta}_{sdw} = A\mathbf{m}\exp(i\varphi)$, where $\mathbf{m}$ is the unit vector of the staggered magnetization and $\mathbf{R} = (x, \mathbf{r}_\perp)$ with x being the chain direction.

The energy density of spin rotations $\mathbf{m}(\mathbf{R})$ in SDWs is not affected by Coulomb forces:

$$W_{spin}\{\mathbf{m}\} = \frac{1}{2N_F}\Big[\tilde{C}_\parallel\,(\partial_x\mathbf{m})^2 + \\ + \tilde{C}_\perp\left((\partial_y\mathbf{m})^2 + (\partial_z\mathbf{m})^2\right) + W_{sa}\Big]. \quad (1)$$

Here $N_F = 2/\pi\hbar v_F$ and v_F are the density of states and the Fermi velocity of the parent metal, $\tilde{C}_\parallel$ and $\tilde{C}_\perp$ are the elastic moduli related to the rotation of the staggered magnetization unit vector $\mathbf{m}$, and W_{sa} is the spin anisotropy energy. Elastic moduli $\tilde{C}_\parallel$ and $\tilde{C}_\perp$ are similar to bare moduli of phase displacements taken without Coulomb interactions, i. e., $C^0_\parallel$ and $C_\perp$ in Eq. (2) below.

The energy for deformations related to phase displacements in both CDW and SDW takes a form

$$W_{chrg}\{\varphi\} = \frac{1}{2N_F}\left[C^0_\parallel\left(\frac{1}{\pi}\,\partial_x\varphi\right)^2 + C_\perp\left(\frac{1}{\pi}\,\nabla_\perp\varphi\right)^2\right] + \\ + W_C\ + W_{str}, \quad (2)$$

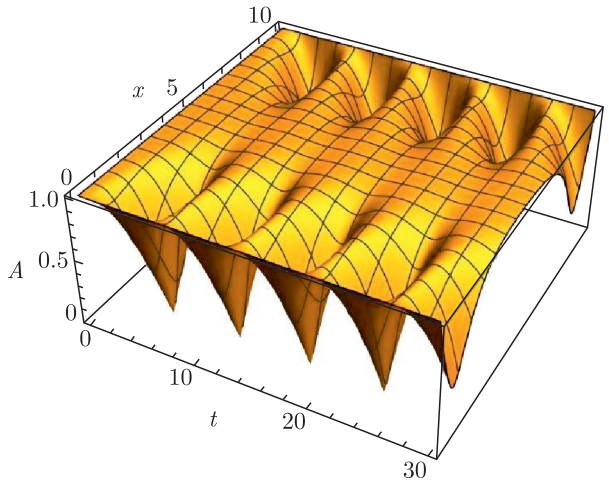

Fig. 1. A time-periodic sequence of phase-slip events seen as a set of nodes of the DW amplitude $A(x,t)$

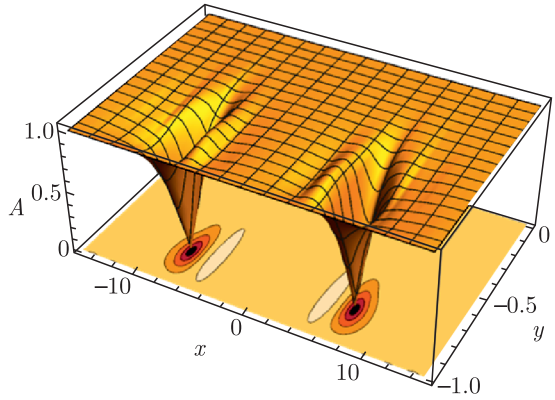

Fig. 2. A static array of dislocations appearing in the electric field applied transversely to the direction x of DW displacements

where dimensionless parameters $C_\parallel^0$ and $C_\perp$ are the normalized compression and share moduli; $C_\perp \propto \propto (AT_cN_Fa_\perp)^2$ is a measure of the interchain coupling related to the transition temperature T_c ($a_\perp$ is the interchain distance); W_{str} is the stress energy from an applied electric potential and/or from disbalance of normal carriers which promote deformation and/or the motion of the DW.

