N.N. Bogoliubov,
"A new method in the theory of superconductivity"

The microscopical theory of superconductivity was developed by Bardeen, Cooper, Schrieffer (BCS), and Bogoliubov in 1957. Originally, a short version of the BCS theory was published in Physical Review Letters [1], while a detailed presentation has appeared later [2]. The BCS theory is based on the assumption that the normal state of a metal is unstable to the formation of bound states of electrons with opposite spins and momenta. This phenomenon was discovered a year earlier by Cooper who considered a system of two electrons with a weak attraction near the Fermi surface [3]. In the BCS theory, a variation approach was used for the wave function where existence of the Cooper pairs in the superconducting state was explicitly taken into account. The calculations were performed for a model Hamiltonian in which electron-phonon interaction was replaced by a direct electron attraction leading to Cooper pair formation near the Fermi surface. To prove the BCS theory, it was required to develop a microscopical theory without using the variational approach for a trial wave function of a predetermined form and to consider electron-phonon interaction explicitly.

Such a theory was proposed by Bogoliubov for the electron-phonon Fröhlich Hamiltonian in the paper [4] [A new method in the theory of superconductivity. I, Soviet Physics JETP **34**, 41 - 46 (1958)] submitted to JETP before publication of the full version of the BCS theory [2]. Studying phenomena of electron-phonon interaction, the author has shown that virtual processes of creation of electron pairs produce divergent contributions. To exclude these "dangerous" diagrams the method of the canonical (u, v)-transformation to new quasi-particles which were superposition of electronic and hole states with opposite spins and momenta was used. Applying the method of compensation of "dangerous" diagrams developed previously by the author in the theory of superfluidity, Bogoliubov has derived a model Hamiltonian for an ideal Fermi-gas of quasi-particles. The spectrum of quasi-particle excitations was described by the equation $E(\mathbf{k}) = \sqrt{[\varepsilon(\mathbf{k})]^2 + [C(\mathbf{k})]^2}$ where $\varepsilon(\mathbf{k})$ is the electronic spectrum in the normal state and $C(\mathbf{k})$ is the gap in the spectrum in the superconducting state. The equations for the gap and the superconducting temperature obtained by the author corresponded to that ones in the model theory of BCS. Thus, introduction of new quasi-particles (which are called sometimes as "bogoliubons") with the spectrum $E(\mathbf{k})$ enables to give a clear physical picture of elementary excitations in the superconducting state as a superposition of an electron and a hole. Application of the model Hamiltonian for quasi-particles also considerably simplifies calculations of various physical properties of superconductors.

In the second publication [5] [A new method in the theory of superconductivity. II, Soviet Physics JETP **34**, 46 - 50 (1958)], using the perturbation theory over the electron-phonon coupling constant the authors derived an effective Hamiltonian with direct electron-electron interaction similar to the model Hamiltonian of BCS. Then, using the method of the canonical (u, v)-transformation to quasi-particles and the method of compensation of "dangerous" diagrams as in Ref. [4], the authors derived equations for the gap and energy of the ground state in the superconducting phase corresponding to obtained in Ref. [4] and the BCS theory. In the third publication [6] [A new method in the theory of superconductivity. III, Soviet Physics JETP **34**, 51 - 55 (1958)] a model of interacting pairs of electrons in terms of Pauli operators was considered. Using the (u, v)-transformation and the method of compensation of "dangerous" diagrams the author derived an equation for the gap in the electronic spectrum in the superconducting state corresponding to the BCS theory. In the monograph [7], besides the presentation of Refs. [4] and [6], a role of the Coulomb interaction of electrons in the superconducting state was studied. It was shown that retardation effects for the electron-phonon interaction results in a considerable decrease of the Coulomb repulsion of electrons due to a logarithmic renormalization of this interaction (the logarithmic Bogoliubov-Tolmachev renormalization). It is worth noting Refs. [8] where Bogoliubov and coauthors have proved that the results obtained for the model BCS Hamiltonian with factorized interaction is asymptotically exact in the thermodynamical limit, $V \to \infty$.

Thus, the introduction of new quasi-particles using the (u, v)-transformation is proved to be an effective method for developing theory of superconductivity both for the original electron-phonon Hamiltonian and for the model Hamiltonian with direct electron-electron interaction. In what follows, the (u, v)-transformation found a wide application in the diagonalization of bilinear operator forms with "anomalous" terms. In particular, this method has been used in studies of inhomogeneous superconductors where (u, v) parameters of the transformation depend on coordinates (Bogoliubov-De Jennes transformation [9]) and in calculation of spin-wave excitation spectrum in antiferromagnets where also "dangerous" diagrams appear.

[1] J. Bardeen, L. Cooper, and J. Schrieffer, Phys. Rev. **106**, 162 (1957).
Microscopic theory of superconductivity.

[2] J. Bardeen, L. Cooper, and J. Schrieffer, Phys. Rev. **108**, 1175 (1957).
Theory of superconductivity.

[3] L. Cooper, Phys. Rev. **104**, 1189 (1956).
Bound electron pairs in degenerate Fermi gas.

[4] N. N. Bogoliubov, Soviet Physics JETP **34**, 41 - 46 (1958).
A new method in the theory of superconductivity. I.

[5] V. V. Tolmachev, S. V. Tiablikov, Soviet Physics JETP **34**, 46 - 50 (1958).
A new method in the theory of superconductivity. II.

[6] N. N. Bogoliubov, Soviet Physics JETP **34**, 51 - 55 (1958).
A new method in the theory of superconductivity. III.

[7] N. N. Bogoliubov, V. V. Tolmachev, D. V. Shirkov, *New method in the theory of superconductivity*, (Consultants Bureau, New York, London, Chapman and Hall, 1959, YII).

[8] Н. Н. Боголюбов, Д. Н. Зубарев, Ю. А. Церковников, Докл. АН СССР, **117**, 788 (1957). К теории фазового перехода; ЖЭТФ **39**, 120 - 129 (1960). Асимптотически точное решение для модельного гамильтониана теории сверхпроводимости; Н. Н. Боголюбов, препринт ОИЯИ Р-511, Дубна, 1960 (ЭЧАЯ.- 1971.- т.1, вып.2.- с.301-364.) К вопросу о модельном гамильтониане в теории сверхпроводимости.

[9] P. G. De Jennes, *Superconductivity of metals and alloys* (W.A. Benjamin, Inc., New York-Amsterdam, 1966).

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