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MACROSCOPIC QUANTUM TUNNELING: FROM QUANTUM VORTICES TO BLACK HOLES AND UNIVERSE

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The process of quantum tunneling of macroscopic objects is well known in condensed matter physics, where the collective variables are used, which describe the collective dynamics of a macroscopic body [1–3]. This approach allows estimating the semiclassical tunneling exponent without consideration of the details of the object structure on the microscopic (atomic) level.

One of the applications of the macroscopic quantum tunneling is the calculation of the quantum creation of the topological objects. Examples are the nucleation quantized vortices in moving superfluids [4], nucleation of Abrikosov vortices in superconductors in the presence of supercurrent [5], and the instanton — the process of creation of the topological charge in quantum field theories [6]. In the process of quantum nucleation of the vortex ring with radius R (a vortex instanton) the collective (macroscopic) dynamically conjugate variables are represented by the area $A = \pi R^2$ of the created vortex ring and (with some factor) its coordinate z along the normal to the ring.

It looks reasonable to apply the approach of macroscopic quantum tunneling also to such macroscopic objects as a black hole. In this case the corresponding collective variables [7] are the area of the event horizon $A = 4\pi R^2$ and its dynamically conjugate variable — the gravitational coupling K (we use the gravitational coupling $K = 1/(4G)$, where G is the Newton "constant").

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Since in both cases one of the collective variables is represented by the corresponding area, this suggests that there can be some thermodynamic analogy between the vortex ring and the black hole. It was shown in Ref. [8] that quantized vortices in Fermi superfluids have many common properties with the black holes. In particular, there is an analog of the Hawking temperature for the moving vortex ring, see Eq. (4.1.9) in Ref. [8]:

$$T_H = \frac{\hbar v_F}{4\pi R} \ln \frac{R}{r_c}, \tag{1}$$

where v_F is Fermi velocity, and r_c is the radius of the singularity — the vortex core radius. In Fermi supefluids, the core size is the analog of the Planck length, which determines singularity inside the black hole.

The temperature in Eq. (1) looks similar to the Hawking temperature of black holes:

$$T_H = \frac{\hbar c}{4\pi R}. \tag{2}$$

The analogy with black holes is supported by the behavior of fermionic quasiparticles living in the vortex core. They occupy bound states: the Caroli–de Gennes–Matricon states [9]. Due to the motion of the vortex ring, the fermions are excited from the bound states to the continuous spectrum by the process of quantum tunneling. The tunneling exponent reproduces the thermal nucleation with the analog of Hawking temperature in Eq. (1). If to extend this analogy to the black hole, then the Hawking radiation from the black hole can be considered as the quantum tunneling of particles from the bound state inside the black hole singularity to the continuous spectrum outside the event horizon. The Hawking radiation as semiclassical tunneling was considered in Refs. [10–14] and in many following papers.

The analogy between the vortex rings in Fermi superfluids with their fermion zero modes in the vortex core and the black hole concerns both the individual processes of the particle creation by quantum tunneling from the object to the external world and the related process of macroscopic quantum tunneling of the whole macroscopic object. In this paper, we discuss the processes of microscopic and macroscopic quantum tunneling related to the black hole and de Sitter Universe using experience with the objects in condensed matter, where we know physics both on macro and micro scales. The plan of the paper is the following.

In Sec. II of the full text, we consider the macroscopic quantum tunneling of the Schwarzschild black hole to the Schwarzschild white hole of the same mass using inverse Newton constant $K = 1/4G$ as dynamic and thermodynamic variable. Introduction of the variying K modifies the first law for the Schwarzschild black hole thermodynamics:

$$dS_{BH} = -AdK + \frac{dM}{T_{BH}},$$

where M is the black hole mass, $A = 4\pi R^2$ is the area of horizon, and $T_{\rm BH}$ is Hawking temperature in Eq. (2). From this first law, it follows that the dimensionless quantity M^2/K is the adiabatic invariant, which in principle can be quantized if to follow the Bekenstein conjecture [15].

