APPENDIX

In Fig. 6 there is drawn the cone of particles scattered into an angle θ relative to the direction *AO* of the incident particles. Obviously any azimuthal angle φ is equally probable. In the present case the direction of the primary beam lies in the plane *P* of the emulsion; therefore the dip angle β of a particle can be measured by the arc CF.

The probability that a particle has a dip angle $\beta \leq \beta_0$ is given by the ratio of the arc *CF* to the arc *CD*: $p = CF/CD$. Obviously p decreases with increasing θ.

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Oscillations in a Fermi Liquid

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Different types of waves that can be propagated in a Fermi liquid, both at absolute zero and at non–zero temperatures, are investigated. Absorption of these waves is also considered.

THE present paper is devoted to the study of the propagation of waves in a Fermi liquid, and proceeds from the general theory of such liquids developed by the author.[1] These phenomena in a Fermi liquid should be distinguished by a large singularity, connected primarily with the impossibility of propagation in it of ordinary hydrodynamic sound waves at absolute zero. The latter circumstance is already evident from the fact that the path length, and therefore the viscosity of a Fermi liquid, tends to infinity for $T \to 0$, as a result of which the sound absorption coefficient increases without limit.

It is shown, however, that in a Fermi liquid at absolute zero other waves can be propagated; these differ in nature from ordinary sound, and we shall call them waves of "zero sound".

Initially, theproblem of vibrations in a Fermi liquid was considered by Gol'dman[2] in application to an electron gas with Coulomb interaction between the particles. The problem of a gas with uncharged particles, considered in detail here for liquids, was first considered in the research of Klimontovich and Silin,[3] and later in a series of works of Silin.[4-6] There, the gas was considered to be slightly non–ideal, with an interaction satisfying the conditions of applicability of perturbation theory.

1. VIBRATIONS IN A FERMI LIQUID AT ABSOLUTE ZERO

We begin with the investigation of those vibrations at absolute zero which do not involve the spin characteristics of the liquid. This means that not only the equilibrium distribution function n_0, but also the "perturbing" function

$$n = n_0 + \delta n\,(\mathbf{p}) \qquad (1)$$

is independent of the spin variables. At absolute zero, n_0 is a step function which is broken off at the limiting momentum $p = p_0$. *

The energy of the quasi–particles (elementary excitations) is a function of n, i.e., the form of the function $\epsilon\,(p)$ depends on the form of $n\,(p)$. By analogy to (1), we write it in the form

$$\varepsilon = \varepsilon_0\,(p) + \delta\varepsilon\,(\mathbf{p}), \qquad (2)$$

where the function $\epsilon_0\,(p)$ corresponds to the distribution $n_0\,(p)$. The value of $\delta\epsilon$ itself is connected with δn by a formula of the form (see Ref. 1):

$$\delta\varepsilon\,(\mathbf{p}) = \mathrm{Sp}_{\sigma'}\int f\,(\mathbf{p},\mathbf{p}')\,\delta n' d\tau', \qquad (3)$$

$$d\tau = d^3\mathbf{p}\,/\,(2\pi\hbar)^3.$$

Inasmuch as δn is assumed to be independent of the spin variable, the operation Sp is applied only to the scattering amplitude f. But the scalar function $\mathrm{Sp}_{\sigma'}f$ can contain the spin operator σ only in the form of the product $\sigma\,[\mathbf{pp}']$ of two axial vectors: σ and $[\mathbf{pp}']$ (we do not consider expressions containing two products of components of σ, since for spin 1/2, as is well known, they reduce to expressions containing σ in the zeroth or first degree). But this product is not invariant to a time reversal and therefore cannot enter into the invariant quantity $\delta\epsilon$. Thus σ drops out completely and $\delta\epsilon$ is shown to be independent of the spin variable.

