ЖЭТФ, 2022, том 162, вып. 5 (11), стр. 680–685

DIMENSIONLESS PHYSICS: CONTINUATION

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Received August 21, 2022,
revised version August 21, 2022
Accepted for publication September 09, 2022

DOI: 10.31857/S0044451022110086
EDN: KZDZIL

Several approaches to quantum gravity (including the model of superplastic vacuum; Diakonov tetrads emerging as the bilinear combinations of the fermionis fields [1–4]; BF-theories of gravity; and effective acoustic metric [5, 6]) suggest that in general relativity the metric must have dimension 2, i.e. $[g_{\mu\nu}] = 1/[L]^2$, irrespective of the dimension of spacetime. This leads to the "dimensionless physics" discussed in the review paper [7]. We continue to exploit this issue.

Elasticity tetrads. The 3 + 1-dimensional vacuum crystal is the plastic (malleable) medium [8], described in terms of the elasticity tetrads [9–12]:

$$E^a_\mu = \frac{\partial X^a}{\partial x^\mu}\,, \tag{1}$$

where equations $X^a(x) = 2\pi n_a$ are equations of the (deformed) crystal planes. The functions X^a play the role of the geometric $U(1)$ phases and are dimensionless. The elasticity tetrads play the role of the gauge fields (translation gauge fields) and have the same dimension 1 as the dimension of gauge fields:

$$[E^a_\mu] = \frac{1}{[L]}\,. \tag{2}$$

The dimension n of quantity A means $[A] = [L]^{-n}$, where $[L]$ is dimension of length. The matrix E^a_μ is not necessarily quadratic. The extension of tetrads to the rectangular vilebein is considered in Ref. [13].

Elasticity tetrads in Eq.(1) give rise to the metric, which is the bilinear combination of tetrads:

$$g_{\mu\nu} = \eta_{ab} E^a_\mu E^b_\nu\,. \tag{3}$$

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The metric $g_{\mu\nu}$ has dimension $n = 2$, while the contravariant metric $g^{\mu\nu}$ has dimension $n = -2$:

$$[g_{\mu\nu}] = \frac{1}{[L]^2}\ ,\ [g^{\mu\nu}] = [L]^2\,. \tag{4}$$

The tetrad determinant has dimension $n = 4$ in the 4-dimensional spacetime and dimension $n = N$ in the N-dimensional spacetime, where the dimensions of the metric elements are the same as in Eq.(4):

$$[e] = [\sqrt{-g}\,] = \frac{1}{[L]^N}\,. \tag{5}$$

Eq.(5) makes the spacetime integration dimensionless:

$$[\int d^N x\sqrt{-g}\,] = [1] = 0\,, \tag{6}$$

which leads to the dimensionless Lagrangian $\mathcal{L}$:

$$[S] = [\int d^N x\sqrt{-g}\,\mathcal{L}] = [\int d^N x\sqrt{-g}\,]\cdot[\mathcal{L}] = [1]\cdot[1] = [1]\,. \tag{7}$$

Classical dynamics of particle is described by action:

$$S = M\int ds\,, \tag{8}$$

where with Eq.(3) the interval is dimensionless:

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu\ ,\ [s^2] = \frac{1}{[L]^2}\cdot[L]^2 = [1] = 0\,. \tag{9}$$

The variation of action gives the Hamilton–Jacobi equation in terms of the contravariant metric:

$$g^{\mu\nu}\partial_\mu S\partial_\nu S + M^2 = 0\,. \tag{10}$$

Since the action and the interval are dimensionless, the mass M in Eq.(8) is also dimensionless, $[M] = [1] = 0$, for any dimension N of spacetime.

In the spacetime crystal, the interval between the events is counted in terms of the lattice points, and this is the geometric reason why the interval is dimensionless. One may say that dynamics comes from geometry. In the Diakonov theory, the interval determines the dynamics of the particle, rather than the geometric distance, i.e. the geometry follows from dynamics.

