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COLLECTIVE SPIN MODES IN FERMI LIQUIDS WITH SPIN-ORBIT COUPLING

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Spin-orbit coupling (SOC) plays an important and, sometimes decisive, role in many condensed matter systems, including two-dimensional (2D) electron and hole gases in semiconductor heterostructures [1,2], non-centrosymmetric normal metals [3] and superconductors [4,5], bismuth tellurohalides [6], a variety of iridates and vanadates [7], surface/edge states of three-dimensional (3D)/2D topological insulators [8–12], conducting states at oxide interfaces [13], 2D transition metal dichalcogenides (TMD) [14,15], graphene on TMD substrates [16], atomic Bose [17,18] and Fermi [19,20] gases in simulated non-Abelian magnetic fields, etc. Coupling between electron spins and momenta leads to a number of fascinating consequences, such as the electric-dipole spin resonance (EDSR) [21,22], current-induced spin polarization [23–25], persistent spin helices [26–28], quantum spin [29–31] and anomalous Hall effects [32–34], to name just a few. An interesting and still largely open question is the interplay between spin-orbit and electron-electron interactions. Such interplay gives rise to new phases of matter, e. g., topological Mott insulator [35,36], gyrotropic and multipolar orders in normal metals [37], helical Fermi liquid (FL) [38] Gor'kov – Rashba superconductor with mixed singlet-triplet order parameter [39], topological Kondo insulators [40], etc. It also affects in a nontrivial way many physical phenomena, e. g., optical conductivity [41,42], plasmon spectra [43–45], RKKY interaction [46–48], non-analytic behavior of the spin susceptibility [49–51], etc., and gives rise to spin-dependent electron-electron interaction [52].

In this paper, we review recent progress in theoretical understanding and experimental observation of a new type of collective spin modes in 2D FLs with SOC. Such modes is perhaps the most direct manifestation of an interplay between spin-orbit and electron-electron interactions, as their existence hinges on both components being present. Unlike the conventional Silin mode in a partially spin-polarized FL [53] these modes exist even in the absence of an external magnetic field; in addition, they modify in a qualitative way the Silin mode if both SOC and magnetic field are present. As long as SOC is weak, the new modes correspond to oscillations of the magnetization which are decoupled from the oscillations of charge. The origin of the new modes can be traced to the effective spin-orbit magnetic field, which depends on the orientation and magnitude of the electron momentum, and also on the position of electron valley in the Brillouin zone (for multi-valley systems, such a graphene with proximity-induced SOC). Some of these modes have already been observed experimentally in $Cd_{1-x}Mn_xTe$ quantum wells (in the presence of the magnetic field) [54–59], and in the surface state of a three-dimensional (3D) topological insulator (TI) Bi_2Se_3 (in zero magnetic field); [60] however, many

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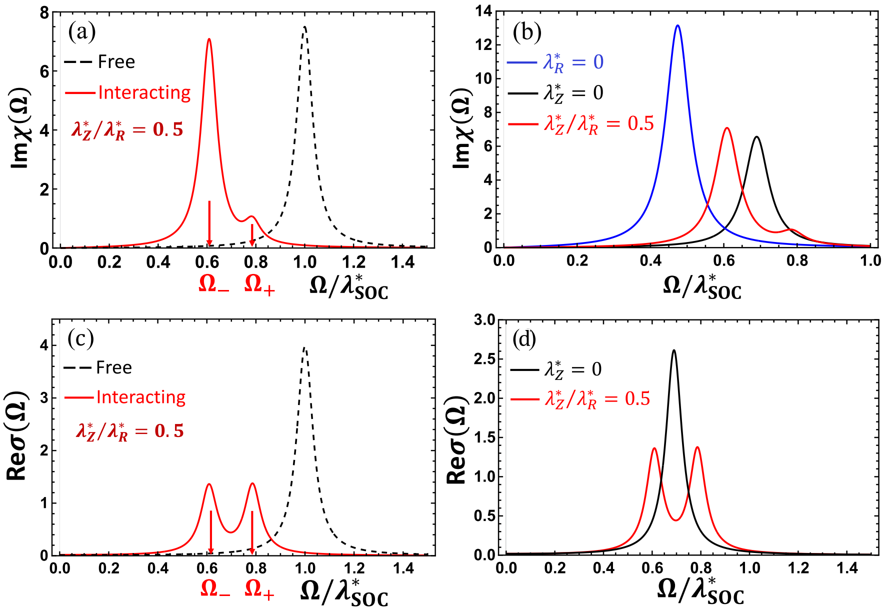

