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ON THE MICROSCOPIC APPROACH TO THE ANDREEV CURRENT...

P. I. Arseev [a*], ***N. S. Maslova*** [b], ***Yu. M. Bilinskii*** [a]

[a] *P. N. Lebedev Physical Institute of the Russian Academy of Sciences 119991, Moscow, Russia*

[b] *M. V. Lomonosov Moscow State University 119991 Moscow, Russia*

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It was shown how we can describe microscopically the Andreev current in a uniform way for a contact with direct coupling between N and S leads and with intermediate chain of atoms (multilayer system) inside the contact. Considering various types of connection of the normal lead to external thermal bath we reproduce various nonequilibrium distributions at the edge of the normal lead. It was shown what type of connection to the external reservoir corresponds to the classical result of Blonder,Klapwijk and Tinkham. Also we discuss difference in equilibrium and non equilibrium proximity effect, and it is clarified that the Andreev current arises due to the nonequilibrium effects which is much larger than the equilibrium one.

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1. INTRODUCTION

One of the interesting manifestations of superconductivity is macroscopic quantum tunneling effects. In the case of a contact between two superconductors it is Josephson effect at first. In a junction between superconductor and normal metal macroscopic quantum nature of superconducting state also reveals as the Andreev current (These phenomena attracted attention rather long ago — see review by K.Likharev [1]). Usually qualitative explanation of the Andreev current in NS contacts is based on the process of the Andreev reflection [2] at the boundary described as reflection and transmission of quasiparticles at the tunneling barrier. In the widely cited paper by Blonder,Klapwijk and Tinkham (BTK) [3] one dimensional scattering model was used to write down the equation for the current. In this theory normal metal in the vicinity of the contact do not suppose to be in thermal equilibrium. That means that electron distribution function in the N reagion are not the Fermi function depended only on the energy of the particles. The assumption of the authors of [3] was that electrons moving to the contact obey Fermi distribution

$$n_{\rightarrow}(\omega) = n_F(\omega - eV)$$

while reflected and moving out electrons are non equilibrium

$$n_{\leftarrow}(\omega) = An_F(\omega + eV) + Bn_F(\omega - eV),$$

whith coefficients A and B calculated in the scattering theory. The question how this non equilibrium distribution forms if at some distance there should be thermal equilibrium distribution was out of consideration in that paper. Contrary to the scattering approach to the tunneling transport there is also a method based on nonequilibrium Green's functions [4]. In this approach a system is placed between two thermal baths with different chemical potentials and all transport characteristics and nonequilibrium distributions in the intermediate system are calculated using Green's functions. This formulation of the problem seems to be much more adequate for experimental setups. One of the first papers in this direction was written rather long ago [5], but this approach became widespread later in the 90s.

For superconductor structures there were rather many papers in which quasiclassical approach was used.

* E-mail: ars@td.lpi.ru

In dirty metals quasiclassical equations are simplified to the Usadel equations [6] which look easier than the Gorkov equations. Last years people were interested in complicated multiterminal problems, but initially some papers were devoted to a simple NS contact [8] [7].

Nowadays superconducting hybrid structures with very small dimensions are fabricated. To our opinion it is useful to return to the initial microscopic picture of the contacts not only to repeat the BTK result, but to try to understand for what systems it is valid, and how it is connected with microscopis language and nonequilibrium Green's function approach.

For contacts with superconductors such microscopic approach was used by Cuevas, Martin-Rodero and Levy-Yeyati in [9].

We present here theoretical approach for one dimensional or quasi one-dimensional (planar) contacts based on nonequilibrium diagram technique similar to [9] but taking into account all changes in electron properties at the edges of the contact. Besides we do not suppose that normal metal is a thermal bath itself, which allows to show the origin of nonequilibrium electron distribution. So a more general system than in [9] is considered with some intermediate region between the external reservoirs. This part of the present paper complements the old paper of the authors [10] in which normal current characteristics were considered in the NS contact with additional atomic state between the leads.