The kinetic equation for a Fermi liquid has the form:

$$\frac{\partial n}{\partial t} + \frac{\partial n}{\partial \mathbf{r}}\frac{\partial \varepsilon}{\partial \mathbf{p}} - \frac{\partial n}{\partial \mathbf{p}}\frac{\partial \varepsilon}{\partial \mathbf{r}} = I\,(n), \qquad (4)$$

where $I\,(n)$ is the integral of collisions between quasi–particles. The number of collisions is proportional to the square of the width of the diffusion zone, so that at absolute zero, $I(n) = 0$. Substituting (1) and (2) in (4), and considering that n_0 and ϵ_0 do not depend on r, we get

$$\frac{\partial \delta n}{\partial t} + \frac{\partial \delta n}{\partial \mathbf{r}}\frac{\partial \varepsilon_0}{\partial \mathbf{p}} - \frac{\partial \delta\varepsilon}{\partial \mathbf{r}}\frac{\partial n_0}{\partial \mathbf{p}} = 0,$$

and assuming δn and $\delta\epsilon$ to be proportional to

$$e^{-i\omega t + i\mathbf{kr}},$$

$$(\mathbf{kv} - \omega)\,\delta n = \mathbf{kv}\,\frac{\partial n_0}{\partial \varepsilon}\,\delta\varepsilon, \qquad (5)$$

where we have introduced the velocity of the quasi-particles $\mathbf{v} = \partial\epsilon_0\,/\,\partial\mathbf{p}$. In view of the absence of the δ–function $\partial n_0/\partial\epsilon$ from the right hand side of this equation, there actually enter in them only the values of all quantities taken at the limit $p = p_0$ of the (unperturbed) Fermi distribution. We introduce a new notation for what follows:

$$F = \mathrm{Sp}_{\sigma'}f\,(\mathbf{p},\mathbf{p}')\,4\pi p^2 dp\,/\,(2\pi\hbar)^3\,d\varepsilon. \qquad (6)$$

Then we can write Eq. (3) in the form:

$$\delta\varepsilon = \iint F\,\delta n' d\varepsilon' do'\,/\,4\pi.$$

Here only the $\delta n'$ are functions changing rapidly with ϵ'. Therefore, we can rewrite this expression in the form:

$$\delta\varepsilon = \int F\nu' do'\,/\,4\pi, \qquad (7)$$

where the function

*To avoid excessive complication of our study, we limited ourselves to the simplest and most important caseof an energy spectrum with an occupied region represented by a uniform sphere of radius p_0.

$$\nu(\mathbf{n}) = \int \delta n(\mathbf{p})\, d\varepsilon \tag{8}$$

has been introduced which depends only on the direction n of the vector p, and the function $F(\mathbf{p}, \mathbf{p}')$ is taken on the boundary of the (unperturbed) Fermi distribution; here F depends only on the angle χ between p and p′.

We note for what follows that the relation found in Ref. 1, which connects the actual mass m of the particles with the effective mass m^* of the quasi–particles, can, with the help of the function $F(\chi)$ be written in the form

$$\overline{F \cos\chi} = (m^*/m) - 1, \tag{9}$$

where the bar denotes averaging over the directions (in the derivation of this relation, we assume in (6) that $\epsilon = p^2/2m^*$). The equation for the velocity of ordinary sound c can be put in the form

$$\overline{F} = 3nm^*c^2/p_0^2 - 1. \tag{10}$$

Let us substitute (7) in Eq. (5) and integrate the latter over $d\epsilon$. This gives

$$(\mathbf{kv} - \omega)\nu = -\mathbf{kv}\int F\nu'\, do'/4\pi.$$

Let us take the direction of k as the polar axis, and let the angles θ, φ define the direction of the momentum p (and the direction of v coinciding with it) relative to this axis. Also, we introduce the propagation velocity $u = \omega/k$ of this wave, and the notation $\eta = u/v$, so that we can finally write the resultant equation in the form

$$(\eta - \cos\theta)\,\nu(\theta, \varphi) = \cos\theta \int F(\chi)\,\nu(\theta', \varphi')\, do'/4\pi. \tag{11}$$

This integral equation defines the principal velocity of propagation of the waves and the form of the function $\nu(\theta, \varphi)$ in them. The latter has the following graphic meaning. The fact that δn is proportional [as is evident from Eq. (5)] to the derivative $\partial n_0/\partial\epsilon$ means that the change of the distribution function for vibrations reduces to the deformation of the boundary of the Fermi surface (a sphere in the undisturbed distribution). The integral of (8) represents the magnitude of the displacement (in energy units) of this surface in the given direction n.