The Coulomb part of the energy, W_C, comes from the local charge density related to the DW displacements: $n_c = e\rho_c\partial_x\varphi/\pi a_\perp^2$, where ρ_c and $\rho_n = 1-\rho_c$ are the normalized densities of the condensate and of the normal carriers. The Coulomb interactions drastically affect the charged phase deformations of dislocations greatly increasing their energy and stretching the shape in the chains' x-direction, as we shall remind below. Vaguely, the combined effect of Coulomb interactions and the screening results in hardening of the effective compressibility [38] which vanishes at T_c and diverges at low T with freezing out of normal carriers. The effective compressibility $C_\parallel$ hardens with growing $r_\perp$ (starting from $C_\parallel^0$ at shortest interchain distances) as $C_\parallel \sim r_\perp^2/r_0^2$ beyond the screening length of the parent metal, $r_\perp > r_0 \sim 1\,\text{Å}$, until it saturates beyond the screening length $r_{scr} = r_0/\sqrt{\rho_n}$ at the value which grows activationally with lowering T. At $r_\perp > r_{scr}$

$$C_\parallel^0 \Rightarrow C_\parallel = \rho_c/\rho_n,$$

$$C_\parallel \propto \rho_c \ \propto A^2 \propto (T_c - T) \text{ at } T \to T_c$$

and

$$C_\parallel \propto \rho_n^{-1} \propto \exp(\Delta/T) \text{ at } T \to 0.$$

The resulting big energy of dislocations at small ρ_n does not prevent their creation which just requires for bigger applied potentials, but the Coulomb energy changes drastically the energy dependence of the dislocation on its position Y (with respect to a counterpart or a surface). While the spin-vortex energy per its unit length is a standard $W_V \sim T_c\ln(Y/a_\perp)$, for the dislocation as the charged phase vortex, this form of the energy is reached only when the Coulomb interaction is screened at $Y \gg r_{scr}$, where $W_D \sim \sim T_c(r_{scr}/r_0)\ln(Y/r_{scr})$ with the energy scale being greatly enhanced as r_{scr}/r_0.

Coulomb interactions become even more important in an intermediate (wide at low T) region $r_0 < r_\perp < < r_{scr}$ governed by the nonlocal, due to unscreened Coulomb interactions, elasticity with $C_\parallel \sim r_\perp^2/r_0^2$. For dislocations with $r_{scr} \gg Y \gg r_0$, a curious confinement law is established with $W_D \sim T_cY/r_0$ meaning a constant force acting upon the D-line.

The energies $W_V(Y)$ and $W_D(Y)$ are similar only near T_c at small A (hence, at the vanishing gap Δ) when the abundant free carriers screen Coulomb interactions already at shortest distances. Otherwise, at the developed gap $\Delta > T$, the energy of dislocations is greatly enhanced with respect to that of vortices which brings about the option of their splitting into combined half-integer vortices. In SDWs, the Coulomb enhancement of the dislocation energy plays a principal role bringing to life a special combined topological object: the half-integer dislocation $\varphi \to \varphi + \pi$ accompanied by the 180° rotation $\mathcal{O}_\pi$ of the staggered magnetization $\mathbf{m} \to -\mathbf{m}$. Indeed, the SDW order parameter $\boldsymbol{\eta} = \mathbf{m}\cos(Qx+\varphi)$ allows for the following three types of self-mapping $\boldsymbol{\eta} \to \boldsymbol{\eta}$ associated with vorticities ν_φ and ν_m.

(i) Normal dislocation $D_{2\pi}$: $\varphi \to \varphi + 2\pi \equiv \varphi$ and $\mathbf{m} \to \mathbf{m}$; $\nu_\varphi = 1$ and $\nu_m = 0$.

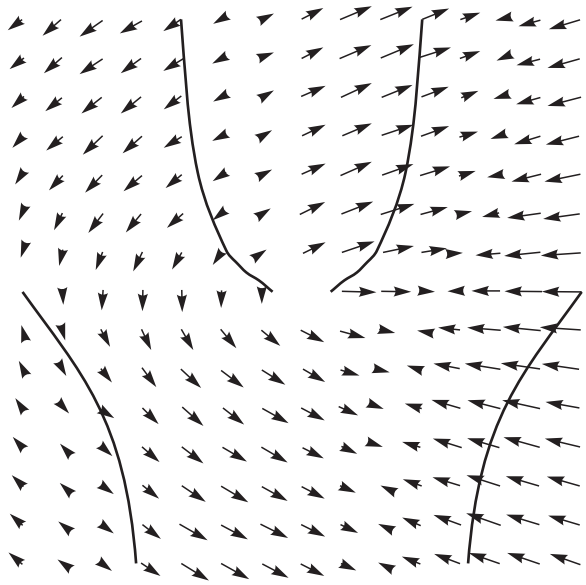

Fig. 3. Vector-field $\mathbf{m}$ for the half-integer dislocation combined with the semi-vortex of the magnetization. Solid lines indicate constant DW phases. Due to the presence of the half-integer dislocation, the number of sites changes between the upper and the lower rows from 2.5 periods $(6+6+3)$ to 2 periods $(7+5)$

(ii) Normal $\mathbf{m}$-vortex $V_{2\pi}$: $\mathbf{m} \to \mathcal{O}_{2\pi}\mathbf{m} \equiv \mathbf{m}$ and $\varphi \to \varphi$; $\nu_\varphi = 0$ and $\nu_m = 1$.