Scalar fields. The quadratic terms in the action for the scalar field Φ in the N-dimensional spacetime are:

$$S = \int d^N x \sqrt{-g} \left(g^{\mu\nu}\nabla_\mu \Phi^* \nabla_\nu \Phi + M^2 |\Phi|^2\right) . \quad (11)$$

From Eqs. (4) and (6) it follows that the scalar field is dimensionless, $[\Phi] = [1] = 0$, for arbitrary spacetime dimension N. This universal zero dimension differs from the N-dependent dimension $n = (N-2)/2$ of scalar fields in the conventional approach.

Wave function. Expanding the Klein-Gordon equation over $1/M$ one obtains the non-relativistic Schrödinger action for the wave function ψ:

$$\Phi(\mathbf{r}, t) = \frac{1}{\sqrt{M}} \exp\left(iMt/\sqrt{-g^{00}}\right) \psi(\mathbf{r}, t) , \quad (12)$$

$$S_{\rm Schr} = \int d^3x dt \sqrt{-g}\, \mathcal{L} , \quad (13)$$

$$2\mathcal{L} = i\sqrt{-g^{00}}\, (\psi \partial_t \psi^* - \psi^* \partial_t \psi) + \frac{g^{ik}}{M} \nabla_i \psi^* \nabla_k \psi . \quad (14)$$

The normalization condition for the wave function is:

$$\int d^3r \sqrt{\gamma}\, |\psi|^2 = 1 , \quad (15)$$

where $\sqrt{\gamma} = \sqrt{-g}\sqrt{-g^{00}}$ is the determinant of the space part of the metric. This corresponds to the particle number conservation in the nonrelativistic quantum mechanics, see e.g. Eq.(13) in Ref. [14].

Since the dimension of this determinant is $[\sqrt{\gamma}] = \frac{1}{[L]^3}$, the wave function is dimensionless. This is distinct from the conventional Schrödinger equation without gravity, where the dimension of ψ is $[\psi] = [L]^{-(N-1)/2}$ for the N dimensional spacetime. Inclusion of gravity provides the natural zero dimension for the probability amplitude in quantum mechanics, $[\psi] = 0$, for any spacetime dimension.

The same result can be obtained from overlap of the quantum states, which is naturally dimensionless:

$$< \mathbf{r} \,|\, \mathbf{r}' > = \frac{1}{\sqrt{\gamma}}\, \delta(\mathbf{r} - \mathbf{r}') . \quad (16)$$

Then for the wave function

$$\psi(\mathbf{r}) = < \mathbf{r} \,|\, \psi > \, , \ | \psi > = \int d^{N-1}r \sqrt{\gamma}\, \psi(\mathbf{r}) | \, \mathbf{r} > , \quad (17)$$

one obtains Eq.(15) for normalization:

$$1 = < \psi \,|\, \psi > = \int d^{N-1}r \sqrt{\gamma}\, |\psi|^2 . \quad (18)$$

From Eq.(18) it follows that the wave function is dimensionless, which is the consequence of the presence of the metric field. This demonstrates the connection between quantum mechanics and general relativity.

The action (13) and Lagrangian (14) do not contain $\hbar$. The role of $\hbar$ in the conventional relation between the energy levels and frequency, $E_m - E_n = \hbar\omega_{mn}$, is now played by $\sqrt{g^{00}}$ in the red shift equation $M_m - M_n = \sqrt{g^{00}}\, \omega_{mn}$ [15]. The dimensional metric leads to the difference between the dimensional frequency, $[\omega_{mn}] = 1/[L]$, and the dimensionless mass:

$$[M] = [\sqrt{g^{00}}][\omega] = [L] \cdot \frac{1}{[L]} = [1] = 0 . \quad (19)$$