We at once note that it follows from the form of Eq. (11) that the real (only the undamped vibrations are of interest to us) value of η ought to exceed 1, i.e., the propagation velocity of the waves satisfies the inequality

$$u > v. \tag{12}$$

As an example, let us investigate the case in which the function $F(\chi)$ reduces to a constant (we denote it by F_0). The integral on the right hand side of Eq. (11) does not depend on the angles θ, φ in this case. Therefore the desired function ν has the form (we omit the exponential factor):

$$\nu = \text{const} \cdot \cos\theta / (\eta - \cos\theta). \tag{13}$$

The limiting Fermi surface has the form of a surface of revolution, elongated in the forward direction of the propagation of the wave, and flattened in the opposite direction. For comparison, let us point out that the ordinary sound wave corresponds to a function ν of the form $\nu = \text{const} \cdot \cos\theta$ which represents the displacement of the Fermi surface as a whole, without a change in shape.

For the determination of the velocity u, we substitute Eq. (13) in (11) and get

$$\frac{F_0}{4\pi}\int_0^\pi \frac{\cos\theta}{\eta - \cos\theta}\, 2\pi \sin\theta\, d\theta = 1.$$

Carrying out the integration, we find the following equation, which determines in implicit form the velocity of the wave for a given value of F_0:

$$\varphi(\eta) \equiv \frac{\eta}{2}\ln\frac{\eta+1}{\eta-1} - 1 = \frac{1}{F_0}. \tag{14}$$

The function $\varphi(\eta)$ decreases monotonically from $+\infty$ to 0 for a change of η from 1 to ∞, always remaining positive. It then follows that the waves under consideration can exist only for $F_0 > 0$. Inasmuch as the function F is proportional to the scattering amplitude, taken with opposite sign (at the angle 0°), of the quasi–particles with one another [(see Ref. 1)], then the latter must be negative, which corresponds to the mutual collision of quasi–particles. However, it must be emphasized that this conclusion applies only to the case F = const. If the function $F(\chi)$ is not constant (and at the same time is not small compared with unity; see below), then propagation of zero sound is in general possible, bor both attractive and repulsive interactions of the quasi–particles.

For $\eta \to \infty$: $\varphi(\eta) \approx 1/3\eta^2$. Therefore, large F_0 corresponds to $\eta = \sqrt{F_0/3}$. In the opposite case of $F_0 \to 0$, we find that η tends toward unity according to the relation

$$\eta - 1 \sim e^{-2F_0}. \tag{15}$$

The latter case has much more general value. It corresponds to zero sound in an almost ideal Fermi gas for arbitrary form of the function $F(\chi)$. Actually, an almost ideal gas corresponds to a function F which is small in absolute magnitude. It is seen from Eq. (11) that in this case η will be close to unity and the function ν will be significantly different from zero only for small angles θ. On this basis, and being concerned only with this range of angles, we can replace the function F in the integral on the right side of Eq. (11) by its value for $\chi = 0$ (for $\theta \to 0$ and $\theta' \to 0$, $\chi \to 0$ also). As a result, we again recover Eqs. (13) and (15) with the constant F_0 replaced by $F(0)$ (this result coincides with that obtained earlier by Silin[4]).

We note that in a weakly non–ideal Fermi gas, the velocity of zero sound exceeds the velocity of ordinary sound by a factor of $\sqrt{3}$. Actually, for the former, we have $\eta \approx 1$, i.e., $u \approx v$. For the velocity of ordinary sound we get from Eq. (10) (neglecting the term $\bar{F}$ in it and setting $m^* \approx m$): $c^2 \approx p_0{}^2/3m^2 = v^2/3$