As in the case of the semiclassical consideration of the Hawking radiation in terms of the quantum tunneling, we shall use the Painlevé – Gullstrand coordinate system [16, 17] with the metric

$$ds^2 = -dt^2(1 - \mathbf{v}^2) - 2dt\, d\mathbf{r}\cdot\mathbf{v} + d\mathbf{r}^2. \quad (3)$$

Here the vector $v_i(\mathbf{r}) = g_{0i}(\mathbf{r})$ is the shift velocity — the velocity of the free-falling observer, who crosses the horizon. In condensed matter, the analog of this metric is the so-called acoustic metric [18] emerging for quasiparticles in moving superfluids, where the shift velocity v_i is played by superfluid velocity. The analogs of the black hole and white hole horizons described by this metric can be also reproduced in the Dirac and Weyl topological semimetals, where the horizon takes place on the boundary between different types of Dirac or Weyl materials [19–21]. For the Schwartzschild black hole one has

$$\mathbf{v}(\mathbf{r}) = \mp\hat{\mathbf{r}}\sqrt{\frac{M}{2rK}} = \mp\hat{\mathbf{r}}\sqrt{\frac{2MG}{r}}, \quad (4)$$

where sign "−" is for a black hole (flow in), and sign "+" is for a white hole (flow out). For the fully static black hole, $\mathbf{v} = 0$ (no flow).

From the Euclidean action for the black hole, it follows that K and A serve as dynamically conjugate variables. The quantum tunneling exponent is usually determined by the imaginary part of the action on the classical trajectory $A(K)$, which transforms the black hole to white hole at fixed $M = \text{const}$:

$$p \propto \exp(-I_{BH\to WH}),$$
$$I_{BH\to WH} = \int\limits_C A(K')\, dK'. \quad (5)$$

Along this trajectory, the variable K' changes from K to the branch point at $K' = \infty$, and then from $K' = \infty$ to $K' = K$ along the other branch, where the area $A(K') < 0$. The integral gives the tunneling exponent of the transition from the black hole to the white hole with the same mass M:

$$I_{BH\to WH} = 2\pi M^2 \int\limits_K^\infty \frac{dK'}{K'^2} = 2\pi\,\frac{M^2}{K}. \quad (6)$$

The tunneling exponent in Eq. (6) can be expressed in terms of the black hole entropy $S_{BH} = A/4G$, and it is twice the black hole entropy, which enters the probability of transition:

$$p \propto \exp(-2\pi M^2/K) = \exp(-2S_{BH}). \quad (7)$$

The factor 2 has the important consequence. The quantum tunneling can be considered as random thermodynamic fluctuation, and the latter can be expressed in terms of the difference in entropy before and after transition [22]. This suggests that $p \propto \exp(S_{WH} - S_{BH})$, and thus from Eq. (7) one has $S_{WH} - S_{BH} = -2S_{BH}$, i. e., the entropy of the white hole is with minus sign the entropy of the black hole:

$$S_{WH}(M) = -S_{BH}(M) = -\frac{A}{4G}. \quad (8)$$

Then one obtains that the temperature of the white hole is also negative.

While the negative temperatures is the well known phenomenon, the negative entropy looks strange. Anyway, the black hole states with negative entropy have been considered in Ref. [23], where it has been suggested that appearance of negative entropy may indicate a new type instability, see also Ref. [24]. Such super-low entropy of white hole can be also seen as an example of a memory effect discussed in Ref. [25], i. e. the entropy of the white hole is negative, since this state remembers that it is formed from the black hole by the quantum tunneling.

Here we show that there are different ways to calculate the entropy of the white hole, and all of them support its negative sign.

In Sec. III of the full text, we use the following way of calculations. We consider three different types of the hole objects: black hole, white hole, and the fully static intermediate state. The probability of tunneling transitions between these three macroscopic states is found using singularities in the coordinate transformations between these objects. The black and white holes are described by the Painlevé–Gullstrand coordinates with opposite shift vectors in Eq. (4), while the intermediate state is described by the static Schwarzschild coordinates with $\mathbf{v} = 0$. The singularities in the coordinate transformations lead to the imaginary part in the action, which determines the tunneling exponent. For the white hole the same negative entropy is obtained, while the intermediate state — the fully static hole — has zero entropy.

In Sec. IV of the full text, we consider the electrically charged black hole, the Reissner–Nordström (RN) black hole with two horizons, inner and outer. We calculated the entropy of RN black hole and the corresponding temperature of the thermal Hawking radiation using several different approaches. These are:

(i) The method of semiclassical tunneling, which is used for calculation of the Hawking temperature.

(ii) The cotunneling mechanism — the coherent sequence of tunneling at two horizons, each determined by the corresponding Hawking temperature.