2. ONE DIMENSIONAL MODEL

The one dimensional model of NS contact is shown in Fig. 1 and can be described by the following Hamiltonian $\hat{H}$

$$\hat{H} = \sum_{i\sigma} \mu a^{+}_{i\sigma} a_{i\sigma} + \sum_{i\geq 1,\sigma} t(a^{+}_{i,\sigma} a_{i+1,\sigma} + \text{H.c.}) + \\ + \sum_{i\geq 1,\sigma} t(c^{+}_{i,\sigma} c_{i+1,\sigma} + \text{H.c.}) + \sum_{i\geq 1,\sigma} (\Delta c^{+}_{i\uparrow} c^{+}_{i\downarrow} + \text{H.c.}) + \\ + T(c^{+}_{1,\sigma} a_{1,\sigma} + \text{H.c.}) \quad (1)$$

In the both parts of the chain we number the sites from one to the left or to the right from the contact. The first line describes normal metal, the second line — superconductor and term in the third line describes the tunneling between normal and superconducting parts of the chain. We assume in what follows that all energies are counted from the chemical potential of superconductor, so in the equation (1) $\mu_{sp} = 0$, T — is the tunneling matrix amplitude between N and S parts. The hopping matrix element t in the chain is supposed to be the same in the superconducting and normal part

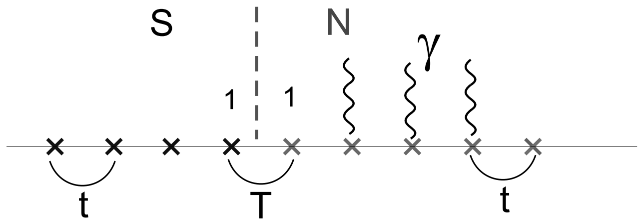

Fig. 1. One dimensional NS system with added relaxation in the normal part shown as a weak connection to external bath at each site (wavy lines)

of the chain. (Later we consider also modified formulas with different t_S, t_N). So the choice $T = t$ evidently corresponds to the «ideal transparency» of the contact. Operators c_i correspond to electrons in superconductor and a_i — to the normal metal in the site representation.

Tunneling current through the contact has the usual form (electrical current is eI):

$$I = iT\left[\langle c^{+}_{1} a_1\rangle - \langle a^{+}_{1} c_1\rangle\right] = T\left[G^{<}_{NS} - G^{<}_{SN}\right]. \quad (2)$$

Though the current is expressed only in terms of normal Green's functions, calculations with superconductors inevitably incorporate anomalous functions. It is convenient to use well known matrix representation in order to simplify calculations with superconductors

$$\Gamma^{R}(\omega) = \begin{pmatrix} G^{R}(\omega) & -F^{R}(\omega) \\ F^{+R}(\omega) & G^{A}(-\omega) \end{pmatrix}, \quad (3)$$

$$\Gamma^{<}(\omega) = \begin{pmatrix} G^{<}(\omega) & -F^{<}(\omega) \\ F^{+<}(\omega) & G^{>}(-\omega) \end{pmatrix}. \quad (4)$$

We use the following definitions of Keldysh functions (here t and t' lie on the Keldysh contour):

$$\begin{aligned} G_{nm}(t,t') &= -i\langle T_c c_{n\uparrow}(t) c^{+}_{m\uparrow}(t')\rangle, \\ F_{nm}(t,t') &= i\langle T_c c_{n\uparrow}(t) c_{m\downarrow}(t')\rangle, \\ F^{+}_{nm}(t,t') &= i\langle T_c c^{+}_{n\downarrow}(t) c^{+}_{m\uparrow}(t')\rangle. \end{aligned} \quad (5)$$

In this representation tunneling interaction is proportional to the unity matrix

$$\hat{T} = T\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (6)$$

Let us first calculate matrix Green's function for the normal and superconducting semi infinite chains. For infinite normal metal chain:

$$G^{0R}_{nm}(\omega) = \\ = a\int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} dk e^{ika(n-m)} \frac{1}{\omega + i\delta + \mu - 2t\cos(ka)}, \quad (7)$$

$$G^{0A}_{nm}(\omega) = \left[G^{0R}_{mn}(\omega)\right]^*, \quad (8)$$

further on $\omega = \omega + \mu$

Exact calculation of integral in (7) gives

$$G^{0R}_{nm}(\omega) = -\frac{i}{2t}\frac{1}{cos(\phi)}\times \\ \times\left[(-i)^{|n-m|}e^{i|n-m|\phi} - (-i)^{(n+m)}e^{i(n+m)\phi}\right], \quad (9)$$