In the general case of an arbitrary dependence of $F(\chi)$, the solution of Eq. (11) is not well defined. In principle, it permits the existence of different types of zero sound, which are distinguished from one another by the angular dependence of their amplitude $\nu(\theta, \varphi)$, and which are propagated with different velocities. Along with the axially symmetric solutions of $\nu(\theta)$, asymmetric solutions can also exist. In these ν has an azimuthal factor $e^{\pm im\varphi}$ (m = integer)

Thus, for a function $F(\chi)$ of the form

$$F = F_0 + F_1 \cos\chi \tag{16}$$
$$= F_0 + F_1(\cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\varphi - \varphi'))$$

solutions can exist with

$$\nu \sim e^{\pm i\varphi}.$$

Actually, substituting Eq. (16) in (11) and carrying out the integration over $d\varphi'$ (assuming in this case that $\nu = f(\theta)e^{i\varphi}$), we obtain

$$(\eta - \cos\theta) f = \frac{F_1}{4}\cos\theta\sin\theta\int_0^\pi \sin^2\theta' f' d\theta'.$$

Thence,

$$\nu = \text{const}\cdot\frac{\sin\theta\cos\theta}{\eta - \cos\theta}e^{i\varphi}. \tag{17}$$

Conversely, substituting this expression in the equation, we obtain the relation

$$\int_0^\pi \frac{\sin^3\theta\cos\theta}{\eta - \cos\theta}d\theta = \frac{4}{F_1}, \tag{18}$$

which determines the dependence of the propagation velocity on F_1. The integral on the left side of the equation falls off monotonically with increase in the function η. Therefore its maximum possible value is achieved for $\eta = 1$. Computing the integral, we find that the corresponding (the least achieved) value of F_1 is 6. Thus, propagation of the asymmetric wave of the form (17) is possible only for $F_1 > 6$.

Turning to a real Fermi liquid—the liquid He^3—it is reasonable to attempt to approximate the unknown function $F(\chi)$ by the two term expression (16). We can determine the coefficients F_0 and F_1 entering into it by means of the relations

$$F_0 = 3mm^*c^2/p_0^2 - 1, \quad F_1/3 = m^*/m - 1$$

[see Eqs. (9) and (10)], knowing the values of the effective mass m^* and the velocity of ordinary sound c. We can derive the first from experimental data on the temperature dependence of the entropy (in the lowest temperature region). From the data available at present,[7] we get $m^* = 1.43\, m$ (m is the mass of the He^3 atom). For the velocity c, we get 195 m/sec from the data of Walters and Fairbank[8] on the compressibility of liquid He^3. Finally, p_0 is obtained directly from the density of the liquid:

$$p_0/\hbar = 0.76\times10^8 \text{ cm}^{-1}.$$

On the basis of these data, we obtain

$$F_0 = 5.4; \quad F_1 = 1.3. \tag{19}$$

From these values, we can draw a conclusion about the fact that in liquid He^3 the propagation of asymmetric zero sound is impossible. For symmetric zero sound, the solution of the equation with the function $F(\chi)$ from (16) and (19)* leads to the value $\eta = 1.83$, when we obtain $u = v = 1.83 p_0/m^* = 206$ m/sec.

The possibility of the propagation of waves in a Fermi liquid at absolute zero means that its energy spectrum can automatically possess a "Bose branch" in the form of phonons with energy $\epsilon = up$. However, one must say that it would be incorrect to introduce corrections corresponding to this branch in the thermodynamic quantities of the Fermi liquid, inasmuch as it has a much higher power of the temperature (T^3 in the heat capacity) than the departures from the approximate theory developed in Ref. 1.

2. VIBRATIONS OF A FERMI LIQUID AT TEMPERATURES ABOVE ZERO

For low, but non–zero, temperatures, mutual collisions of quasi–particles take place in the Fermi liquid. The number of these collisions is proportional to T^2. The corresponding relaxation time (the free path time) is $\tau \sim 1/T^2$. The character of the waves propagated in the liquid naturally depends fundamentally on the relations between their frequency and the reciprocal of the relaxation time.

For $\omega\tau \ll 1$ (which is actually equivalent to the condition of the shortness of the free path length of the quasi–particles in comparison with the wave–length λ), the collisions succeed in establishing thermodynamic equilibrium in each (small in comparison with λ) element of volume of the liquid. This means that we are dealing with ordinary hydrodynamical sound waves, propagated with a velocity c.