Weyl and Dirac fermions. The dimensional tetrads $[E^a_\mu] = 1/[L]$ are obtained directly from the zero dimension of wave functions, which gives rise to the dimensionless Weyl and Dirac fields, $[\Psi] = 0$, in the action:

$$S = \int d^4x\, e\, e^\mu_a \bar{\Psi} \gamma^a \nabla_\mu \Psi , \quad (20)$$

where e is the tetrad determinant. Since the action is dimensionless, then assuming that the quantum field operators Ψ are dimensionless, one obtains $[e\, e^\mu_a] = 1/[L]^3$, which gives the dimensional tetrads:

$$[e^\mu_a] = [L] \, , \, [E^a_\mu] = \frac{1}{[L]} \, , \ [e] = \frac{1}{[L]^4} . \quad (21)$$

This is in agreement with the Diakonov theory [1–4], where tetrads emerge as the bilinear combinations:

$$E^a_\mu \propto < \bar{\Psi} \gamma^a \nabla_\mu \Psi > \, , \, [E^a_\mu] = \frac{1}{[L]} , \quad (22)$$

and metric $g_{\mu\nu}$ is the quadrilinear combination of the fermionic fields, $< \bar{\Psi}\Psi\bar{\Psi}\Psi >$. This approach also allows the rectangular vilebein [13], where spin a and coordinate μ spaces have different dimensions.

The Hamiltonian for massless Dirac fermions has dimension 1, i.e. the same as the dimension of frequency:

$$H = \int_{x_0 = {\rm const}} d^3r\, e\, e^i_a \bar{\Psi} \gamma^a \nabla_i \Psi \, , \ [H] = [\omega] = \frac{1}{[L]} . \quad (23)$$

The dimension of the Hamiltonian does not coincide with the dimension of mass M, which is dimensionless.

Gauge fields. The action for the $U(1)$ gauge field in the N-dimensional spacetime is:

$$S \sim \int d^N x \sqrt{-g}\, g^{\mu\nu} g^{\alpha\beta} F_{\mu\alpha} F_{\nu\beta} . \quad (24)$$

In case of the conventional dimensionless tetrads, the action in Eq.(24) is dimensionless only for $N = 4$.

With dimensionful tetrads the action (24) is dimensionless for arbitrary N, since

$$[g^{\mu\nu}] = [L]^2 \ , \ [F_{\mu\nu}] = \frac{1}{[L]^2} \ , \ [\sqrt{-g}] = \frac{1}{[L]^N} \,. \quad (25)$$

Acoustic metric also has dimension 2. The effective acoustic metric [5,6] describes propagation of sound in a non-homogeneous flowing fluid and also phonons in moving superfluids and other Goldstone modes, such as magnons and collective modes of magnon Bose condensate [16]. The action for Goldstone mode (the phase ϕ of the Bose condensate) is similar to the action (11):

$$S = \int d^4x \sqrt{-\tilde g}\, \tilde g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi \,. \quad (26)$$

From the action (26) it follows that the effective contravariant metric $\tilde g^{\mu\nu}$ has the conventional dimension -2, i.e. $[\tilde g^{\mu\nu}] = [l]^2$. This is also seen form the effective interval in terms of hydrodynamic variables [6,17]:

$$d\tilde s^2 = \tilde g_{\mu\nu} dx^\mu dx^\nu = \frac{n}{ms}[-s^2dt^2 + (dx^i - v^i dt)(dx^i - v^i dt)] \,. \quad (27)$$

Here n is the density of atoms in the liquid; m is the mass of the atom; s is the speed of sound; and v^i is the velocity of the liquid, which coincides with the shift vector N^i in the Arnowitt-Deser-Misner formalism. Using the conventional dimensions of hydrodynamic variables one obtains the dimension 2 for the covariant metric:

$$[\tilde g_{\mu\nu}] = [n] \cdot \frac{1}{[m]} = \frac{1}{[l]^3} \cdot [l] = \frac{1}{[l]^2} \,, \quad (28)$$

and the dimensionless interval. The dimension of avoustic metric follows from the dynamics of the superfluid: geometry comes from dynamics.