where $\sin\phi(\omega) = \omega/2t$

It is possible to emulate the behavior of a finite chain by using the infinite chain with infinitely strong point defects $U \to +\infty$ added at site 0. Then in matrix form Green's functions for semi infinite chain gives by

$$\Gamma^R_{nm}(\omega) \equiv \\ \equiv \Gamma^{0R}_{nm}(\omega) - \Gamma^{0R}_{n0}(\omega)\left(\Gamma^{0R}_{0,0}(\omega)\right)^{-1}\Gamma^{0R}_{0,m}(\omega). \quad (10)$$

For the normal metal chain which begins from the site number 1 this equation gives

$$G^R_{1m}(\omega) = -\frac{i}{t}(-i)^{(m-1)}e^{im\phi}. \quad (11)$$

If we add in the Hamiltonian (7) interaction with a bath at each site of the normal chain (which is shown symbolicaly as wavy lines in Fig. 1) it can be proved, that we have simply to replace $\omega \to \omega + i\gamma$ Futher on we shall use only single site diagonal Green's functions G_{11}, F_{11} and omit this site index (11) using matrix representation

$$\hat{\Gamma}^{0R}_N(\omega) = \begin{pmatrix} G^{0R}_{11}(\omega) & 0 \\ 0 & G^{0A}_{11}(-\omega) \end{pmatrix} =$$

$$= -\frac{i}{t}e^{-\chi}e^{i\phi}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (12)$$

$$\sin(\phi)\,\mathrm{ch}(\chi) = \frac{\omega}{2t}, \quad \cos(\phi)\,\mathrm{sh}(\chi) = \frac{\gamma}{2t}.$$

For the superconductor chain we also start from Green's functions of infinite chain:

$$G^{0R}_{nm}(\omega) = a\int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} dk e^{ika(n-m)}\times \\ \times\frac{\omega + 2t\cos(ka)}{(\omega+i\delta)^2 - (2t\cos(ka))^2 - \Delta^2}, \quad (13)$$

$$F^{0R}_{nm}(\omega) = a\int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} dk e^{ika(n-m)}\times \\ \times\frac{\Delta}{(\omega+i\delta)^2 - (2t\cos(ka))^2 - \Delta^2}. \quad (14)$$

Similar calculations for the superconductor give for $\omega < \Delta$

$$\Gamma^{0R}_S(\omega) = \frac{-2e^{-\chi_0}}{(2t)^2\,\mathrm{sh}(\chi_0)}\begin{pmatrix} \omega & -\Delta \\ \Delta & -\omega \end{pmatrix}, \quad (15)$$

where

$$\mathrm{sh}(\chi_0) = \frac{\sqrt{\Delta^2-\omega^2}}{2t},$$

$$\mathrm{ch}(\chi_0) = \sqrt{1 + \frac{\Delta^2-\omega^2}{(2t)^2}}.$$

In terms of the matrix functions (3,4) the current (2) is written as

$$I = T^2\left[\Gamma^<_N\Gamma^{0A}_S - \Gamma^{0R}_S\Gamma^<_N + \Gamma^R_N\Gamma^{0<}_S - \Gamma^{0<}_S\Gamma^A_N\right]_{11}, \quad (16)$$

where the index (11) here means that at the end of the calculations we have to know only the element (11) of this matrix expression. In this formula Green's functions of superconductor are the initial ones without interaction with the normal part taking into account. But Green's functions of the normal chain should be calculated exactly with full account for the tunneling processes to the superconductor.