If $\omega\tau \gg 1$, then, on the contrary, the collisions do not play essential roles in the process of the propagation of the vibrations, and we will have the waves of zero sound considered in the preceding section.

In both these limiting cases, the propagation of waves is accompanied by a comparatively weak absorption. In the intermediate region, $\omega\tau \sim 1$, the absorption is very strong and isolation of the different types of waves as undamped processes is not possible here.

*These computations were carried out by A. A. Abrikosov and I. M. Khalatnikov.

One can easily obtain the temperature and frequency dependence of the absorption coefficient γ in the region of ordinary sound with the aid of the known formula for the absorption of sound (see Ref. 9, for example), according to which γ is proportional to the square of the frequency and to the viscosity coefficient*. Inasmuch as the viscosity of a Fermi liquid is proportional to $1/T^2$ [10], then we find that

$$\gamma \sim \omega^2/T^2 \quad \text{for} \quad \omega \ll 1/\tau. \tag{20}$$

Absorption in the region of zero sound differs essentially in its character from absorption of ordinary sound. In the latter, the collisions cannot lead to a dissipation of the energy "into the noise" of the distribution, which is changed only by the sound vibrations as such. This is connected with the circumstance already mentioned, that a distribution changed in this fashion remains in thermodynamic equilibrium in each element of the volume. Therefore, the absorption of ordinary sound is connected with the effect of the collisions on the distribution function itself.

In the region of zero sound the collisions lead to absorption "into the background" of the distribution which is changed only by the vibrations themselves, which in this case are not in thermodynamic equilibrium (inasmuch as the form of the limiting Fermi surface is deformed). This change in the distribution function does not depend on the frequency, and therefore the absorption coefficient will not depend on the frequency either. The dependence of γ on the temperature is determined by its proportionality to the number of collisions, i.e.,

$$\gamma \sim T^2 \quad \text{for} \quad \varkappa T/\hbar \gg \omega \gg 1/\tau. \tag{21}$$

The upper limit of the region of applicability of this formula is determined by the inequality $\hbar\omega \ll \varkappa T$ ($\varkappa$ is Boltzmann's constant), which allows a classical consideration of collisions. We recall that the inequality assumed here,

$$\varkappa T/\hbar \gg 1/\tau,$$

i.e.,

$$\hbar/\tau \ll \varkappa T$$

(smallness of the quantum uncertainty of the energy of quasi–particles in comparison with $\varkappa T$), must

*The contribution to γ from second viscosity and thermal conductivity is proportional to a much higher power of T and is therefore inconsiderable.

hold since it is the condition of applicability of everything generally developed in the theory of the Fermi liquid.[1]

The determination of the absorption coefficient of zero sound in the frequency range $\hbar\omega \gtrsim \varkappa T$ requires quantum consideration. The corresponding calculations can be simplified if we develop them in such a way that we express the desired "quantum" absorption coefficient in terms of the "classical" from Eq. (21).

The absorption of sound quanta takes place in the collisions of quasi-particles. If we denote by ϵ_1 and ϵ_2 the energies of the quasi-particles before and after collisions, then at a given frequency ω, they are connected by the law of conservation of energy

$$\varepsilon_1 + \varepsilon_2 + \hbar\omega = \varepsilon_1' + \varepsilon_2'.$$

In addition to the collisions, we must also consider the inverse collisions, which are accompanied by the emission of sound quanta. Taking into consideration the well known properties of the collision probabilities of Fermi particles, we find that the total rate of decrease of the number of sound quanta as a result of collisions is given by the expression

$$\iiiint w(\mathbf{p}_1, \mathbf{p}_2; \mathbf{p}_1', \mathbf{p}_2')\{n_1 n_2 (1-n_1')(1-n_2') - n_1' n_2' (1-n_1)(1-n_2)\} \times \delta(\mathbf{p}_1' + \mathbf{p}_2' - \mathbf{p}_1 - \mathbf{p}_2 - \hbar\mathbf{k}) \times \delta(\varepsilon_1' + \varepsilon_2' - \varepsilon_1 - \varepsilon_2 - \hbar\omega)\, d\tau_1 d\tau_2 d\tau_1' d\tau_2'. \quad (22)$$

The delta functions in the integrand allow the satisfaction of the laws of conservation of energy and momentum.