(iii) Combined object $D_\pi V_\pi$: $\varphi \to \varphi \pm \pi$ and $\mathbf{m} \to \mathcal{O}_\pi\mathbf{m} = -\mathbf{m}$; $\nu_\varphi = \pm 1/2$ and $\nu_m = \pm 1/2$.

In the last case, both the orientational factor $\mathbf{m}$ and the translational one $\cos(Qx+\varphi)$ change the sign, but their product in $\boldsymbol{\eta}$ stays invariant, as it is demonstrated schematically in Fig. 3. Remind that at a given position, the energies of vortices depend on their winding numbers proportional to $\nu^2_{\varphi,m}$

We must compare the energies of objects (i) and (iii) under the requirement of the charge conservation, i. e., preserving the total phase vorticity $\sum \nu_\varphi$. For magnetic vortices the total vorticity $\sum \nu_m$ does not need to be conserved but in the bulk it must be kept zero; otherwise, the energy of noncompensated vortices diverges logarithmically at large distances. For phase dislocations, the energy divergence is not a limitation since it is compensated by driving potentials. Hence, the only decomposition path for the conventional dislocation of the case (i) to the two pairs of half-vortices of the case (iii) is $D_{2\pi} \Rightarrow \{D_\pi, V_\pi\} + \{D_\pi, V_{-\pi}\}$. If the energy parameters for both D and V were the same, then the dissociation cost is zero: $\nu^2_{2\pi} = 1 \Rightarrow 4\nu^2_\pi = 1$, and the result will depend on a tiny balance of similar coefficients. But with dominating Coulomb energy of dislocations,

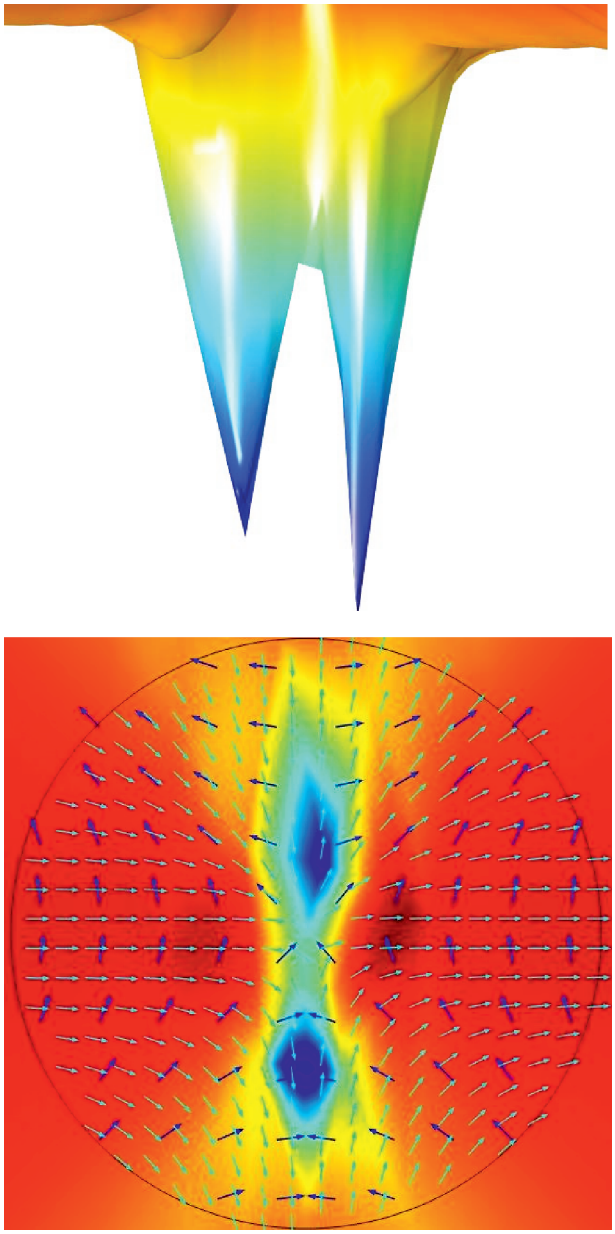

Fig. 4. Results of the modeling of the evolution from a single integer dislocation to the split pair of half-integer combined vortices: (upper panel) the order parameter amplitude $A(x,y)$; (lowere panel) vector fields of the CDW phase and of the spin rotation on the background of the density plot for the amplitude

as expected at low T, the magnetic vortex energy can be neglected, then the decomposition gains nearly half of the energy, $(\nu_{2\pi} = 1)^2 \Rightarrow 2(\nu_\pi = 1/2)^2 = 1/2$, which makes it inevitable.

Figure 4 demonstrates results of our modeling of the evolution from a single phase-only integer vortex (the dislocation) to the split pair of half-integer combined vortices.