Note, that of course we can change this choice vice versa and can use completely symmetric expression as well

$$I = T^2\Big([R]^{-1}\Gamma^{0<}_N\Gamma^A_S - \Gamma^R_S\Gamma^{0<}_N[A]^{-1} + \\ +\Gamma^R_N\Gamma^{0<}_S[A]^{-1} - [R]^{-1}\Gamma^{0<}_S\Gamma^A_N\Big)_{11}.$$

For our problem it is easier to use unperturbed Green's functions of superconductor because for energies in the superconducting gap (if $eV < \Delta$) all $\Gamma^{0<}_S = 0$. This simplifies significantly all calculations. Exact retarded function of the normal metal is equal to

$$\Gamma^R_N = \left[1 - \Gamma^{0R}_N T^2\Gamma^{0R}_S\right]^{-1}\Gamma^{0R}_N \equiv [R]^{-1}\Gamma^{0R}_N,$$

here short notation $[R]^{-1}$ is used which corresponds to the sum of the perturbation series on the tunneling interaction. Using the unperturbed Green's function of the normal and superconducting chain we obtain

$$[R]=\left[1-iC\begin{pmatrix}\omega & -\Delta\\ -\Delta & \omega\end{pmatrix}\right],$$

where

$$C=\frac{1}{2t}\left(\frac{T^2}{t^2}\frac{e^{-\chi_0}}{\mathrm{sh}(\chi_0)}\right)e^{i\phi}e^{-\chi}. \qquad (17)$$

In the same notations, the exact $\Gamma_N^{<}$ is equal to

$$\Gamma_N^{<}=[R]^{-1}\,\Gamma_N^{0<}\,[A]^{-1}+\Gamma_N^R T^2\Gamma_S^{0<}\Gamma_N^A.$$

Let us look at the anomalous part of the normal Green's function appeared due to the proximity effect. The Andreev current is determined only by this part and is in essence the Josephson current appeared due to the induced anomalous part:

$$\begin{aligned}[][\Gamma_N^{<}]_{12}&=\\ &=-i\Big\{C\Delta[n(\omega-eV)-n(\omega+eV)]-\\ &-C^2\Delta\omega[n(\omega-eV)+n(\omega+eV)]\Big\}\times\\ &\times\left\{|(1-iC\omega)^2+C^2\Delta^2|^2\right\}^{-1}. \qquad (18)\end{aligned}$$

We choose the chemical potential of the superconductor to be zero and the chemical potential of the normal metal is shifted by the applied voltage $\mu_N=eV$. So the appearence of $n(\omega+eV)$ is connected with the element (22) of the matrix Green's function (4) since

$$G^{>}(-\omega)\propto(n(-\omega)-1)=$$

$$=n_F(-\omega-eV)-1=-n_F(\omega+eV).$$

Very interesting feature of eq.(18) is that it describes two different contributions to proximity effect. The second term proportional to C^2 corresponds to equilibrium proximity effect. And the first one is proportional to C but appears only in nonequilibrium situation, when $eV\neq 0$. Since $C\propto T^2$ the nonequilibrium contribution is usually much larger than equilibrium one for small transparency of the contact.

We should like to note that only nonequilibrium proximity effect is responsible for the Andreev current appearance.

If we are interested only in the Andreev current in the gap, let us suppose further that

$$\frac{\Delta}{2t}\ll 1,\quad \omega\le\Delta.$$

The current (16) is

$$\begin{aligned}I=\int\frac{d\omega}{2\pi}\frac{\Delta^2}{t^2}\left(\frac{T^2}{t^2}\frac{e^{-\chi_0}}{\mathrm{sh}(\chi_0)}\right)^2&\times\\ \times\frac{\cos^2(\phi)}{|(1-iC\omega)^2+C^2\Delta^2|^2}&\times\\ \times[n(\omega-eV)-n(\omega+eV)]&, \qquad (19)\end{aligned}$$

where C given by Eq.(17) is some function of ω and contains information about the tunneling amplitude and density of states at the edges of the contact.

For small $\gamma/2t$ and $\Delta/2t$ it simplifies to

$$C=\left(\frac{T^2}{t^2}\right)\frac{1}{\sqrt{\Delta^2-\omega^2}}$$

and the current density $I(\omega)$ becomes

$$I(\omega)=\frac{4\Delta^2\left(\frac{T^2}{t^2}\right)^2}{\left[1+\left(\frac{T^2}{t^2}\right)^2\right]^2(\Delta^2-\omega^2)+4\left(\frac{T^2}{t^2}\right)^2\omega^2}. \qquad (20)$$