(iii) The calculation of the macroscopic quantum tunneling from the RN black hole to the RN white hole using the method of singular coordinate transformations.

(iv) The adiabatic change of the fine structure constant α to zero. This adiabatic process transforms the RN black hole to the Schwarzschild black hole, which does not contradict to the conservation of the charge Q. When α slowly decreases to zero, the two horizons move slowly with conservation of the charge number Q and mass M. Finally, the inner horizon disappears and the black hole at $\alpha = 0$ becomes neutral. In such slow process, the entropy does not change and is the same as the entropy of the neutral black hole. Since the states with different Q can be obtained by the adiabatic transformations, this suggests that entropy of the RN black hole does not depend on charge Q. This is supported by the other approaches, which give the same result.

So, the correlations between the inner and outer horizons lead to the total entropy and to the temperature of Hawking radiation, which depend only on mass M of the black hole and do not depend on the black hole charge Q:

$$S_{BH}(Q, M) = S_{BH}(Q = 0, M). \tag{9}$$

This deviation from the conventional area law can be ascribed to the correlated contributions of both horizons to entropy.

The full agreement between the results of different approaches confirms the validity of the methods used in this paper. In particular, this demonstrates that some singular coordinate transformations violate the general covariance in general relativity: they transform the initial state to the physically (thermodynamically) different state. This corresponds to the spontaneously broken symmetry with respect to the general coordinate transformations, which leads to the existence of the non-equivalent degenerate states with the same energy (black hole and white hole). While the physical laws are invariant under the singular coordinate transformations, the degenerate states are not: they transform into each other under these transformations.

All this also supports the statement that the (anti)symmetry between the black and white holes can be extended to their entropy and temperature. The Schwarzschild black hole and the Schwarzschild white whole are described by the metrics with opposite shift vectors. The shift vector changes sign under time reversal, which transforms a black hole into a white hole. The absence of the time reversal invariance for each of these holes makes these states nonstatic, but still the metric is stationary (time independent), and thus the entropy and temperature can be well defined. The Schwarzschild black hole and the Schwarzschild white hole have the opposite entropies, $S_{WH}(M) = -S_{BH}(M)$, and the opposite Hawking temperatures, $T_{WH}(M) = -T_{BH}(M)$. For the intermediate static hole with $\mathbf{v} = 0$ the time reversal symmetry is not violated, and this object has zero temperature and zero entropy, $S_{static} = T_{static} = 0$.

It is interesting to consider the other objects including the Kerr black and white holes, where time reversal symmetry is violated by rotation. In this case, the coordinate transformations produces the singularity in action not only in $\int M\,dt$, but also in $\int J\,d\phi$, where J is angular momentum and ϕ is the polar coordinate. The proper coordinate system can be found in Refs. [26, 27]. The recent discussion of the Painlevé–Gullstrand forms and their extensions can be found in Refs. [28, 29]. In Ref. [30], the entropy of the Kerr black hole was obtained using the method of the adiabatic transformation. The result is similar to that for the RN black hole: the entropy depends only on the mass M of the black hole:

$$S_{BH}(J,M) = S_{BH}(J=0,M). \tag{10}$$

The consideration can be extended to the other black holes with several horizons [31–33], in particular to the Reissner – Nordström – de Sitter black hole with the cosmological event horizon. In Ref. [32], the entropy of the Reissner – Nordström black hole in Eq. (9) is reproduced in the asymptotic limit of infinite cosmological horizon.

In Sec. V of the full text, the entropy and temperature of the expanding de Sitter Universe are considered. We show that as distinct from the black hole physics, the de Sitter thermodynamics is not determined by the cosmological horizon. The effective temperature of the de Sitter spacetime differs from the conventional Hawking temperature $T_H = H/2\pi$, which follows from the formal semicalssical calculation of the tunneling rate across the cosmological horizon (H is the Hubble parameter). In particular, atoms in the de Sitter Universe experience thermal activation corresponding to the local temperature, which is twice larger than the Hawking temperature, $T_{loc} = 2T_H = H/\pi$ [34]. The same double Hawking temperature describes the decay of massive scalar field in the de Sitter Universe [35–37].