The known SDW crystals possess low symmetries which originates the spin anisotropy in all three directions. Being small, the anisotropy will not affect the arrangement in a vicinity of the D-line, but at large distances from the D-line, the π-rotation of spins will be concentrated in space within the Neél domain wall, as it has been outlined already in [13]. The wall will form a string (a plane in 3D) which tempts to confine the two combined objects.

Unusually, there will be no confinement within a quite wide region $r_\perp < r_{scr}$, where the linear law, rather than conventional logarithmic one, for the D-line energy takes place as we have sketched above. Here, the total energy gain with respect to the normal D-line is $-E_C N/2$, where $E_C \sim T_c a_\perp/r_0$ and $N = Y/a_\perp$ is the number of chains separating combined vortices. This repulsive anti-confinement energy directly overruns the energy lost due to domain wall formation $W^A_{spin} = w^A N$, both having the similar N-dependence. Usually $w^A \sim 1$ K/chain $\ll E_C \sim 10^1$ K, so the net interaction between the two semivortices is strongly repulsive.

Beyond the screening length $r_\perp > r_{scr}$, the Coulomb energy slows down while the W^A_{spin} keeps growing linearly, then the total energy gain of two objects with respect to one D-line is

$$W = -E_0 \ln N + w^A N, \quad E_0 \sim T_c \frac{r_{scr}}{r_0} = \frac{T_c}{\sqrt{\rho_n}}.$$

Hence there is an equilibrium distance between the semi-vortices, $N_{eq} \sim E_0/w^A \propto 1/\sqrt{\rho_n}$, which diverges with freezing out of the screening when $\rho_n \to 0$. Already at accessibly low temperatures, the string length may reach the sample width which is typically about 1 μm.

3. Two-fluid hydrodynamics for collective and normal variables in density waves.

Related cases of vortices in superconductors and dislocations in DWs [32–34] are described usually within GL-like models and their time-dependent (TDGL) generalizations. While being consistent for the explicitly gauge-invariant theory of superconductors [39, 40], this approach fails for DWs and other electronic crystals. Working at all steps explicitly with interfering order parameter and normal electrons, employing the chiral invariance, and taking the account for the related quantum anomaly have allowed us [24] to construct the treatable theory without descending to the burdens of microscopic calculations.

The equations of motion for the CDW phase certify the local equilibrium of forces (elastic, Coulomb, frictional) with no relation to the charge conservation. Instead, the last is usually preserved automatically by construction of the charge and the current densities:

$$n_c = \frac{1}{\pi}\,\rho_c \partial_x \varphi, \quad j_c = -\frac{1}{\pi}\,\rho_c \partial_t \varphi \tag{3}$$

(to be compared with $j_{sc} \propto \rho_{sc}\partial_x \vartheta$ for the current dependence on the phase ϑ in superconductors).

The normalized (to $T = 0$) condensate density or the phase rigidity $\rho_c(A)$ has a property that $\rho_c(0) = 1$ and $\rho_c(A) \sim A^2 \to 0$ at $A \to 0$. The expressions (3) can ensure the charge conservation automatically indeed, but only if the CDW amplitude is invariable: $A(t, x) = \text{const}$, otherwise

$$\frac{dn_c}{dt} = \partial_t n_c + \partial_x j_c = \frac{1}{\pi}\,(\partial_x \rho_c \partial_t \varphi - \partial_t \rho_c \partial_x \varphi) \neq 0. \tag{4}$$

The charge conservation $dn_c/dt = 0$ is violated in the current-carrying state ($\partial_t \varphi \neq 0$) if ρ_c is not space homogeneous ($\partial_x \rho_c \neq 0$) and in the strained (charged) state $\partial_x \varphi \neq 0$ if ρ_c varies in time, $\partial_t \rho_c \neq 0$. It is ultimately necessary to take into account the normal carriers explicitly, without integrating them out prematurely. At first sight, that would require for descending to the fully microscopic theory with its notorious complications even in linear or gapless regimes [33, 41]. Still, there is a way [24] to keep the phenomenology which is based on the knowledge of chiral invariance and transformation, importantly taking into account the chiral anomaly.