This formula is in agreement with the BTK equation (see Ref. [9] as well). Of course in any case the Andreev current is proportional to T^4, the square of a «tunneling transparency», because it appears only due to the proximity effect. Note once more, that it is only nonequilibrium part of the proximity effect which makes contribution to the Andreev current. Tunneling conductivity for small bias is equal to

$$\frac{dI}{dV}(0)=I(\omega=0)=\frac{4\left(T^2/t^2\right)^2}{\left[1+(T^2/t^2)^2\right]}. \qquad (21)$$

The case of ideal transparency is formulated in this model as a condition $T=t$ which corresponds to the ideal metal chain one half of which is superconducting. In this case the current (20) does not depend on energy ω inside the gap:

$$\frac{dI}{dV}(\omega)=2, \qquad (22)$$

as in the paper [3].

Of course it is not obligatory that normal and superconducting parts are the same material, so hopping matrix elements t in the Hamiltonian (1) can be different for the normal and supercondcuting chains. In this case the current (20) looks like

$$\begin{aligned}I(\omega)&=\\ &=\frac{4\Delta^2\left(\frac{T^2}{t_St_N}\right)^2}{\left[1+\left(\frac{T^2}{t_St_N}\right)^2\right]^2(\Delta^2-\omega^2)+4\left(\frac{T^2}{t_St_N}\right)^2\omega^2}. \qquad (23)\end{aligned}$$

Formally for $T^2 = t_S t_N$ we again reproduce the case of ideal transparency, but it is doubtful that it makes some physical sense.

So we see that the system, in which BTK consideration is literally valid, is the NS contact with very small relaxation rate to equilibrium state spatially spread from the contact to infinity along the normal chain. In this case far from the contact we have thermal equilibrium distribution in the normal metal with the chemical potential $\mu = eV$ and in the vicinity of the contact this distribution is distorted so, that BTK non equilibrium distribution needed for the Andreev current appears.

3. CONTACT WITH INTERMEDIATE REGION

In this section we consider a more general tunneling system with some intermediate region as it is shown in Fig. 2. Superconductor is connected to one dimensional chain, but this chain consists of finite number of sites L. The other end of the chain is connected to massive thermal bath with γ_0 be the tunneling rate to the bath. For the unperturbed (without interaction with the superconductor) retarded Green's function we obtain

$$G_{11}^{0R}(\omega) = -\frac{i}{t}\frac{1}{sch_{L+1}(\phi)} \times \times \left[sch_L(\phi) - (-1)^{L+1}\left(\frac{\gamma_0}{t}\right) \times \times \frac{sch_1^2(\phi)}{sch_{L+1}(\phi) + \left(\frac{\gamma_0}{t}\right) sch_L(\phi)} \right]. \quad (24)$$

This is the Green's function in site representation for the edge site (site number 1) of the normal chain. For brevity we introduced the notation

$$sch_L(\phi) = e^{-iL\phi} - (-1)^L e^{iL\phi}.$$

Denominator of the G^R describes resonances in the intermediate system. Without relaxation these resonances are simply size quantised states in a finite chain determined by the condition $sch_L(\phi) = 0$. Clearly there are two different limiting cases

$$\chi L = (\gamma/2t)L \gg 1 \quad \text{and} \quad \chi L = (\gamma/2t)L \ll 1.$$

(Remind that χ is determined by the intensity of the spread along the chain relaxation γ (12).) In the first case

$$sch_L(\phi) \simeq e^{L\gamma/2t} \gg 1,$$

and we return to the semi-infinite chain, because uniformly distributed along the chain relaxation γ almost completely «takes away» electron flow before it reaches the opposite end and the second contact to reservoir plays no role. Contrary to this in the second case we have a system of several sites (atoms) placed between the contacts which serve as thermal baths.

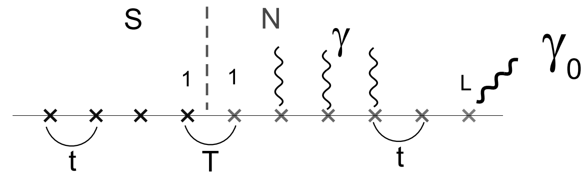

Fig. 2. Tunneling system with an intermediate region. Normal chain is of L sites length and at the end it is connected to a massive reservoire. The tunneling rate from the last site L to the reservoire is denoted as γ_0

For $L = 1$ the general formula (24) gives exactly the Green's function of a single site (atom).

$$G_{11}^{0R}(\omega) = \frac{1}{\omega + i\gamma_0}.$$

For $L = 2$ Eq. (24) gives the function for two sites and so on.