The quantum tunneling process, which leads to the decay of the composite particle in the de Sitter vacuum, occurs fully inside the cosmological horizon and is fully determined by the local temperature T_{loc}. The unconventional thermodynamics of the de Sitter vacuum follows from the specific geometry of the de Sitter expansion and is not related to the existence of the cosmological horizon. The weakening of the role of the cosmological horizon in the de Sitter Universe is confirmed by the proper consideration of the Hawking radiation, and macroscopic quantum tunneling. The free energy of the fluctuations of the matter fields also corresponds to the local temperature T_{loc}. It is not restricted by the region inside the horizon, i. e., it is also not related to the existence of the cosmological horizon. All this raises the question of the role of the cosmological horizon and Hawking temperature in the pure de Sitter vacuum.

The decay of the composite particles, which are excitations above the de Sitter vacuum, does not directly lead to the decay of the vacuum itself. However, it is instructive to consider the de Sitter state as the thermodynamic state, which contains the thermal matter with the local temperature T_{loc}. Then the interaction between the thermal matter and the dark energy during the evolution of the Universe leads to the decay of the vacuum energy density ρ_V and of the Hubble parameter H according to the following power law [38]:

$$H \sim E_{Pl}\left(\frac{t_{Pl}}{t}\right)^{1/3}, \tag{11}$$

$$\rho_V \sim E_{Pl}^4\left(\frac{t_{Pl}}{t}\right)^{2/3}. \tag{12}$$

Here the Planck time $t_{Pl} = G^{1/2}$ and Planck energy $E_{Pl} = 1/t_{Pl}$ are introduced. Such power law decay is discussed in different approaches. It is similar to that in Eq. (192) in Ref. [39] (see also Ref. [40]) and in Eq. (109) in Ref. [41]. The time scale of the decay of the de Sitter expansion, which follows from Eq. (12), $t_Q = E_{Pl}^2/H^3$, corresponds to the time at which de Sitter state looses coherence [42].

On the other hand, the possibility of the decay of the pure de Sitter vacuum due to Hawking radiation remains unclear and requires the further consideration [43]. This does not mean that the de Sitter vacuum is stable: this only means that the Hawking radiation alone does not lead to instability, i. e. the de Sitter vacuum is stable with respect to the decay via the Hawking radiation. The Hawking radiation does not lead to the change of the vacuum energy density, which generates the de Sitter expansion. This means that even if the pair creation takes place, the de Sitter expansion immediately dilutes the produced particles, and thus there is no vacuum decay in the de Sitter spacetime.

There are many other mechanisms, not related to the Hawking radiation, which could lead to the decay of the de Sitter spacetime [44–51], including the infrared instability, instability due to the dynamic effects of a certain type of quantum fields, instability towards spontaneous breaking of the symmetry of the de Sitter spacetime or the instability towards the first order phase transition in the vacuum, etc. But in most cases, either the de Sitter vacuum is not perfect, i. e., there are deviations from the exact de Sitter and the de Sitter symmetry is lost, or the vacuum energy is fine-tuned, i. e., the cosmological constant problem is ignored. The de Sitter instability, which avoids fine tuning, but uses the special vector field in Dolgov scenario [52], is in Ref. [53].

The problem of the dynamical stability of the de Sitter vacuum is directly related to the cosmological constant problem. The q-theory [54] demonstrates the solution of the problem in thermodynamics: in the equilibrium Minkowski vacuum the cosmological constant is nullified due to thermodynamics. However, it remains unclear whether the de Sitter state relaxes to the equilibrium. This depends on the stability of the de Sitter vacuum. If the de Sitter attractor is not excluded in dynamics, then the only possibility to solve the dynamical cosmological problem within the q-theory is to assume that the Big Bang occurred in the part of the Universe, which is surrounded by the equilibrium environment [55]. In this case, any perturbation of the vacuum energy by the Big Bang, even of the Planck scale order, will inevitably relax to the equilibrium Minkowski vacuum with zero cosmological constant. This relaxation does not require any fine-tuning, since it is dictated by the equilibrium environment.

In conclusion, the macroscopic quantum tunneling elaborated in the early works by S. V. Iordansky, A. M. Finkel'shtein, and E. I. Rashba in Landau Institute allows studying similar processes in cosmology. The probability of the processes of macroscopic quantum tunneling of cosmological objects is extremely small. However, the theoretical consideration of these processes allows making conclusions on entropy and temperature of the cosmological objects, which are rather unexpected.

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