The Lagrangian $H - i\hbar\partial_t$ for electrons in 1D CDW has a form

$$\mathcal{L} = \begin{pmatrix} -i\hbar\partial_t - i\hbar v_F \partial_x + \Phi - v_F A_x & \Delta e^{i\varphi} \\ \Delta e^{-i\varphi} & -i\hbar\partial_t + i\hbar v_F \partial_x + \Phi + v_F A_x \end{pmatrix}, \tag{5}$$

where Φ and A_x are the scalar and the vector potentials. The chiral transformation $\psi_\pm \longrightarrow \psi_\pm \exp(\pm i\varphi/2)$ [2] brings the wave function Ψ to the local frame of a distorted CDW phase. It eliminates the phase factors in non-diagonal elements in $\mathcal{L}$, but in expense of additional parts in Φ and A_x:

$$\begin{gathered} \Phi \longrightarrow V = \Phi + \frac{\hbar v_F}{2}\,\partial_x \varphi, \\ A_x \longrightarrow A^*_x = A_x + \frac{\hbar}{2v_F}\,\partial_t \varphi, \end{gathered} \tag{6}$$

$$E_x \longrightarrow E_x^* = -\partial_x V + \partial_t A_x^* = \\ = E_x - \frac{\hbar v_F}{2}\left(\partial_x^2 - \frac{1}{v_F^2}\,\partial_t^2\right)\varphi. \quad (7)$$

The additions to Φ and A_x from x- and t-derivatives of the phase are naturally interpreted as the Fermi energy shift $\delta E_F = v_F \delta P_F$ following the Fermi momentum shift $\delta P_F = \hbar\partial_x\varphi/2$ under the CDW phase deformations. Under the CDW phase deformation and the applied electric field E_x, the electrons experience the chiral invariant potentials V and A_x^* and the longitudinal force E_x^*.

At first sight, we have arrived at the transparent picture of a 1D Dirac semiconductor with the gap $2\Delta = 2A\Delta_0$ under the effective electric field (7), and it looks straightforward to exclude the fermions to arrive at an effective action $S\{\Phi, \varphi, A\} = \int W dx\, dt$. And here we arrive at puzzling contradictions.

(i) In view of Eq. (7), the free energy W should contain the potential and the phase only in the invariant combination V. Having choosen the phase as $(\hbar v_F/2)\partial_x\varphi = -\Phi$, the effective potential V disappears from the action, then no density and no polarization are perturbed with respect to Φ. This contradicts to basic properties of the CDW in both the semiconductor with respect to electrons and the metal in the collective behavior.

(ii) We definitely expect W to contain the term proportional to $\rho_c(\partial_x\varphi)^2$, where ρ_c is a collective density responsible for the phase rigidity. But treating the Lagrangian (5) perturbatively with respect to the effective electric field (7), we evidently should get

$$\delta W_n = -\frac{\epsilon(k,\omega)}{8\pi e^2}(E_x^*)^2 = \\ = -\frac{\epsilon(k,\omega)}{8\pi e^2}\left(-\frac{\hbar v_F}{2}\,\frac{\partial^2\vartheta}{\partial x^2} + E_x\right)^2, \quad (8)$$

where ϵ is the dielectric susceptibility as a function of the wave number k and the frequency ω (the last will be neglected here for shortness). At small k,

$$\epsilon = \epsilon_\Delta + \frac{1}{(lk)^2}, \quad \frac{1}{l^2} = \frac{\rho_n}{r_0^2}, \quad \rho_n = \frac{dn}{d\zeta}N_F^{-1}. \quad (9)$$

Here n is the concentration of free electrons related to their chemical potential ζ and ρ_n is the normalized density of states which also will be found to be the normal density. The dielectric constant $\epsilon_\Delta \propto (r_0\xi_0)^{-2}$ collects the polarization of electrons gapped by the CDW while ρ_n comes from conducting carriers thermally excited or injected [42] above the gap providing the finite screening length $l = r_0/\sqrt{\rho}_n$. Both contributions to δW_n bring drastic contradictions.

a) The term with ϵ_Δ in Eq. (9) yields to (8) the forth order gradients of the phase $(\partial_{xx}^2\varphi)^2$ instead of the expected second order $(\partial_x\varphi)^2$.

b) The singularity in k in the term with ρ_n seems at first to serve fortunately by canceling excess gradients leading to the contribution proportional to $(\partial_x\varphi)^2$: $\delta W_{\rho n} = -\rho_n(\hbar v_F/4\pi)\,(\partial_x\varphi + \pi N_F\Phi)^2$. This expression brings two confusions with respect to the expected $W_{\rho c} = \rho_c(\hbar v_F/4\pi)\,(\partial_x\varphi)^2$. The signs in expressions for $W_{\rho n}$ and $W_{\rho c}$ are opposite, and also temperature dependencies are conflicting among coefficients ρ_n (expected to rise from zero at $T = 0$ up to 1 at T_c) and ρ_c (expected to fall to zero at T_c starting from 1 at $T = 0$).