Equation (24) is written for the chemical potential equal to the site energy level, but can be easily modified for general case. If chemical potential does not coincide with atom level ε, then ω should be replaced by $\omega - \varepsilon$.

Now we have some new functions G_{11}^{0R} and consequently new equations for $C(\omega)$. But this is the only needed modification, all equations of the previous section are valid with replacement by the new function $C(\omega)$. For example for a single site in the contact

$$C = -\frac{i}{(\omega - \varepsilon_0) + i\gamma_0}\left(\frac{T^2}{t_S^2}\frac{e^{-\chi_0}}{2\,\mathrm{sh}(\chi_0)}\right). \quad (25)$$

This value of C leads for $\Delta \ll t_S$ to simplified expressions analogous to (21)

$$\frac{dI}{dV}(0) = I(\omega = 0) = \frac{4(t_S^2/\gamma_0^2)\left(T^2/t_S^2\right)^2}{\left[1 + (t_S^2/\gamma_0^2)\left(T^2/t_S^2\right)^2\right]^2}. \quad (26)$$

Conductivity at the edge of the gap in this case does not depend on T and band width $2t_S$

$$\frac{dI}{dV}(\Delta) = I(\omega = \Delta) = \frac{1}{2}\frac{\gamma_0^2}{\gamma_0^2 + \Delta^2}. \quad (27)$$

If γ_0 is small enough so, that $\gamma_0/t_S \ll (T/t_S)^2 \le 1$, then conductivity (26) is suppressed compared to (21),

$$\frac{dI}{dV}(0) = \frac{4(t_S^2\gamma_0^2)}{T^4}. \quad (28)$$

But it is possible also that $\gamma_0/t_S \ll 1$, but $\gamma_0/t_S \gg (T/t_S)^2$ for small tunneling transparency. Then

$$\frac{dI}{dV}(0) = \frac{4T^4}{(t_S^2\gamma_0^2)}. \tag{29}$$

Compared to (21), a resonant enhancing factor (t_S^2/γ_0^2) appears. Equations (26), (27) shows that tunneling conductivity is less then in the direct NS contact and increases with bias increasing even in the case $T = t_S$.

Let us look briefly at the chain with two atoms ($L = 2$), because it is the simplest example of intermediate system with several levels. For two atoms we have

$$G_{11}^{0R}(\omega) = \frac{\omega - \varepsilon + i\gamma}{(\omega - \varepsilon)^2 - t^2 + i\gamma_0\omega}, \tag{30}$$

$$C = i\frac{\omega + i\gamma_0}{\omega^2 - t^2 + i\gamma_0\omega}\left(\frac{T^2}{t_s^2}\frac{e^{-\chi_0}}{2\,\mathrm{sh}(\chi_0)}\right). \tag{31}$$

Substituting this value of the function C in our equations we get

$$\frac{dI}{dV}(0) = \frac{4(\gamma_0^2/t_S^2)\left(T^2/t^2\right)^2}{\left[1 + (\gamma_0^2/t_S^2)\,(T^2/t^2)^2\right]^2}, \tag{32}$$

$$\frac{dI}{dV}(\Delta) = \frac{1}{2}\frac{\gamma_0^2 t^4}{(\gamma_0^2 + \Delta^2)\left((\Delta^2 - t^2)^2 + \gamma_0^2\Delta^2\right)}. \tag{33}$$

We see that for γ_0 and t there are critical values, determined by the condition

$$\frac{\gamma_0}{t^2} = \frac{t_S}{T^2}.$$

For γ_0 less than this critical value the conductivity is suppressed while for large values we have some enhancement. Equation (33) shows that we return to the single atom situation for $t \gg \Delta$ when discrete levels are out of the superconducting gap. For $t \simeq \gamma_0 \ll \Delta$

$$\frac{dI}{dV}(\Delta) \simeq \frac{\gamma_0^2 t^4}{\Delta^6} \tag{34}$$

is negligibly small under these conditions.