In the integral (22), the essential values of the energy are only those in the region of diffuseness of the Fermi distribution. In this region, the expressions under the integral sign are changed strongly only by multipliers which contain $n(\epsilon)$. Furthermore, it should be noted that the angular integrals in (22) are practically unchanged in the transition from the "classical" region

$$\hbar\omega \ll \varkappa T\,.$$

to the "quantum" region

$$\hbar\omega \gg \varkappa T.$$

In view of this fact, it will be sufficient for us to calculate the integral

$$J = \iiiint \{n_1 n_2 (1-n_1')(1-n_2') - n_1' n_2' (1-n_1) \times (1-n_2)\, \delta(\varepsilon_1' + \varepsilon_2' - \varepsilon_1 - \varepsilon_2 - \hbar\omega)\, d\varepsilon_1 d\varepsilon_2 d\varepsilon_1' d\varepsilon_2',$$

taken only over the energy. Then, substituting

$$n(\varepsilon) = [e^{(\varepsilon-\mu)/\varkappa T} + 1]^{-1}$$

and introducing the notation

$$x = (\varepsilon - \mu)/\varkappa T, \quad \xi = \hbar\omega/\varkappa T,$$

we get (omitting the factor T^3)

$$J = \iiiint_{-\infty}^{+\infty} \frac{(1-e^{-\xi})\,\delta(x_1' + x_2' - x_1 - x_2 - \xi)\, dx_1 dx_2 dx_1' dx_2'}{(e^{x_1}+1)(e^{x_2}+1)(1+e^{-x_1'})(1+e^{-x_2'})}.$$

In view of the rapid convergence of the integral, the region of integration can be extended from $-\infty$ to $+\infty$.

For integration purposes, we transform to the variables x_1, x_2, y_1, y_2, where $y = x - x'$. Integration over x_1 and x_2 is elementary and gives

$$J = (1-e^{-\xi}) \times \iiiint_{-\infty}^{+\infty} \frac{\delta(y_1 + y_2 + \xi)\, dx_1\, dx_2\, dy_1\, dy_2}{(e^{x_1}+1)(e^{x_2}+1)(1+e^{-x_1+y_1})(1+e^{-x_2+y_2})}$$

$$= (1-e^{-\xi}) \iint_{-\infty}^{+\infty} \frac{y_1 y_2 \delta(y_1 + y_2 + \xi)\, dy_1\, dy_2}{(1-e^{y_1})(1-e^{y_2})}$$

$$= -(1-e^{-\xi}) \int_{-\infty}^{+\infty} \frac{y(\xi + y)\, dy}{(e^{y}-1)(e^{-y-\xi}-1)}$$

$$= \int_{-\infty}^{+\infty} y(\xi + y)\left\{\frac{1}{e^{y}-1} - \frac{1}{e^{y+\xi}-1}\right\} dx.$$

For calculation of the resulting difference of two diverging integrals, we introduce as an intermediate the finite lower limit $-\Lambda$ and write:

$$J = \int_{-\Lambda}^{+\infty} \frac{y(\xi + y)}{e^{y}-1}\, dy - \int_{-\Lambda+\xi}^{+\infty} \frac{y(y-\xi)\, dy}{e^{y}-1}$$

$$= 2\xi \int_{-\Lambda}^{\infty} \frac{y\,dy}{e^{y}-1} - \int_{-\Lambda+\xi}^{-\Lambda} \frac{y(y-\xi)\, dy}{e^{y}-1}.$$