General relativity. Let us consider the GR action on example of q-theory – the class of theories which avoid the cosmological constant problem. The huge contributions of zero point energy to the cosmological constant is cancelled in the equilibrium state of the vacuum due to thermodynamics [18–20]. For the particular q-theory on “brane” the action is [21,22]:

$$S = -\int d^4x \sqrt{-g} \left[\epsilon(q) + \frac{R}{16\pi G_N(q)} + \Lambda_0 + \mathcal{L}^M[\psi, q]\right] + \mu \int d^4x \; n \ , \ q = \frac{n}{\sqrt{-g}} \,. \quad (29)$$

Here n is the 4D analog of the particle density in the quantum vacuum (density of the "spacetime atoms"), which has the same dimension 4 as tetrad determinant

$$[n] = [\sqrt{-g}] = \frac{1}{[L]^4} \,, \quad (30)$$

q is the vacuum variable, and μ plays the role of the chemical potential in the vacuum thermodynamics. In the expanding Friedmann-Robertson-Walker universe:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -d\tau^2 + a^2(\tau) d\mathbf{r}^2 \ , \ H(\tau) = \frac{da/d\tau}{a(\tau)} \,, \quad (31)$$

where τ is the conformal time; $a(\tau)$ is the scale factor; and $H(\tau)$ is time-dependent Hubble parameter. The scale factor $a(\tau)$ has dimensions 1, $[a(\tau)] = \frac{1}{[L]}$, while the following quantities are dimensionless:

$$[q] = [\mu] = [\epsilon] = [R] = [G_N] = [\Lambda_0] = = [H] = [\tau] = [\psi] = [M] = [1] = 0 \,. \quad (32)$$

Some of the dimensionless quantities can be fundamental, or correspond to some integer valued topological invariants. For example, the "chemical potential" μ may correspond to the topological invariant, $\mu = \pm 1$, which changes sign at the Big-Bang quantum phase transition [20]. Since masses of particles are dimensionless, and there is no fundamental mass scale, one can choose any convenient mass as a unit mass.

Note also that the dimensionless interval in Eq.(9) does not mean the existence of the fundamental length, such as Planck length. First, because the gravitational coupling $1/G_N$ is not necessarily fundamental. Second, in the model of the superplastic vacuum there is no equilibrium value of the distance between the neighbouring lattice points. As distinct from the solid state crystals, arbitrary deformations of the vacuum crystal are possible. In Diakonov model [1] the metric is emergent, and on the fundamental level the distance between the spacetime points is not determined.

Unruh and Hawking. In terms of the dimensionful metric, the acceleration is dimensionless [7]:

$$a^2 = g_{\mu\nu} \frac{d^2x^\mu}{ds^2} \frac{d^2x^\nu}{ds^2} \,, \quad (33)$$

$$[a^2] = [g_{\mu\nu}][x^\mu][x^\nu] = \frac{1}{[l]^2} \cdot [l]^2 = [1] = 0 \,. \quad (34)$$

This leads to the dimensionless Unruh temperature:

$$T_{\rm U} = \frac{a}{2\pi} \ , \ [T_{\rm U}] = [1] = 0 \,. \quad (35)$$

The Gibbons-Hawking temperature of the cosmological horizon is also dimensionless, as follows from Eq.(32):

$$T_H = \frac{H}{2\pi} \ , \ [T_H] = [H] = [1] = 0 \,. \quad (36)$$

Eqs. (35) and (36) look fundamental: they do not contain parameters. However, for the temperature of the Hawking radiation from the black hole horizon,