So varying the parameters of intermediate system (several atoms chain) we theoretically can enhance Andreev conductivity in some limited bias region. But the more general situation is that the Andreev current is suppressed.

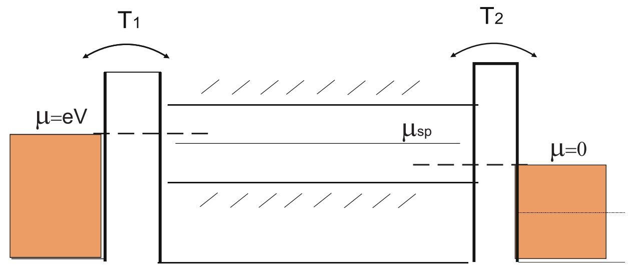

Fig. 3. The NSN structure with superconductor connected only to the two normal leads. The position of the chemical potential of superconductor between chemical potentials of the normal leads is determined by the current conservation law and is not symmetric for different tunneling amplitudes T_1 and T_2

4. NSN JUNCTION

Up to now it was supposed that the chemical potential of superconductor is fixed. For example the superconductor is grounded and relaxation in superconductor is quick enough to fix $\mu = 0$.

In a double junction NSN system we can not set $\mu = 0$ in superconductor. We control only the difference of chemical potentials of the normal contacts by applying some bias. Such system is shown in Fig. 3. Previous consideration shows that the Andreev current is determined only by the difference of chemical potentials of N and S system at a contact. For the energies in the superconductor gap and enough long superconductor this current «does not feel» other contact at the second end of the superconductor. Of course this is seeming uncertainty because of the following. If superconductor is under some external fields, then we have to take into account that superconducting order parameter also changed [11]. In our case the main effect is in changing the phase of Δ. This additional phase changes should be rewritten as additional voltage bias using gauge transformation. Then using the self consistency equations for superconductor these phase changes can in principal be connected with the external bias. But it is known, that this procedure is completely equivalent to the current conservation law. Thus the current conservation condition

$$I_1(eV - \mu) = I_2(\mu),$$

where I_1 and I_2 are the Andreev currents (19) in the left and right junctions, is the condition which determines the value of μ in superconductor. So in the NSN system the current and all properties of intermediate system is also completely determined by the two external baths as in a normal system, but with additional complication connected with indirect procedure of phase determination.

5. CONCLUSION

The microscopic approach of [9] was extended to a more general situation when normal metal in NS one dimensional contact is not supposed to be a reservoir. Normal chain in our calculations is connected to external thermal bath in different ways. This makes it possible to describe the nonequilibrium distribution of electrons in the vicinity of the contact and to determine the dependence of the tunneling characteristics on the method of connecting the normal metal to the external reservoir. So we found that BTK solution for the Andreev current corresponds to spatially spread along the chain weak connection to external thermal bath at each site.

Within our approach we have unified description of the contacts of various types: from direct NS contact to a contact with some normal multilevel system in between the S and N sides. This intermediate system, as we have seen for a single and double level examples, mostly suppresses the Andreev current, although there are some parameter relations that can increase Andreev conductivity in some bias range.

We should like to pay attention to the following. Quasiparticles themselves of course give no contribution to the Andreev current for energies inside the superconducting gap, so the «scattering approach» by BTK is only an indirect method to calculate proximity effect and to describe anomalous parts induced in the normal metal. If we calculate the proximity effect directly using Green's function methods, we see that besides equilibrium contribution to the proximity effect there is also a nonequilibrium part. To our opinion it is less discussed, that proximity effect consists of two contributions and the nonequilibrium contribution is much larger than the equilibrium one. Within microscopic approach based on nonequilibrium diagram technique we see that the Andreev current appears only due to the nonequilibrium proximity effect and is in essence the «induced Josephson current».

This approach can be easily extended to the case of a planar contact. In the mixed representation for Green's functions we can use functions $G_{mn}(p, \omega)$ where p is a momentum along the plane of the contact and m, n are site numbers in perpendicular direction. The only modification required is to add the momentum integral in all formulas for the current.

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