These contradictions can be traced back to the notion of the chiral anomaly, and they can be cured by properly taking this anomaly into account. The chiral anomaly is ubiquitous in the premature linearization of electronic spectra. First, in the course of the linearization, the control is lost of the position of the bottom of the electronic band, hence of the difference between actions of the external potential Φ and of the Fermi energy shifting δE_F which wrongly appear to be additive. Moreover, the whole energy cost of the chiral transformation δW_{CT} is lost. This energy can be captured from the non-linearized spectrum of the normal metal if we consider the chiral transformation perturbation $\delta n_{CT} = \partial_x\varphi/\pi$ as a redistribution of the total particle density accompanying this transformation:

$$\delta W_{CT} = \frac{\hbar v_F}{4\pi}(\partial_x\varphi)^2 + \frac{\Phi}{\pi}\partial_x\varphi, \quad (10)$$

where the first term is the density perturbation cost $(\delta n_{CT})^2/2N_F$ and the second term is its potential energy $\delta n_c\Phi$. Beyond these physical arguments [43, 44], the derivation of the chiral anomaly in the spirit of the field-theory procedure of regularization of fermionic determinants was demonstrated for CDWs [45] and for SDWs [45, 46], recall also the special field-induced SDWs [47].

Bringing together the nonperturbative contribution (10) and the perturbative one (8), we get

$$W_{tot} = \delta W_{\rho n} + \delta W_{CT} = \\ = \rho_c\left(\frac{\hbar v_F}{4\pi}(\partial_x\varphi)^2 + \frac{\Phi}{\pi}\partial_x\varphi\right) - \rho_n\frac{\Phi^2}{\pi\hbar v_F}, \quad (11)$$

where $\rho_c = 1 - \rho_n$. The above equation correctly manifests the expected dependencies in T and A yielding also the important relation $\rho_c + \rho_n = 1$.

The total charge density becomes

$$n_{tot} = \frac{\partial W_{tot}}{\partial \Phi} = \frac{1}{\pi}\rho_c \partial_x \varphi - \rho_n \Phi N_F = \\ = \frac{1}{\pi}\partial_x \varphi - \rho_n V N_F. \quad (12)$$

The above illustrative discussion was valid in lowest bilinear approximation in gradients and potentials for the constant amplitude A. Below, we shall suggest a general nonlinear scheme necessary for modeling configurations with vortices.

The above relations written for the limit $A = \text{const}$ and for small deviations of n_e and n_h can be generalized by extending the energy functional as $\int W\, dx$ with

$$W = \frac{\hbar v_F}{4\pi}\left[\kappa_x(\partial_x A)^2 + \kappa_\perp(\nabla_\perp A)^2 + \kappa_\perp A^2(\nabla_\perp \varphi)^2\right] + \\ + \left\{\frac{\hbar v_F}{4\pi}(\partial_x \varphi)^2 + \frac{1}{\pi}\Phi\partial_x\varphi\right\} + \left(\Phi + \frac{\hbar v_F}{2}\partial_x\varphi\right) n + \\ + F(A, n_e, n_h) - \frac{\epsilon_{host} a_\perp^2}{8\pi}(\nabla\Phi)^2, \quad (13)$$

where ϵ_{host} is the dielectric permittivity of the host crystal.

Here the parameter $\kappa_\perp$ is the CDW share modulus coming from the interchain coupling of CDWs, and the on-chain rigidity $\kappa_x \sim 1$ (to be put $\kappa_x = 1$); $F(A, n_e, n_h)$ is a free energy as a function of the normalized gap value $A = \Delta/\Delta_0$ and of the concentration of normal carriers: electrons n_e and holes n_h with $n = n_e - n_h$. The equilibrium value A_{eq} is connected with n_e and n_h via the minimum of $F(A, n_e, n_h)$ in such a way that $A_{eq}(n_e, n_h)$ vanishes when $n_{e,h}$ or, better say, their chemical potentials $\pm\zeta$ surpass critical values, hence the metallic phase with $A = 0$ is restored. The terms in brackets $\{\ldots\}$ are originated by the chiral anomaly of Eq. (10) coming from background deformations of the CDW phase, the next term (proportional to n) comes from the energy of intrinsic electrons in the combined potential V.

Assuming the dissipative regime for both φ and A, functional derivatives of (13) yield equations for the time evolution and the Poisson equation for Φ:

$$\partial_x^2 A + \kappa_\perp \nabla_\perp^2 A + \kappa_\perp A(\nabla_\perp\varphi)^2 - \frac{\partial F}{\partial A} = \gamma_A \partial_t A, \quad (14)$$

$$\partial_x^2\varphi + \pi N_F \partial_x \Phi + \pi\partial_x n + \\ + \kappa_\perp \nabla_\perp\left(A^2 \nabla_\perp \varphi\right) = \gamma_\varphi \partial_t, \quad (15)$$

$$\frac{1}{\pi}\,\partial_x\varphi + n = -N_F r_0^2 \nabla^2 \Phi. \quad (16)$$

Here $\gamma_\varphi = \gamma A^2$, $\gamma = \text{const}$ and $\gamma_A = \text{const}$ are the damping coefficients; γ_φ is related to the sliding CDW conductivity σ_{CDW} as $\gamma_\varphi \sigma_{CDW} = N_F e^2/a_\perp^2 = 1/4\pi r_0^2$.