$T_{\rm BH} = 1/8\pi G_N M$, situation is different. Although the Hawking temperature is dimensionless ($[T_{\rm BH}] = [1]$, since $[G_N] = [M] = [1]$), it does not look fundamental, since it depends on the dimensionless parameter G_N. The same concerns the Bekenstein-Hawking entropy:

$$S_{\rm BH} = \frac{A}{4G_N}\,. \tag{37}$$

It is dimensionless due to dimensionless horizon area:

$$dA = \sqrt{dS^{ik}dS_{ik}}\ ,\ [A] = [1] = 0\,, \tag{38}$$

$$A = \int_0^{\pi} d\theta \int_0^{2\pi} d\phi\, \sqrt{g_{\phi\phi}g_{\theta\theta}}\ ,\ [g_{\phi\phi}] = [g_{\theta\theta}] = [A] = 0\,. \tag{39}$$

The Bekenstein-Hawking entropy (37) determines the black hole thermodynamics, but similar to the Hawking temperature it does not look as fundamental, since it contains the gravitational coupling $1/G_N$. Also it is not clear why the microscopic degrees of freedom responsible for the black hole entropy should be characterized by the Planck length [23]. In the superplastic vacuum [8] the Planck length scale is absent, since there is no equilibrium value of the distance between the lattice points: this vacuum can be arbitrarily deformed.

On the other hand, since the area is dimensionless, one may suggest that the entropy of the black hole horizon can be expressed in terms of the area only:

$$S_{\rm BH} = \eta A\ ,\ [\eta] = [S_{\rm BH}] = [A] = [1] = 0\,. \tag{40}$$

Here η is some fundamental dimensionless parameter, like the topological invariant. In this case one may take the point of view that Einstein's gravity equations can be derived solely from thermodynamics [24]. The constant of proportionality η between the entropy and the area determines gravitational coupling $1/G_N = 4\eta$. In this thermodynamic approach, $1/G_N$ becomes fundamental due to the fundamentality of the parameter η.

However, in the thermodynamic approach to gravity there is the "species problem" [25]: the gravitational coupling G_N may depend on the number of fermionic and bosonic quantum fields [26–28]. This destroys many conjectures, which are based on positivity of the gravitational coupling [29], and prevents $1/G_N$ to be the fundamental parameter. But this "no-go theorem" can be avoided, if $1/G_N$ is the quantum number related to symmetry and/or topology. Then the parameter $1/G_N$ does not depend on interaction between gravity and quantum field, though it may experience jumps during the topological quantum phase transitions. This takes place in topological materials when one varies the parameters of interaction [30, 31] and may take place when the Big Bang is crossed [20].

Einstein-Cartan, Barbero-Immirzi, Nieh-Yan and topology. Topological invariants relevant for the quantum vacuum are known in the crystalline matter [10, 11, 32] and can be extended to the superplastic vacuum. The topology in the crystalline quantum vacua is enriched due to the dimensional elasticity tetrads in Eq.(1), which come from the geometric $U(1)$ phases. This topological approach may take place in the Einstein-Cartan-Sciama-Kibble theory, which is expressed in terms of tetrads, and thus is more fundamental than the conventional Einstein gravity based on metric. Such type of gravity emerging in superplastic crystals has been discussed in Ref. [33]. The action in the Einstein-Cartan gravity can be expressed in terms of the differential forms, which contain the elasticity tetrads as the translational gauge fields:

$$S_{\rm EC} \sim \epsilon_{abcd} \int d^4x\, E^a \wedge E^b \wedge R^{cd}\,. \tag{41}$$

This action is dimensionless because the one-form tetrad has dimension 1, $[E^a_\mu] = \frac{1}{[L]}$, while the curvature two-form R^{ab} has dimension 2:

$$[R^{ab}_{\mu\nu}] = \frac{1}{[L]^2}\,. \tag{42}$$

With the dimensional elasticity tetrads the topology of the $3+1$ crystalline phases [10, 11, 32] may provide the fundamental topological prefactor in Eq.(41), with $1/G_N$ as integer or fractional topological number.