Keeping in mind that we shall transform to the limit $\Lambda \to \infty$, we neglect e^y in the denominator of the second of the integrals. The first we rewrite in the form

$$\int_{-\Lambda}^{\infty} \frac{y\,dy}{e^y-1} = \int_0^{\infty} \frac{y\,dy}{e^y-1} + \int_{-\Lambda}^{0} \frac{y\,dy}{e^y-1}$$

$$= \frac{\pi^2}{6} + \int_{-\Lambda}^{0} \left(\frac{y}{1-e^{-y}} - y\right) dy.$$

$$= \frac{\pi^2}{6} + \int_0^{\Lambda} \frac{y\,dy}{e^y-1} + \frac{\Lambda^2}{2}.$$

Carrying out reductions and then transforming to $\Lambda \to \infty$, we finally obtain

$$J = (2\xi\pi^2/3)(1 + \xi^2/4\pi^2).$$

The desired absorption coefficient γ is proportional to J. The coefficient of proportionality between them is so determined that for $\xi \ll 1$, $\gamma = \gamma_{cl}$. We then obtain:

$$\gamma = \gamma_{cl}\,[1 + (\hbar\omega/2\pi\varkappa T)^2] \quad \text{for} \quad \hbar\omega \gtrless \varkappa T. \tag{23}$$

Considering that $\gamma_{cl} \sim T^2$, we find that in the limit of high frequencies:

$$\gamma \sim \omega^2 \quad \text{for} \quad \hbar\omega \gg \varkappa T, \tag{24}$$

i.e., the absorption coefficient remains proportional to the square of the frequency, but does not depend on the temperature. We note that the transition from the formula for "low" to the formula for "high" frequencies takes place at

$$\hbar\omega \sim 2\pi\varkappa T,$$

(and not $\hbar\omega \sim \varkappa T$).* The result of (24 refers, in particular, to the zero sound of all frequencies at the absolute zero of temperature.

3. SPIN WAVES IN A FERMI LIQUID

In addition to a consideration of zero sound in Sec. 1, which does not involve the distribution of spins, in a Fermi liquid at absolute zero, waves of other types can also be propagated. These we call spin waves.*

In this section, we denote by K the function

$$K = f(\mathbf{p}, \mathbf{p}')\,4\pi p^2\,dp\,/\,(2\pi\hbar)^3\,d\varepsilon, \tag{25}$$

in which the operator Sp is not used. In the calculation of exchange interaction between the quasi–particles, this function contains terms which are proportional to the product $\sigma\sigma'$, i.e., it has the form:[1]

$$K = {}^1/_2 F(\chi) + {}^1/_2 G(\chi)\,\sigma\sigma' \tag{26}$$

[F coincides with the function (6) used above].

In place of Eq. (11) we have now

$$(\eta - \cos\theta)\,\nu = \cos\theta\,\mathrm{Sp}_{\sigma'} \int F\nu'\,do'/4\pi. \tag{27}$$

In addition to the solutions $\nu(n)$ considered earlier, which do not depend on the spin, this equation also has a solution of the form

$$\nu = \mu(\mathbf{n})\,\sigma. \tag{28}$$

Substituting (26) and (28) in (27), completing the operation Sp and dividing both sides of the equation by σ, we get

$$(\eta - \cos\theta)\,\mu = \cos\theta \int G\mu'\,do'/16\pi. \tag{29}$$

We see that for each of the components of the vector μ, we obtain an equation which differs from (11) only by the replacement of F by $G/4$. Therefore, all the further calculations of Sec. 1 can immediately be applied to the spin waves.

In the real liquid He^3, we can determine from available experimental data on its magnetic susceptibility only the mean value of $\overline{G}$, which was pointed out previously—1.9. Inasmuch as this quantity is negative, then (in view of the results of Sec. 2) it is most probable that the propagation of spin waves in liquid He^3 is not possible. Such a conclusion, however, is in no sense categorical.

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*Considering the frequencies $\omega \gg \varkappa T/\hbar$, we at the same time assume satisfaction of the inequality

$$\hbar\omega \ll \varkappa T_0$$

(T_0 is the temperature of degeneration of the Fermi distribution). In the opposite case, particles from the "depth" of the Fermi distribution take part in the absorption and all the theory developed here would become inapplicable.

*The equation for spin waves in weakly non–ideal Fermi gas was considered by Silin.[6]

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