In spite of a superficial similarity, Eqs. (14)–(16) show striking differences with respect to commonly use TDGL ones. Thus, in Eq. (14) the conventional term $\kappa_\perp A(\nabla_\perp\varphi)^2$ does not find its partner with the x-derivative of φ as if the longitudinal gradient of φ does not suppress the amplitude — the phase-slip nodes would not appear then. In Eqs. (15) and (16), the terms containing $\partial_x\varphi$, $\partial_{xx}\varphi$, and $(\partial_x\varphi)^2$ are not multiplied by A^2, so the attempt to present them as derivatives of Ψ will bring singularities proportional to $1/A$ and $1/A^2$. Contrary to conventional GL-type equations, now Eqs. (14)–(16) are nonanalytic in the order parameter $\eta = A\exp(i\varphi)$ or, in other words are singular in its amplitude. Nevertheless, there are hidden cancellations allowing to compensate for singularities, even if implicitly, which is vitally important for allowance of space- and spaciotemporal vortices.

There are some technical challenges in numerical implementations of the quoted above equations which one commonly does not meet working within conventional GL approaches, see, e. g., Refs. [35,48]. The first is the control of compensations at $A \to 0$ in expressions for total charges, currents, and the condensate energy bringing to action the hidden function of the condensate density ρ_c. The second is the entanglement in dependencies of thermodynamic functions and their derivatives on A and on n or ζ: approaching of n or ζ to critical values should eliminate the energy minimum over A at $A \neq 0$ opening the metallic state, e. g., in the vortex core.

The best, and may be the only analytically transparent, advancing is possible with the simplest Landau type expression for F:

$$F(n, A) = \frac{n^2}{2N_F} + \frac{1}{2}\left[-\tau + \left(\frac{n}{n_{cr}}\right)^2\right] \times \\ \times (A\Delta_0)^2 N_F + \frac{b}{4}A^4\Delta_0^2 N_F, \quad (17)$$

where $a, b \sim 1$. In Eq. (17), the first term is the contribution of the normal metal, while two other terms give the Landau type expansion in the order parameter with the proximity $\tau \propto (1 - T/T_c)$ to the transition temperature being shifted by presence of the normal carriers which critical concentration is n_{cr}.

We have performed the numerical modeling employing the following combination of equations: Eq. (13) for A with F given by Eq. (17), Eq. (14) for φ, and the dissipative equation for $\mathbf{m}$ generated by the func-

tional (1). The vortices can be spontaneously generated only if equations are written in invariarian variables, rather than in the economical form for non-unique phase φ and the angle θ. The order parameter $\boldsymbol{\eta}_{SDW} = A\mathbf{m}\exp(i\varphi)$ is written as $\boldsymbol{\eta}_{SDW} = (u+iv)\{p,q\}$. For the CDW or for frozen alligned spins in SDW, the invariant variables are just u and v. For a SDW with a spin vorticity, the allowed varibles were taken as a set of four bilinear combinations $\{\alpha,\beta,\gamma,\delta\} = \{up, uq, vp, vq\}$ imposing the apparent constraint $\alpha\delta = \beta\gamma$. Examples of resulting calculations are shown in Figs. 1, 2, and 4.

4. Conclusions.

A necessity of semi-vortices in conventional antiferromagnets in presence of frozen-in host lattice dislocations was understood already by I. E. D. [13]. In the SDW, the semi-vortices become the objects of the lowest energy created in the course of phase slip process; the normal dislocation must split into two objects of the combined topology with the repulsion between them. The combined topological objects, where the spin rotations are coupled to DW displacements, are stabilized by lowering the Coulomb energy of dislocations. This combination effectively reduces the SDW period.

Exploiting the chiral transformation and understanding the role of the chiral anomaly allows formulating a phenomenological theory in terms of equations for the DW complex order parameter, the electric potential, and the concentration of normal carriers. This approach resolves the problem of violation of the conservation law for condensed carriers rising dangerously for nonstationary inhomogeneous regimes; that allows to model consistently such strongly nonlinear effects as phase slips and nucleation and propagation of phase vortices.

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REFERENCES

1. S. Brazovskii, I. Dzyaloshinskii, and N. Kirova, Sov. Phys. JETP **54**, 1209 (1981).

2. S. Brazovskii and I. Dzyaloshinskii, Sov. Phys. JETP **44**, 1233 (1976).

3. S. Brazovskii, I. Dzyaloshinskii, and S. P. Obukhov, Sov. Phys. JETP **45**, 814 (1977).

4. S. Brazovskii and N. Kirova, *Soviet Science Revies*, ed. by I. M. Khalatnikov, Harwood Acad. Publ., New York (1984), Vol. A5, p. 99.