The same can be valid for the dimensionless parameter in the Barbero-Immirzi action:

$$S_{\rm BI} \sim \int d^4x\, E^a \wedge E^b \wedge R_{ab}\,. \tag{43}$$

Eq.(43) looks similar to the Nieh-Yan term in the action, see e.g. Ref. [34]. Due to dimensional tetrads the prefactors in the Nieh-Yan and in the Barbero-Immirzi actions are dimensionless, and thus can be fundamental [7]. It is not excluded that these parameters are the topological invariants similar to that in topological insulators, semimetals and superconductors [10].

The dimensional metric and tetrads appear also in such topological field theories as the BF-theoryl. For example, the composite metric (Schönberg-Urbantke metric [35–40]) is formed by triplet of the 2-form fields:

$$\sqrt{-g}g_{\mu\nu} = \frac{1}{12}e_{abc}e^{\alpha\beta\gamma\delta}B^a_{\mu\alpha}B^b_{\beta\gamma}B^c_{\delta\nu}\,. \tag{44}$$

The 2-forms in the BF action $\int B \wedge F$ have dimension 2, $[B] = [F] = 1/[L]^2$. Then the composite metric in Eq.(44) has also dimension 2, $[g_{\mu\nu}] = 1/[L]^2$. In the same way the two-form field B can be represented as the bilinear combination of the tetrads [37]: $B = E\wedge E$. These one-form tetrads have dimension 1, $[E^a_\mu] = 1/[L]$.

Arnowitt-Deser-Misner (ADM) formalism [41] is used for the Hamiltonian formulation of general relativity. Let us consider this formalism and its application using the dimensional metric. One has the following metric elements and their dimensions:

$$g_{ik} = \gamma_{ik} \ , \ [\gamma_{ik}] = \frac{1}{[L]^2} \, , \quad (45)$$

$$g_{0i} = N_i = \gamma_{ik} N^k \ , \ [N_i] = \frac{1}{[L]^2} \ , \ [N^i] = 0 \, , \quad (46)$$

$$g_{00} = \gamma_{ik} N^i N^k - N^2 = N^i N_i - N^2 \ , \ [N] = \frac{1}{[L]} \, , \quad (47)$$

$$g^{00} = -\frac{1}{N^2} \ , \ [g^{00}] = [L]^2 \, , \quad (48)$$

$$g^{0i} = \frac{N^i}{N^2} \ , \ [g^{0i}] = [L]^2 \, , \quad (49)$$

$$g^{ik} = \gamma^{ik} - \frac{N^i N^j}{N^2} \ , \ [\gamma^{ik}] = [L]^2 \, , \quad (50)$$

$$\sqrt{-g} = N\sqrt{\gamma} \ , \ [\sqrt{\gamma}] = \frac{1}{[L]^3} \, , \quad (51)$$

$$\gamma^{ik}\gamma_{kl} = \delta^i_l \, . \quad (52)$$

Here N and N^i are lapse and shift functions correspondingly, and γ_{ik} are space components of metric.

The ADM formalism allows to consider dynamics in curved space in terms of the Poisson brackets. Let us consider this on example of Poisson brackets for the classical $3+1$ electrodynamics in curved space:

$$\{A_i(\mathbf{r}), D^k(\mathbf{r}')\} = \delta^k_i \delta(\mathbf{r} - \mathbf{r}') \, , \quad (53)$$

which in terms of the gauge invariant fields is:

$$\{B^i(\mathbf{r}), D^k(\mathbf{r}')\} = e^{ikl}\nabla_l \delta(\mathbf{r} - \mathbf{r}') \, . \quad (54)$$