Fig. 1. Theoretical predictions for the zero-field electron spin resonance (ESR) and electric-dipole spin resonance (EDSR) in graphene with proximity-induced spin-orbit coupling (SOC). (a) ESR signal. Vertical axis: the imaginary part of the dynamical spin susceptibility. The frequency on the horizontal axis is scaled with $\lambda^*_{\rm SOC} = \sqrt{\lambda_{\rm R}^{*2} + \lambda_{\rm VZ}^{*2}}$, where $\lambda^*_{\rm R}$ and $\lambda^*_{\rm VZ}$ are (renormalized) couplings of the Rashba and valley-Zeeman (VZ) types of SOC, respectively. $\Omega_\pm$ are the resonance frequencies, given by Eqs. (5.7a) and (5.7b) in the main text. Dashed line: non-interacting system. Red solid line: a two-valley Fermi liquid (FL) with parameters $F_0^a = -0.5500$, $F_1^a = -0.2750$, $F_2^a = -0.1375$, $H_0 = -0.5000$, $H_1 = -0.2500$, and $H_2 = -0.1250$. The ratio $\lambda^*_{\rm VZ}/\lambda^*_{\rm R} = 0.5$. The choice of FL parameters is the same for all panels of the figure. (b) ESR signal in a FL for several values of $\lambda^*_{\rm VZ}/\lambda^*_{\rm R}$, as indicated in the legend. (c) EDSR signal. Vertical axis: the real part of the optical conductivity. Dashed line: non-interacting system. Solid line: FL. (d) EDSR signal in a FL for two values of $\lambda^*_{\rm VZ}/\lambda^*_{\rm R}$, as indicated in the legend. To account for smearing of the resonances by disorder, we added a damping term, $-\delta\hat{n}(\mathbf{k},t)/\tau_s$, to the right-hand side of the kinetic equation (2.8) in the main text. In all panels of Fig. 1, $1/\tau_s = 0.04\lambda^*_{\rm SOC}$, where $\lambda^*_{\rm SOC} = \sqrt{\lambda_{\rm R}^{*2} + \lambda_{\rm VZ}^{*2}}$. For $\lambda^*_{\rm R} = 15.0$ meV and $\lambda^*_{\rm VZ} = 7.5$ meV, the spin relaxation time is $1/\tau_s = 1$ ps. Reprinted with permission from Ref. [61]. Copyright 2021 by the American Physical Society

more predictions are still awaiting their experimental confirmation.

We discuss collective spin modes in three types of real-life systems: i) a 2D electron gas (2DEG) with Rashba and/or Dresselhaus SOC, ii) graphene with proximity-induced SOC, and iii) a Dirac helical state on the surface of a 3D TI. If SOC and/or magnetic field are weak, i. e., the corresponding energy scales are much smaller than the Fermi energy, collective modes in systems i) and ii) can be analyzed with the single- or two-valley versions of the Fermi-liquid (FL) theory, respectively.

The paper is organized as follows. Sec. I provides a brief introduction into the subject. In Sec. II.A, we introduce three systems considered in the rest of the paper: i) a 2D electron gas (2DEG) with Rashba and/or Dresselhaus SOC, ii) graphene with proximity-induced SOC, and iii) a Dirac helical state on the surface of a 3D TI. In Sec. II.B, we describe the single- and two-valley FL theories, which will be applied to the cases of a 2DEGs and graphene, provided that the corresponding energy scales are much smaller than the Fermi energy. In Sec. II.C, we explain why a FL theory cannot be applied to the cases of arbitrarily strong SOC and/or magnetic field. The reason is that the theory cannot be confined to to a narrow interval of energies near the Fermi energy, where a quasiparticle description is applicable, but involves states far away from the Fermi surface. Sec. III serves as a short reminder of collective modes in a FL without SOC, in general, and of the Silin modes, in particular. In Sec. IV, we discuss collective spin modes in a 2DEG. Sec. IV.A describes how the FL theory is applied to the case of Rashba/Dresselhaus SOC. In Sec. IV.B, we show that the FL kinetic equation for a 2DEG with Rashba and/or Dresselhaus SOC and in the presence of the magnetic field can be mapped onto an effective tight-binding model for an artificial one-dimensional (1D) lattice, whose sites are labeled by the projections of the angular momentum. Within this mapping, the Rashba energy splitting plays a role of the on-site energy, while the Zeeman and Dresselhaus terms describe "hopping" between the nearest and next-to-nearest neighbors. These terms form a "conduction band", which is just the continuum of particle-hole excitations with spin-flips. The role a FL interaction is to produce "defects", both of the on-site and bond types, and the collective modes arise as bound states due to such defects. In Sec. VI.C, we illustrate how this mapping works for the case of a 2DEG with Rashba SOC and in the presence of the magnetic field, using the s-wave approximation for the Landau function. Sec. V deals with collective spin modes in Dirac systems. In Sec. V.A we apply a two-valley version of the FL theory to graphene with proximity-induced SOC. In Sec. V.B, we derive the spectrum of inter-band spin excitations in a Dirac surface state within the ladder approximation. In Sec. VI, we discuss the spatial dispersion of collective spin modes. Sec. VII is devoted to damping due to both disorder and electron-electron interaction. In Sec. VIII, we discuss both the current and future experiment. Sec. VIII. summarizes the results of a series of Raman experiments on $\mathrm{Cd}_{1-x}\mathrm{Mn}_x\mathrm{Te}$. In Sec. VIII.B, we provide a summary of recent Raman spectroscopy of a collective spin mode on the surface of $\mathrm{Bi}_2\mathrm{Se}_3$. Sec. VIII.C contains the theoretical predictions for the electron spin resonance (ESR) and electric-dipole spin resonance (EDSR) measurements on graphene with proximity-induced SOC, both in zero and strong (compared to SOC) magnetic field. In zero field, we predict that both ESR and EDSR signals consist of two rather than one peak, provided that both Rashba and valley-Zeeman types are present, see Fig. 1. Our conclusions are given in Sec. IX.

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