5. S. Brazovskii, AIP Conf. Proc. **1134**, 74 (2009).

6. I. Dzyaloshinskii, Nobel Symposium **24**, 143 (1973).

7. S. A. Brazovskii, I. E. Dzyaloshinskii, and I. M. Krichever, Sov. Phys. JETP **56**, 212 (1982).

8. S. A. Brazovskii, I. E. Dzyaloshinskii, and I. M. Krichever, Phys. Lett. A **91**, 40 (1982).

9. I. E. Dzyaloshinskii and I. M. Krichever, Sov. Phys. JETP **56**, 908 (1982).

10. I. E. Dzyaloshinskii and I. M. Krichever, Sov. Phys. JETP **58**, 1031 (1983).

11. N. Kirova and S. Brazovskii, J. de Phys. IV **9**, Pr10-119 (1999).

12. N. Kirova and S. Brazovskii, J. de Phys. IV **10**, Pr3-183 (2000).

13. I. E. Dzyaloshinskii, JETP Lett. **25**, 98 (1977).

14. G. E. Volovik and V. P. Mineev, JETP Lett. **24**, 561 (1976).

15. M. C. Cross and W. F. Brinkman, J. Low Temp. Phys. **27**, 683 (1977).

16. I. A. Lukyanchuk and M. E. Zhitomirsky, Supercond. Rev. **1**, 207 (1995).

17. G. E. Volovik, PNAS **97**, 2431 (2000).

18. L. Radzihovsky and A. Vishwanath, Phys. Rev. Lett. **103**, 010404 (2009).

19. S. Brazovskii, Physica B **404**, 482 (2009).

20. Y. G. Rubo, Phys. Rev. Lett. **99**, 106401 (2007).

21. V. P. Mineev, Low Temperature Physics/Fizika Nizkikh Temperatur **39**, 1056 (2013).

22. S. I. Anisimov and I. E. Dzyaloshinskii, Sov. Phys. JETP **31**, 773 (1970).

23. I. E. Dzyaloshinskii and G. E. Volovik, Ann. Phys. **125**, 67 (1980).

24. S. Brazovskii and N. Kirova, Ann. Phys. **403**, 184 (2019).

25. P. Monceau, Adv. Phys. **61**, 325 (2012).

26. G. Grüner, Rev. Mod. Phys. **60**, 1129 (1988).

27. G. Grüner, Rev. Mod. Phys. **66**, 1 (1994).

28. *Charge Density Waves in Solids*, ed. by L. Gor'kov and G. Grüner, Elsevier Sci. Publ., Amsterdam (1990).

29. S. Brazovskii and N. Kirova, JETP **129**, 659 (2019).

30. N. P. Ong, G. Varma, and K. Maki, Phys. Rev. Lett. **52**, 663 (1984).

31. N. P. Ong and K. Maki, Phys. Rev. B **32**, 6582 (1985).

32. L. P. Gor'kov, JETP Lett. **38**, 87 (1983).

33. L. P. Gor'kov, see [28], p. 403.

34. L. P. Gor'kov, Sov. Phys. JETP **59**, 1057 (1984).

35. I. Batistic, A. Bjelis, and L. Gor'kov, J. de Phys. **45**, 1049 (1984).

36. T. Yi, N. Kirova, and S. Brazovskii, Eur. Phys. J., Special Topics **222**, 1035 (2013).

37. T. Yi, A. Rojo Bravo, N. Kirova, and S. Brazovskii, J. Supercond. Nov. Mag. **28**, 1343 (2015).

38. S. N. Artemenko, A. F. Volkov, and A. N. Kruglov, Sov. Phys. JETP **64**, 906 (1986).

39. B. I. Ivlev and N. B. Kopnin, Adv. Phys. **33**, 47 (1984).

40. N. Kopnin, *Theory of Nonequilibrium Superconductivity*, Oxford Univ. Press, U.K. (2001).

41. S. Artemenko and A. Volkov, see [28], p. 365.

42. . S. Brazovskii, N. Kirova, H. Requardt, F. Ya. Nad, P. Monceau, R. Currat, J. E. Lorenzo, G. Grbel, and Ch. Vettier, Phys. Rev. B **61**, 10640 (2000).

43. S. Brazovskii, J. de Phys. I **3**, 2417 (1993).

44. S. Brazovskii, J. de Phys. IV **3**, C2-267 (1993).

45. I. V. Krive, A. S. Rozhavsky, and V. A. Rubakov, JETP Lett. **46**, 121 (1987).

46. K. Sengupta and N. Dupuis, Phys. Rev. B **61**, 495 (2000).

47. V. M. Yakovenko and H.-S. Goan, Phys. Rev. B **58**, 10648 (1998).

48. S. M. Gol'berg, N. B. Kopnin, and M. I. Tribel'skii, Sov. Phys. JETP **67**, 812 (1988).