Here $\mathbf{B}$ is magnetic field, and the vector $\mathbf{D}$ is the electric induction of the quantum vacuum (electric displacement field). The electric induction $\mathbf{D}$ is expressed in terms of the electric field $E_i = F_{0i}$:

$$D^k = \frac{1}{\alpha}\frac{\sqrt{\gamma}}{N}\gamma^{ik} E_i \, . \quad (55)$$

Here α is the dimensionless fine structure constant, which determines the dielectric constant – the electric permittivity of the relativistic quantum vacuum, $\epsilon_{\rm vac}$, and the magnetic permeability of the vacuum, $\mu_{\rm vac}$:

$$\epsilon_{\rm vac} = \frac{1}{\mu_{\rm vac}} = \frac{1}{\alpha} \, . \quad (56)$$

In spite of the dimensional metric, electric inductance $\mathbf{D}$ has the same dimension 2 as electric field $\mathbf{E}$:

$$[D^i] = [E_i] = \frac{1}{[L]^2} \, . \quad (57)$$

This follows from Eqs.(45), (48) and (51) for dimensions of the ADM metric elements in $3+1$ spacetime.

The corresponding quadratic Hamiltonian for the electromagnetic field is:

$$H = \int \frac{d^3r}{2}\frac{N}{\sqrt{\gamma}}\gamma_{ik}\left(\alpha D^i D^k + \frac{1}{\alpha} B^i B^k\right) \, . \quad (58)$$

The Hamiltonian has dimension 1, i.e. $[H] = 1/[L]$. Both the Hamiltonian in Eq.(58) and the Poisson bracket in Eq. (54) do not contain the gauge potentials. The gauge potentials also do not enter the Poisson brackets for charged particle, $\{p_i, p_j\} = qF_{ij}$ and $\{p_i(\mathbf{r}), D^k(\mathbf{r}')\} = -q\delta^k_i\delta(\mathbf{r} - \mathbf{r}')$, where q is the dimensionless electric charge of the particle in terms of the electric charge of the electron.

The quantization of electromagnetic field is obtained by the substitution of the Poisson brackets (54) by commutation relations between $\mathbf{D}$ and $\mathbf{B}$. The Poisson brackets in Eqs. (53) and (54) look as fundamental. They do not depend on the metric and do not contain physical parameters of the quantum vacuum. However, the function $\mathbf{D}$ in Eq.(55) breaks this fundamentality. It is the phenomenological variable, which describes the response of the vacuum to electric field. This response contains the electromagnetic coupling $1/\alpha$, which is not fundamental because of the corresponding "species problem": it depends on the fluctuating bosonic and fermionic fields in the quantum vacuum, and is space-dependent. While the gravitational coupling $1/G_N$ can be fundamental due to topology, there are no topological invariants which could support the fundamentality of the electromagnetic coupling $1/\alpha$. This is in favour of the scenario in which the quantum electrodynamics is the effective low-energy theory, where for example the gauge fields emerge as the bilinear combinations of the fermionic fields, or/and the gauge fields emerge in the vicinity of the topologically stable Weyl points in the fermionic spectrum [17,42–44]. This, however, does not exclude the other possible pre-quantum and pre-spacetime theories, see Ref. [45] and references therein.

Conclusion. Several approaches to quantum gravity (including the model of superplastic vacuum; Diakonov tetrads emerging as the bilinear combinations of the fermionis fields; BF-theories of gravity; and effective acoustic metric) suggest that in general relativity the metric has dimension 2, i.e. $[g_{\mu\nu}] = 1/[L]^2$, irrespective of the dimension of spacetime. One consequence of such dimension of the metric is that the wave function in quantum mechanics is dimensionless, $[\psi(x)] = [1] = 0$. This also leads to the dimensionless quantum fields.

On the other hand, if one starts with the conjecture that in quantum mechanics the wave function is naturally dimensionless, one obtains dimension 2 for metric. This suggests the close connection between quantum mechanics and general relativity.

Acknowledgements. This work has been supported by the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (Grant Agreement No. 694248).

The full text of this paper is published in the English version of JETP.

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